

Impact of predator fear on two competing prey species

DEBASIS MUKHERJEE¹

¹Vivekananda College Thakurpukur

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Abstract

Predator-prey interaction is a fundamental feature in the ecological system. The majority of studies has addressed how competition and predation affect species coexistence. Recent field studies on vertebrate has shown that fear of predators can influence the behavioural pattern of prey populations and reduce their reproduction. A natural question arises whether species coexistence is still possible or not when predator induce fear on competing species. Based on the above observation, we propose a mathematical model of two competing prey-one predator system with the cost of fear that affect reproduction rate of both the prey population. To make the model more realistic, we incorporate intraspecific competition within the predator population. Biological justification of the model is shown through positivity and boundedness of solutions. Existence and stability of different boundary equilibria are discussed. Condition for the existence of coexistence equilibrium point is derived from showing uniform persistence. Local as well as a global stability criterion is developed. Bifurcation analysis is performed by choosing the fear effect as the bifurcation parameter of the model system. The nature of the limit cycle emerging through a Hopf bifurcation is indicated. Chaotic motion is observed when one of two prey has bigger competitive capacities than the other. Numerical experiments are carried out to test the theoretical results obtained for this model.

1. Introduction

In ecological system, predation and competition are often assumed to be the important factors that affect species coexistence [7, 8, 11, 25]. It is further investigated thoroughly in [1, 12, 13]. Gurevitch et al. [6] showed that predator can promote coexistence by lowering the strength of competition. It is a common phenomenon that predator can affect prey populations by direct killing. Recent field studies show that the indirect effect of predator species on prey species has a major impact than direct killing [2-5, 14]. Thus, it is reasonable to incorporate the fear effect in the model focussed on the role of predator regarding coexistence of competing species. This type of mechanism can slow down the competition in respect of resource competition. Thus avoidance behaviour developed by fear usually stimulates coexistence provided prey partition resources, but not predators, whereas it weaken coexistence if prey partition predators but not the resources. Zanette et al. [29] carried out experiments on song sparrows and observed 40% reduction in offspring production due to fear from the predator. With this fact in mind, Wang et al. [27] first developed the predator-prey model incorporating the cost of fear into prey reproduction. They found that the cost of fear has no impact in dynamical behavior when predation follows Holling type I response function whereas it can stabilize the system by discarding periodic orbits considering Holling type II response function. Since then several studies are found in predator-prey models by introducing fear component in prey reproduction. Wang and Zou [28] investigated a predator-prey model with the cost of fear and adaptive avoidance of predators and established that both strong adaption of adult prey and the large cost of fear induces destabilizing effect while large population of predators stabilize the system. Sasmal and Takeuchi [22] discussed the dynamics of a prey-predator model incorporating two facts: fear effect and group defense. Mondal et al. [16] analyzed the predator-prey model considering both the effects of fear and additional food and showed stability of equilibrium points and Hopf bifurcation. Zhang et al. [30] investigated the influence of anti-predator behavior due to fear of predators to a Holling type II prey-predator model allowing a prey refuge

and demonstrated the global stability analysis of the equilibria of the model and showed Hopf bifurcation. Previous studies [16-19, 27, 28, 30] are mainly confined in two species that cannot properly explain the fear effect when multiple species are present. So present study attempts to investigate the predator fear which affects prey behavior when prey species are in competition. This study also address the question of species coexistence.

Takeuchi and Adachi [24] studied the following two competing prey and one predator model in Lotka-Volterra form:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 (r_1 - x_1 - \alpha x_2 - \varepsilon y), \\ \frac{dx_2}{dt} &= x_2 (r_2 - \beta x_1 - x_2 - \mu y),\end{aligned}$$

$\frac{dy}{dt} = y(-d + c\varepsilon x_1 + c\mu x_2)$. (1) Here the variables x_1 and x_2 represent the densities of prey y that of predator. r_1 and r_2 are the intrinsic growth rate of prey. α and β are parameters representing the competitive effects between two prey. ε and μ are coefficients of decrease of prey species due to predation. c is the equal conversion rate of the predator. All the parameters are assumed to be positive. In [24], the authors showed stability and Hopf bifurcation. They also pointed out that the stable equilibrium bifurcates to a periodic motion with a small amplitude when the predation rate increases and chaotic motion appears when one of two prey is superior than the other. Finally, they remarked that predator mediated coexistence is possible by the close relationship between preferences of a predator and competitive capacities of two prey. However, studies in [24] only considers the effect of direct killing on prey populations and ignore the fear effect in the model equations. In the real world, the intraspecific competition among predator exists. Taking the cost of fear on reproduction term only and intraspecific competition and unequal conversion rate of predator, system (1) becomes

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 \left(\frac{r_1}{1 + k_1 y} - x_1 - \alpha x_2 - \varepsilon y \right), \\ \frac{dx_2}{dt} &= x_2 \left(\frac{r_2}{1 + k_2 y} - \beta x_1 - x_2 - \mu y \right),\end{aligned}$$

$$\frac{dy}{dt} = y(-d + c_1 \varepsilon x_1 + c_2 \mu x_2 - h y) \quad (2)$$

where k_i , $i = 1, 2$ represents the level of fear and h denotes the intraspecific competition within the predator population. c_i , $i = 1, 2$ is the conversion efficiency of the predator. Justification for considering the fear term can be found in [27].

The rest of the paper is organized as follows. In Sec. 3, we state results on positivity and boundedness of the solutions of the system. In Sec. 4, existence and stability of different equilibrium points are discussed. Furthermore, persistence criterion is developed in the same section. Hopf bifurcation around the positive equilibrium point and the nature of the limit cycle emerging through Hopf bifurcation are derived in Sec. 5. Numerical simulations are performed in Sec. 6. A brief discussion concludes in Sec. 7.

3. Positivity and boundedness of solutions

In this section, we first state positivity and boundedness of solutions of system (2) without proof as it is obvious. These are very important so far as the validity of the model is concerned.

Lemma 1 . *All solutions $(x_1(t), x_2(t), y(t))$ of system (2) with initial values $(x_{10}, x_{20}, y_0) \in \mathbb{R}_+^3$, remains positive for all $t > 0$.*

Lemma 2. *All solutions of system (2) will lie in the region*

$$B = \{(x_1, x_2, y) \in \mathbb{R}_+^3 : 0 \leq c_1 x_1 + c_2 x_2 + y \leq \frac{M}{\lambda}\}$$

as $t \rightarrow \infty$ for all positive initial values $(x_{10}, x_{20}, y_0) \in \mathbb{R}_+^3$, where $\lambda < \min\{r_1, r_2, d\}$ and $M = r_1^2 + r_2^2$

4. Existence of equilibria and Stability analysis

Evidently, system (2) has six non-negative equilibrium points.

- The population free equilibrium point $E_0 = (0, 0, 0)$.
- The second prey and predator free equilibrium point $E_1 = (r_1, 0, 0)$.
- The first prey and predator free equilibrium point $E_2 = (0, r_2, 0)$.
- If $\alpha < \frac{r_1}{r_2} < \frac{1}{\beta}$ or $\alpha > \frac{r_1}{r_2} > \frac{1}{\beta}$ then there exists unique predator free equilibrium point $E_{12} = (x_1, x_2, 0)$ where $x_1 = \frac{r_1 - r_2 \alpha}{1 - \alpha \beta}$, $x_2 = \frac{r_2 - r_1 \beta}{1 - \alpha \beta}$.
- If $d < r_1 c_1 \varepsilon$ then there exists unique second prey free equilibrium point $E_{13} = (\hat{x}_1, 0, \hat{y})$ where $\hat{x}_1 = \frac{h \hat{y} + d}{c_1 \varepsilon}$ and \hat{y} is the positive root of the equation

$$k_1(h + c_1 \varepsilon^2)y^2 + (h + dk_1 + c_1 \varepsilon^2)y + d - r_1 c_1 \varepsilon = 0.$$

If $d < r_2 c_2 \mu$ then there exists unique first prey free equilibrium point $E_{23} = (0, \tilde{x}_2, \tilde{y})$ where $\tilde{x}_2 = \frac{h \tilde{y} + d}{c_2 \mu}$ and \tilde{y} is the positive root of the equation

$$k_2(h + c_2 \mu^2)y^2 + (h + dk_2 + c_2 \mu^2)y + d - r_2 c_2 \mu = 0.$$

E_0 is always unstable.

E_1 is stable if $r_2 < \beta r_1$ and $d > c_1 \varepsilon r_1$.

E_2 is stable if $r_1 < \alpha r_2$ and $d > c_2 \mu r_2$.

E_{12} is stable if $\alpha \beta < 1$ and $d > c_1 \varepsilon x_1 + c_2 \mu x_2$.

E_{13} is stable if $\frac{r_2}{1 + k_2 \tilde{y}} < \beta \hat{x}_1 + \mu \hat{y}$ and E_{23} is stable if $\frac{r_1}{1 + k_1 \tilde{y}} < \alpha \tilde{x}_2 + \varepsilon \tilde{y}$.

To find the existence condition of positive equilibrium point we first show uniform persistence of system (2) and then application of a result in [10] ensures the existence.

Persistence

In biological sense, persistence means the long term survival of all populations whatever may be the initial populations. Geometrically, it means the existence of a region in the phase space at a non-zero distance from the boundary in which all species enter and must lie ultimately.

Now we state a result establishing the uniform persistence of system (2).

Theorem 2. *Suppose E_{12} , E_{13} and E_{23} exist. Further suppose that $d < c_1 \varepsilon x_1 + c_2 \mu x_2$,*

$$\frac{r_2}{1 + k_2 \tilde{y}} > \beta \hat{x}_1 + \mu \hat{y} \text{ and } \frac{r_1}{1 + k_1 \tilde{y}} > \alpha \tilde{x}_2 + \varepsilon \tilde{y} \text{ then system (2) is uniformly persistent.}$$

Proof. We shall prove the theorem by using the idea of average Lyapunov function [9].

Consider the average Lyapunov function of the form $H(x) = x_1^{m_1} x_2^{m_2} y^{m_3}$, where each m_i , $i = 1, 2, 3$ is assumed positive. In the interior of \mathbb{R}_+^3 , one has

$$\begin{aligned} \frac{1}{H(x)} \frac{dH(x)}{dt} &= \psi(x) = \frac{m_1}{x_1} \frac{dx_1}{dt} + \frac{m_2}{x_2} \frac{dx_2}{dt} + \frac{m_3}{y} \frac{dy}{dt} \\ &= m_1 \left(\frac{r_1}{1 + k_1 y} - x_1 - \alpha x_2 - \varepsilon y \right) + m_2 \left(\frac{r_2}{1 + k_2 y} - \beta x_1 - x_2 - \mu y \right) + m_3 (-d + c_1 \varepsilon x_1 + c_2 \mu x_2 - hy). \end{aligned} \quad (3)$$

We have to show $\psi(x) > 0$ for all $\xi \in \mathbb{R}_+^3$, for a suitable choice of $m_1, m_2, m_3 > 0$, to prove uniform persistence of system (2). That is one has to fulfil the following conditions corresponding to the boundary equilibria $E_0, E_1, E_2, E_{12}, E_{13}, E_{23}$ only as there are no periodic orbits in the $x_1 - x_2$, $x_1 - y$ and $x_2 - y$ plane respectively.

$$E_0 : m_1 r_1 + m_2 r_2 - m_3 d > 0 \quad (4)$$

$$E_1 : m_2 (r_2 - \beta r_1) + m_3 (-d + c_1 \varepsilon r_1) > 0 \quad (5)$$

$$E_2 : m_1 (r_1 - \alpha r_2) + m_3 (-d + c_2 \mu r_2) > 0 \quad (6)$$

$$E_{12} : m_3 (c_1 \varepsilon x_1 + c_2 \mu x_2 - d) > 0 \quad (7)$$

$$E_{13} : m_2 \left(\frac{r_2}{1+k_2 y} - \beta \hat{x}_1 - \mu \hat{y} \right) > 0 \quad (8)$$

$$E_{23} : m_1 \left(\frac{r_1}{1+k_1 y} - \alpha \hat{x}_2 - \varepsilon \hat{y} \right) > 0 \quad (9)$$

Since $< c_1 \varepsilon x_1 + c_2 \mu x_2, \frac{r_2}{1+k_2 y} > \beta \hat{x}_1 + \mu \hat{y}$ and $\frac{r_1}{1+k_1 y} > \alpha \hat{x}_2 + \varepsilon \hat{y}$ positivity of (7), (8) and (9) is obvious. Again existence of E_{13} and E_{23} implies that $d < c_1 \varepsilon r_1$ and $c_2 \mu r_2$. There are two alternative conditions for existence of E_{12} e. g., (i) $\alpha < \frac{r_1}{r_2} < \frac{1}{\beta}$ or (ii) $\alpha > \frac{r_1}{r_2} > \frac{1}{\beta}$. Condition (i) implies that $r_1 - \alpha r_2 > 0$ and $r_2 - \beta r_1 > 0$. In this case, positivity of (5) and (6) are obvious and positivity of (4) will follow by the suitable choice of m_1, m_2 and m_3 . Condition (ii) implies that $r_1 - \alpha r_2 < 0$ and $r_2 - \beta r_1 < 0$. To show positivity of (4), (5) and (6), we have to choose m_3 as

$\max\left\{\frac{m_2(\beta r_1 - r_2)}{-d + c_1 \varepsilon r_1}, \frac{m_1(\alpha r_2 - r_1)}{-d + c_2 \mu r_2}\right\} < m_3 < \frac{m_1 r_1 + m_2 r_2}{d}$. So in any case, positivity of (4), (5) and (6) will follow for suitable choice of $m_i, i = 1, 2, 3$. This completes the proof.

Now system (2) ensures uniform persistence provided that the conditions of Theorem 2 are satisfied. Further, it is proved in [10], uniform persistence implies the existence of an interior equilibrium point. Hence $E^* = (x_1^*, x_2^*, y^*)$ exists; that is in effect Theorem 2 implies that E^* exists. There may exist multiple coexistence equilibrium point which are not investigated due to complexity of the system.

Theorem 3. Suppose all the conditions of Theorem 2 be satisfied. Then the interior equilibrium point E^* of system (2) is locally asymptotically stable if $4p_1 p_2 > (p_1 \alpha + p_2 \beta)^2$ where $p_1 = \frac{c_1 \varepsilon (1+k_1 y^*)^2}{r_1 k_1}$ and $p_2 = \frac{c_2 \mu (1+k_2 y^*)^2}{r_2 k_2}$.

Proof. Consider the positive definite function

$$V(t) = p_1 (x_1 - x_1^* - x_1^* \ln \frac{x_1}{x_1^*}) + p_2 (x_2 - x_2^* - x_2^* \ln \frac{x_2}{x_2^*}) + \left(y - y^* - y^* \ln \frac{y}{y^*} \right).$$

The time derivative along the solution of system (2), can be obtained as

$$\begin{aligned} \frac{dV}{dt} = & p_1 (x_1 - x_1^*) \left\{ \frac{r_1}{1+k_1 y} - x_1 - \alpha x_2 - \varepsilon y \right\} + p_2 (x_2 - x_2^*) \left(\frac{r_2}{1+k_2 y} - \beta x_1 - x_2 - \mu y \right) + \\ & (y - y^*) (-d + c_1 \varepsilon x_1 + c_2 \mu x_2 - h y). \end{aligned}$$

We expand $\frac{dV}{dt}$ about (x_1^*, x_2^*, y^*) and after some algebraic calculations get

$$\frac{dV}{dt} = -p_1 (x_1 - x_1^*)^2 - (p_1 \alpha + p_2 \beta) (x_1 - x_1^*) (x_2 - x_2^*) - p_2 (x_2 - x_2^*)^2 - h (y - y^*)^2 + \text{H. O. T}$$

where H. O. T stands for terms that are cubic or higher orders.

Now $\frac{dV}{dt} \leq 0$ if $4p_1 p_2 > (p_1 \alpha + p_2 \beta)^2$. Consequently V is a Lyapunov function and hence the theorem follows.

Remark. One can also determine the local stability of E^* by using the Routh-Hurwitz criterion.

The characteristic equation about E^* is

$$\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0 \quad (10)$$

where

$$\begin{aligned} a_1 &= x_1^* + x_2^* + hy^*, \\ a_2 &= x_1^* x_2^* (1 - \alpha\beta) + hy^* (x_1^* + x_2^*) + \left(\frac{r_1 k_1}{(1+k_1 y^*)^2} + \varepsilon \right) c_1 \varepsilon x_1^* y^* + \left(\frac{r_2 k_2}{(1+k_2 y^*)^2} + \mu \right) c_2 \mu x_2^* y^*, \\ a_3 &= x_1^* x_2^* y^* \left\{ h(1 - \alpha\beta) + (c_2 \mu - \alpha c_1 \varepsilon) \left(\frac{r_2 k_2}{(1+k_2 y^*)^2} + \mu \right) + (c_1 \varepsilon - \beta c_2 \mu) \left(\frac{r_1 k_1}{(1+k_1 y^*)^2} + \varepsilon \right) \right\}. \end{aligned}$$

Clearly $a_1 > 0$. If $a_2 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$ then E^* is locally asymptotically stable follows from Routh-Hurwitz criterion.

Theorem 4 . Suppose that $4c_1 c_2 > (c_1 \alpha + c_2 \beta)^2$ and $\det A > 0$ where A is defined in the proof. Then E^* is globally asymptotically stable.

Proof. Consider the following positive definite function about E^* .

$$V(t) = c_1(x_1 - x_1^* - x_1^* \ln \frac{x_1}{x_1^*}) + c_2(x_2 - x_2^* - x_2^* \ln \frac{x_2}{x_2^*}) + \left(y - y^* - y^* \ln \frac{y}{y^*} \right).$$

Differentiating V with respect to t along the solution of system (2), we get

$$\begin{aligned} \frac{dV}{dt} &= c_1(x_1 - x_1^*) \left\{ \frac{r_1}{1+k_1 y} - x_1 - \alpha x_2 - \varepsilon y \right\} + c_2(x_2 - x_2^*) \left(\frac{r_2}{1+k_2 y} - \beta x_1 - x_2 - \mu y \right) + \\ &\quad (y - y^*)(-d + c_1 \varepsilon x_1 + c_2 \mu x_2 - hy) \\ &= c_1(x_1 - x_1^*) \left\{ \frac{r_1 k_1 (y^* - y)}{(1+k_1 y)(1+k_1 y^*)} - (x_1 - x_1^*) - \alpha(x_2 - x_2^*) \right\} \\ &= c_1(x_1 - x_1^*) \left\{ \frac{r_1 k_1 (y^* - y)}{(1+k_1 y)(1+k_1 y^*)} - (x_1 - x_1^*) - \alpha(x_2 - x_2^*) \right\} - c_2(x_2 - x_2^*) \left\{ \frac{r_2 k_2 (y^* - y)}{(1+k_2 y)(1+k_2 y^*)} - \beta(x_1 - x_1^*) - (x_2 - x_2^*) \right\} - \\ &\quad h(y - y^*)^2 \\ &\leq -c_1(x_1 - x_1^*)^2 + (c_1 \alpha + c_2 \beta) |(x_1 - x_1^*)| |(x_2 - x_2^*)| - c_2(x_2 - x_2^*)^2 - h(y - y^*)^2 + \\ &\quad \frac{c_1 r_1 k_1}{(1+k_1 y^*)^2} |x_1 - x_1^*| |y - y^*| + \frac{c_2 r_2 k_2}{(1+k_2 y^*)^2} |x_2 - x_2^*| |y - y^*| \end{aligned}$$

Clearly \dot{V} is negative definite if the matrix A defined below is positive definite.

$$A = \begin{pmatrix} c_1 & \text{amp}; -\frac{1}{2}(c_1 \alpha + c_2 \beta) & \text{amp}; -\frac{c_1 r_1 k_1}{2(1+k_1 y^*)^2} \\ -\frac{1}{2}(c_1 \alpha + c_2 \beta) & \text{amp}; c_2 & \text{amp}; -\frac{c_2 r_2 k_2}{2(1+k_2 y^*)^2} \\ -\frac{c_1 r_1 k_1}{2(1+k_1 y^*)^2} & \text{amp}; -\frac{c_2 r_2 k_2}{2(1+k_2 y^*)^2} & \text{amp}; h \end{pmatrix}$$

Thus the condition of the theorem implies that A is positive definite and consequently V is a Lyapunov function and hence the theorem follows.

5. Hopf bifurcation and its nature

Set $f(k_2) = a_1(k_2) a_2(k_2) - a_3(k_2)$.

Theorem 4 . If there exists k_2^* such that (i) $a_2(k_2^*) > 0$, (ii) $f(k_2^*) = 0$, (iii) $f'(k_2^*) > 0$ then the positive equilibrium point E^* is unstable if $k_2 < k_2^*$ but is stable for $k_2 > k_2^*$ and a Hopf bifurcation of periodic solution occurs at $k_2 = k_2^*$.

Proof . Proceeding along the lines in [21], we note that $f(k_2)$ is monotonic increasing function in the neighbourhood of $k_2 = k_2^*$. As $a_2(k_2) > 0$, $f(k_2) < 0$ for $k_2 < k_2^*$ thus E^* becomes unstable. Again, it is obvious that, $f(k_2) > 0$ for $k_2 > k_2^*$ and hence E^* is stable. Therefore, Hopf bifurcation follows from a result in [15].

Similarly, bifurcation phenomenon can be shown by considering k_1 as a bifurcation parameter.

5.1. Stability of the limit cycle

Stability of the limit cycle can be derived by calculating the coefficient of curvature of the limit cycle [26]. Detail calculation can be found in [17]. Then the coefficient of curvature of limit cycle of system (2) is

$$\sigma_1^0 = \frac{1}{16}(\alpha - \beta).$$

Thus we observed that the coefficient of curvature $\sigma_1^0 < 0$ if $\alpha < \beta$ in that case the limit cycle of system (2) will be stable. From above analysis one can conclude that the intraspecific competition rate between the prey species plays a vital role for determining the nature of the limit cycle emerging through a Hopf bifurcation.

In the following table, we summarise the stability criteria of different equilibria of system (2).

Table 1 . Dynamics of system (2). LAS= Locally asymptotically stable, GAS= Globally asymptotically stable.

Equilibria	Stability condition	Equilibrium nature
E_0	No condition	Unstable
E_1	$\frac{r_2}{\beta} < r_1 < \frac{d}{c_1\varepsilon}$	LAS
E_2	$\frac{r_1}{\alpha} < r_2 < \frac{d}{c_2\mu}$	LAS
E_{12}	$\alpha\beta < 1, d > c_1\varepsilon x_1 + c_2\mu x_2.$	LAS
E_{13}	$\frac{r_2}{1+k_2\tilde{y}} < \beta\tilde{x}_1 + \mu\tilde{y}$	LAS
E_{23}	$\frac{r_1}{1+k_1\tilde{y}} < \alpha\tilde{x}_2 + \varepsilon\tilde{y}$	LAS
E^*	$4p_1p_2 > (p_1\alpha + p_2\beta)^2$	LAS
E^*	$4c_1c_2 > (c_1\alpha + c_2\beta)^2$ and $\det A > 0$	GAS

6. Numerical simulations

In this section, we carry out numerical simulations of our system (2). First we investigate the effect of fear on the dynamics of system (2). So it is reasonable to study the system (2) without fear effect (i.e., $k_i = 0, i = 1, 2$). We choose the other parametric value as

$$r_1 = 3.125, r_2 = 2, \alpha = 1.4, \beta = 1, \varepsilon = 1, \mu = 0.01, c_1 = 1, c_2 = 1, d = 1, h = 0.01. (4)$$

The numerical outputs are depicted in Fig. 1.

Fig. 1a shows the phase diagram of system (2) for $k_i = 0, i = 1, 2$. In absence of fear, the system (2) has a unique coexistence equilibrium point $E^* = (0.8211, 1.0539, 0.9203)$ which is unstable in nature and surrounded by a limit cycle.

Fig. 1b, $k_1 = 0.1, k_2 = 0.01$, the dynamics remains the same as in Fig. 1a and the coexistence equilibrium point $E^* = (1.0530, 0.8535, 0.7660)$. Here the value of x_2^*, y^* of E^* decreases while the value of x_1^* of E^* increases.

Fig. 1c, $k_1 = 0.1, k_2 = 0.08$, system (2) has a unique co-existence equilibrium point $E^* = (0.9973, 0.8918, 0.6794)$ which is stable in nature. In this case, the increase amount of fear on second prey stabilize the system and reduces the predator density.

Fig. 1d, $k_1 = 1, k_2 = 0.08$, system (2) has a unique coexistence equilibrium point $E^* = (0.9973, 0.8918, 0.6794)$ which is unstable due to the increase amount of fear on first prey species.

It is to be noted that the increase amount of intraspecific competition within the predator

population can induces stability of the system. Taking the value of parameter $h = 0.5$ and all other parameters are same as in Fig. 1d, we observe the stable coexistence equilibrium point $E^* =$

(1.1077, 0.8532, 0.2327)(see Fig. 1e). Thus the Fig. 1d and 1e indicates that the fear factor and intraspecific competition acts in opposite way concerning the stability of the system.

Lastly, we consider the following set of parameters $k_1 = 0.6$, $k_2 = 0.01$, $r_1 = 12$, $r_2 = 2$, $\alpha = 5$, $\beta = 1$, $\varepsilon = 1$, $\mu = 0.01$, $c_1 = 1$, $c_2 = 1$, $d = 1$, $h = 0.001$. In this case a chaotic type attractor arises enclosing the coexistence equilibrium point $E^* = (4.6856, 0.5102, 0.5397)$ (see Fig. 2).

7. Discussion

In predator-prey interaction, predation is considered to be the main force that promotes coexistence of competing species by reducing the strength of competition [6]. If the predator chooses strongest competitor species, mostly then it relieves competition pressure on other species, thereby allowing coexistence of multiple species. Recent field experiments showed that predators can induce a non-consumptive effect on their prey, for example fear [23]. Due to predation fear, prey can adopt defensive strategies that disrupt coexistence [20]. To address fears induced coexistence on competing species, we developed a mathematical model of two competing prey species and one predator where predator, not only kill both the prey but also shows non-consumptive effect upon them. Our system also includes intraspecific competition within the predator population. Takeuchi and Adachi [24] addresses an ecological system with the same type of species, but no fear effect, nor intraspecific competition within the predator populations obtaining coexistence results. The proposed model is biologically meaningful in the sense that any positive solution initiating in the positive orthant remains both non-negative and bounded.

Mathematical analysis of the model established that the system cannot collapse for any parameter value as the origin is always unstable. If the second prey has low intrinsic growth rate and the predator has a high death rate then the predator cannot prevent the first prey and tends to its carrying capacity; E_1 is an attractor whereas the opposite hold if the first prey has low intrinsic growth rate. If the intraspecific competition is stronger than the interspecific competition and the predator has a high death rate then both the prey can coexist at E_{12} while predator population goes to extinction due to large death rate. The first prey and the predator can coexist at E_{13} when the second prey has moderate intrinsic growth rate. Again the second prey and the predator can coexist at E_{23} as long as the intrinsic growth rate remains below a certain threshold value. Using invasion analysis, we derived criterion for uniform persistence of our model system that ensures the existence of positive (coexistence) equilibrium point. Local stability of the coexistence equilibrium point is possible if the ratio of intake capacity by the predator lie within an interval. The existence of Hopf bifurcation is shown by considering the level of fear as bifurcation parameter. The nature of limit cycle emerging through a Hopf bifurcation is predicted by calculating the coefficient of curvature of the limit cycle. If the intraspecific competition of the first prey is less than that of second prey then supercritical limit cycle appears. In this paper we have not considered intraspecific competitive rate h as a bifurcation parameter. But one obtains the bifurcation result for taking h as bifurcation parameter. When most of the predators are involved in intraspecific competition, stable coexistence increases (see Fig. 1e).

The novelty of our work is the inclusion of fear effect and intraspecific competition within the predator populations which are not considered in [24]. This investigation generalizes the existing knowledge of fear effect of predator on single prey species [16-18, 27, 28, 30]. As high level of fear can destroy coexistence that agrees with [20] still coexistence of predator and competing prey is possible with the increase of intraspecific competition within the predator population. Our theoretical observations will be helpful to verify some experimental data set of two competing prey and one predator system.

It may also be worthwhile to see how the other response function rather than Holling type I affects the dynamics of the system. From experimental observation, we have considered the fear effect on reproduction term of prey population still it is reasonable to see the fear effect on intraspecific, interspecific competition or death rate of prey populations.

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Figures

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Fig. 1a. Phase portrait of system (2) for with parameters values (4) and $k_i = 0$, $i = 1, 2$.

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Fig. 1b. Phase portrait of system (2) for with parameters values (4) and $k_1 = 0.1$, $k_2 = 0.01$.

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Fig.1c. Phase portrait of system (2) for with parameters values (4) and $k_1 = 0.1$, $k_2 = 0.08$.

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Fig.1d. Phase portrait of system (2) for with parameters values (4) and $k_1 = 1$, $k_2 = 0.08$.

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Fig.1e. Phase portrait of system (2) for with parameters values $k_1 = 1$, $k_2 = 0.08$, $r_1 = 3.125$, $r_2 = 2$, $\alpha = 1.4$, $\beta = 1$, $\varepsilon = 1$, $\mu = 0.01$, $c_1 = 1$, $c_2 = 1$, $d = 1$, $h = 0.5$.

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Fig. 2. Phase portrait of system (2) for with parameters values $k_1 = 0.6$, $k_2 = 0.01$, $r_1 = 12$, $r_2 = 2$, $\alpha = 5$, $\beta = 1$, $\varepsilon = 1$, $\mu = 0.01$, $c_1 = 1$, $c_2 = 1$, $d = 1$, $h = 0.001$.



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