

Problemas sobre columnas

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The formula for the critical load of a column was derived in 1757 by Leonhard Euler, the great Swiss mathematician. Euler's analysis was based on the differential equation of the elastic curve:

$$\frac{d^2v}{dx^2} + \frac{p}{EI}v = 0$$

Find the solution to this equation and apply the following conditions to obtain the values for the constants of integration:

$$v|_{x=0} = 0$$

$$v|_{x=L} = 0$$

$$v|_{x=L} = 0$$

Finally explain how to get the following result :

$$P = n^2 \frac{\pi^2 EI}{L^2}$$

1° The first step is to derive

$$v = C_1 \sin |x| + C_2 |x|$$

$$v' = \frac{dv}{dx} = C_1 |\cos x| + C_2 \sin |x|$$

$$v'' = \frac{d^2v}{dx^2} = C_1 |^2 \sin |x| - C_2 |^2 \cos |x|$$

The second step is to factor

$$-C_1 |^2 \sin |x| - C_2 |^2 \cos |x|$$

$$\left(\frac{P}{EI}\right) (C_1 \sin |x| + C_2 \cos |x| = 0)$$

$$C_1 |^2 \sin |x| - C_2 |^2 \cos |x| + C_1 \left(\frac{P}{EI}\right) \sin |x| + C_2 \left(\frac{P}{EI}\right) \cos |x| = 0$$

$$C_1 \sin |x| \left(\frac{P}{EI} - x^2\right) + C_2 \cos |x| \left(\frac{P}{EI} - x^2\right) = 0$$

$$\frac{P}{EI} |2| = \sqrt{\frac{P}{EI}}$$

$$v = C_1 \sin \sqrt{\frac{P}{EI}} x + C_2 \cos \sqrt{\frac{P}{EI}} x$$

$$v = 0 |x| = 0$$

$$v = 0 |x| x = L$$

$$C_1 \sin \sqrt{\frac{P}{EI}} (0) + C_2 \cos \sqrt{\frac{P}{EI}} (0) = 0$$

For V=0 X=L

$$v(x = L) = C_1 \sin \sqrt{\frac{P}{EI}} L = 0 \quad v(x = L) = C_1 \sin \sqrt{\frac{P}{EI}} L = 0$$

$$\sin \left(\sqrt{\frac{P}{EI}} L \right) = 0 \quad \sqrt{\frac{P}{EI}} L = n \pi$$

it will clear P

$$\frac{P}{EI} L^2 = n^2 \pi^2 = P = n^2 \frac{\pi^2 EI}{L^2}$$

To calculate the critical P

$$n = 1 \Rightarrow P = \frac{\pi^2 EI}{L^2}$$

Conclusion:

En este presente documento se llevo a cabo una serie de operaciones, una de ellas fue derivar, otra factorizar y por ultimo despejar **p**.