Lecture/Exercise 2- First algorithms

philipp.hauke[1]

[1]Affiliation not available

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Deutsch-Jozsa algorithm

The Deutsch-Jozsa algorithm is historically important as showing a quantum advantage. It is also useful as a subroutine for other quantum algorithms, e.g., Grover's search.

Problem statement

Given $f : \{0, 1\}^n \to \{0, 1\}$, with $f(x) =$ constant ($f(x) = 1\ \forall x$ or $f(x) = 0\ \forall x$) or $f(x) =$ balanced (as many x give 0 as 1). $x = 0, ...2^n - 1$ in binary language, i.e., $x = 00..0, 00..1, ..., 11..1$.

Task of quantum/classical computer: determine if f is constant or balanced.

Available additional resource: a black box (an "Oracle machine") that implements f. So do not need to worry how efficient it is to call f, just how many times we need to call it.

Deterministic classical solution

In the worst case, a deterministic classical algorithm will need ______________ evaluations of f.

Quantum algorithm

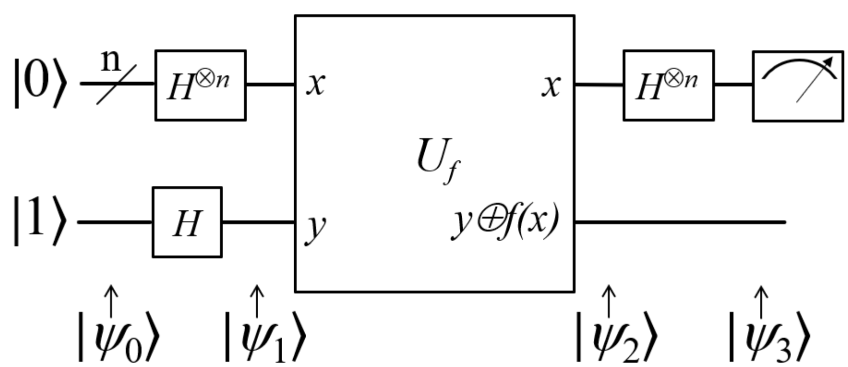

Figure 1: Circuit of the Deutsch-Jozsa algorithm.

0. Initial state: $|\psi_0\rangle =$ ______________ (first n are the register, last one is an "ancilla").

1. Hadamards: $|\psi_1\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle \otimes (|0\rangle - |1\rangle)$

2. Call oracle: $|x\rangle|y\rangle \to |x\rangle|y \oplus f(x)\rangle$ ($\oplus$ is addition modulo 2)

$|\psi_2\rangle =$ ______________________________

Rewrite, using
$|f(x)\rangle - |1 \oplus f(x)\rangle =$ ________________ for $f(x) = 0$, and

$|f(x)\rangle - |1 \oplus f(x)\rangle =$ ________________ for $f(x) = 1$:

$|\psi_2\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1}$ ________ $|x\rangle(|0\rangle - |1\rangle)$

3. Apply another Hadamard to each qubit (from now on, ignore the last qubit): $|\psi_3\rangle = \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} \left[\sum_{y=0}^{2^n-1} (-1)^{x \cdot y} |y\rangle\right] =$ $\frac{1}{2^n} \sum_{y=0}^{2^n-1} \left[\sum_{x=0}^{2^n-1} (-1)^{f(x)} (-1)^{x \cdot y}\right] |y\rangle$ where $x \cdot y = x_0 y_0 \oplus x_1 y_1 \oplus \cdots \oplus x_{n-1} y_{n-1}$ is the sum of the bitwise product.

4. Probability of measuring $|0\rangle^{\otimes n}$:
$|\langle 0|^{\otimes n} |\psi_3\rangle|^2 =$ ________________
$=$ ___ if $f(x)$ =constant
$=$ ___ if $f(x)$ =balanced

==
Which quantum features do you identify?

- ________________________________
- ________________________________
- ________________________________
- ________________________________
- ________________________________

Discussion

The Deutsch-Jozsa algorithm

- produces an answer that is always correct requiring only how many evaluations of f? __________
- yields an "oracle separation" between the classes ____________ A common technique to make proves about complexity classes without having to know how a certain element can actually be realized.
- does not separate the classes ____________ (the problem can be solved efficiently with probabilistic classical computer: to produce the correct answer with a high probability, k evaluations of the function suffice (failing with probability $\epsilon \leq 1/2^k$).