

# Higher Dimensional Hermite-Hadamard Inequality for Semiconvex Functions of Rate $(k_1, k_2, \dots, k_n)$ on the Co-ordinates and Optimal Mass Transportation

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## Abstract

In this paper, we give a new higher dimensional Hermite-Hadamard inequality for a function  $f: \prod_{i=1}^n [a_i, b_i] \subset \mathbb{R}^n \rightarrow \mathbb{R}$  which is semiconvex of rate  $(k_1, k_2, \dots, k_n)$  on the co-ordinates. This generalizes some existing results on Hermite-Hadamard inequalities of S.S. Dragomir. In addition, we explain the Hermite-Hadamard inequality from the point of view of optimal mass transportation with cost function  $c(x, y) = f(y - x) + \sum_{i=1}^n \frac{k_i}{2} |x_i - y_i|^2$ , where  $f(\cdot): \prod_{i=1}^n [a_i, b_i] \rightarrow [0, \infty)$  is semiconvex of rate  $(k_1, k_2, \dots, k_n)$  on the co-ordinates and  $x = (x_1, x_2, \dots, x_n)$ ,  $y = (y_1, y_2, \dots, y_n) \in \prod_{i=1}^n [a_i, b_i]$ . Furthermore, by using the higher dimensional Hermite-Hadamard inequality, we compare the transport cost in different transport models on the sphere  $\mathbb{S}^2$ .

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