Field Theory of Quintessence: Derivation of equation of motion of such scalar Fields

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Abstract

The variant of the quintessence theory is proposed in order to get an accelerated expansion of the Friedmannian Universe in the frameworks of relativistic theory of gravitation.The substance of quintessence is built up the scalar field of dark energy. It is shown, that function V (Φ) , which factorising scalar field Lagrangian (is a scalar field) has no influence on the evolution of the Universe. Some relations, allowing to find explicit dependence Φ on time, were found, provided given function . V (Φ)

Introduction

On the edge between 20th and 21st centuries an outstanding discovery had been made: it was established that expansion of the Universe presently was going with acceleration. To explain this phenomenon, it is sufficient to assume, that in the homogeneous and isotropic Universe (Friedmann Universe) there is a matter having an unusual equation of state:

$$P_{\mathrm{Q}} = -(1-v)r_{\mathrm{Q}}$$

where is the isotropic pressure, P_{Q}, ρ_{Q} is the mass density, and the following restriction] for parameter ν is a consequence of general statements of relativistic theory of gravitation (RTG)

$$0 < v < 2/3$$

A substance having such an equation of state has been called the “quintessence”. It is supposed, that the quintessence is a real scalar field. No wonder, that modeling of the perfect fluid with negative preassure is provided with a rather strange scalar field Lagrangian.
Let us designate the scalar field of quintessence as $\Phi(x^{\mathrm{a}}\ )$and postulate, that its Lagrangian density is given as follows

$$L = -[?] - g \ \ \mathrm{V}(\Phi)\ (I^2)^q \qquad \ldots\ldots\ldots\ldots \ (1)$$

where g = det $g_{\mu\nu}$, $g_{\mu\nu}$ is a metric tensor of the effective Riemannian space , q is a number,
V(Φ)>0 , is some function of the field Φ, and

$$\mathrm{I} = g^{\alpha\beta}\ [?]_\alpha\ \Phi\ [?]_\beta \Phi = g^{\alpha\beta}\ [?]_\alpha \Phi [?]_\beta \Phi$$

where ,[?] is a covariant derivative with respect to metric $g^{\alpha\beta}$

The Lagrangian densities of the form

$$L = -[?] - gV(F)F(I)$$

where F(I) is an arbitrary function of I , have been considered earlier
Our choice follows from intention to get equation of state in the frameworks of the field theory of quintessence, which in their turn can explain the accelerated expansion of the Friedmannian Universe in Relativistic Gravitation theory.

The Equation Of motion for the field $F(x^a)$

According to the variational principle, the equation of motion for the field Φ which has the Lagrangian) can be obtained from the Euler-Lagrange formula

$$[?]L/[?]F - [?]_\mu\ (\ [?]L/[?](_\mu \Phi)\ \ =0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .(3)$$

After substitution of Eq. (1) into Eq. (3) we obtain,

$$(4q-1) dlnV(F)/d(F) I^2 - 8q(2q-1) g^{\alpha\mu} [?]_\alpha\ \Phi\ (\ G^\lambda{}_{\mu\tau}\ \ g^{\tau\beta}\ [?]\ _\lambda [?]_\beta\ \Phi\ -^{\lambda\beta} [?]_\beta \Phi\ [?]_\lambda [?]_\mu \Phi) - 4q(g^{\mu\tau} G^\alpha{}_{\mu\tau} [?]_\alpha \Phi - g^{\lambda\mu} [?]_\lambda [?]_\mu \Phi)$$
$$I = 0$$

Here we have

$$G^\alpha{}_{\mu\tau} = 1/2\ g^{\alpha\beta}([?]_\mu\ \ g_{\beta\tau} + [?]_\tau\ g_{\beta\mu}\ \text{-}[?]_\beta\ g_{\mu\tau})$$

In terms of covariant derivatives [?]α , Eq. (4) has the following form

$$(4q-1)\frac{d\ln V(\Phi)}{d\Phi}I^2+8q(2q-1)g^{\alpha\mu}g^{\lambda\beta}\nabla_\alpha\Phi\nabla_\beta\Phi\nabla_\lambda\nabla_\mu\Phi+$$

$$+4qg^{\lambda\mu}\nabla_\lambda\nabla_\mu\Phi I=0.$$