Singapore Physics Olympiad Training (SPOT) - Waves and Optics

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Syllabus

http://ipho.org/PDF/2015-12-06%20%20Syllabus%20of%20IPhO.pdf

Plan to briefly discuss portions in bold

2.4.2 Waves

Propagation of harmonic waves: **phase as a linear function of space and time**; wave length, wave vector, **phase and group velocities**; exponential decay for waves propagating in dissipative media; transverse and longitudinal waves; the classical Doppler effect. Waves in inhomogeneous media: Fermat's principle, Snell's law. Sound waves: speed as a function of pressure (Young's or bulk modulus) and density, **Mach cone**. Energy carried by waves: proportionality to the square of the amplitude, continuity of the energy flux.

2.4.3 Interference and diffraction

Superposition of waves: coherence, **beats**, standing waves, **Huygens' principle**, interference due to thin films (conditions for intensity minima and maxima only). Diffraction from one and two slits, diffraction grating, **Bragg reflection**.

2.4.4 Interaction of electromagnetic waves with matter

Dependence of electric permittivity on frequency (qualitatively); refractive index; dispersion and dissipation of electromagnetic waves in transparent and opaque materials. Linear polarisation; **Brewster angle**; polarisers; Malus' law.

2.4.5 Geometrical optics and photometry

Approximation of geometrical optics: rays and optical images; a partial shadow and full shadow. **Thin lens approximation**; construction of images created by ideal thin lenses; thin lens equation . Luminous flux and its continuity; illuminance; luminous intensity.

2.4.6 Optical devices

Telescopes and microscopes: magnification and resolving power; diffraction grating and its resolving power; **interferometers**.

Also refer to notes by Jaan Kalda

https://www.ioc.ee/~kalda/ipho/waveopt.pdf

Problems

From the II Estonian-Finnish Olympiad in Physics (2004)

Problem 4

4. Transparent film (6 pts)

A thick glass plate is coated by a thin transparent film. The transmission spectrum of the system is depicted in graph (light falls normal to the plate). The refractive index of the film $n \approx 1.3$. What is the thickness of the film d?

Figure 1: http://www.ioc.ee/~kalda/ipho/E_S2.pdf

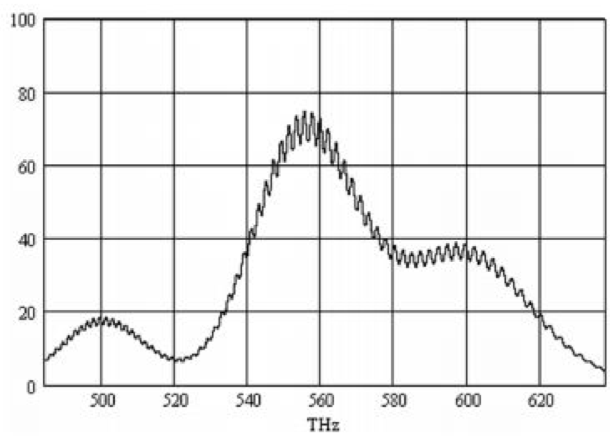


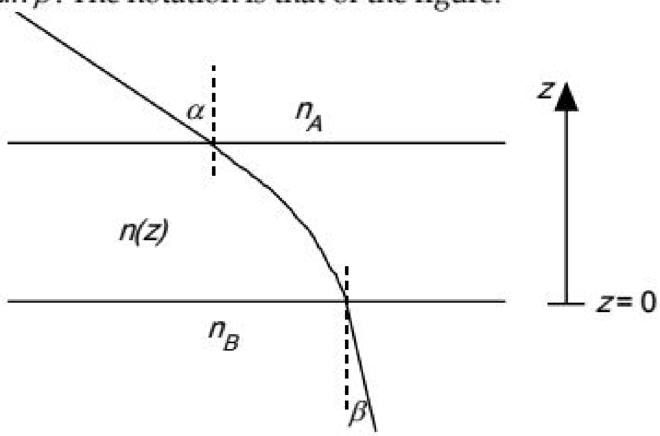
Figure 2: http://www.ioc.ee/~kalda/ipho/E_S2.pdf

From the XV International Physics Olympiad (Sigtuna, Sweden 1984)

Theoretical Problem 1

Problem 1

a) Consider a plane-parallel transparent plate, where the refractive index, n, varies with distance, z, from the lower surface (see figure). Show that $n_A \sin\alpha = n_B \sin\beta$. The notation is that of the figure.



b) Assume that you are standing in a large flat desert. At some distance you see what appears to be a water surface. When you approach the "water" is seems to move away such that the distance to the "water" is always constant. Explain the phenomenon.

c) Compute the temperature of the air close to the ground in b) assuming that your eyes are located 1.60 m above the ground and that the distance to the "water" is 250 m. The refractive index of the air at 15 ˚C and at normal air pressure (101.3 kPa) is 1.000276. The temperature of the air more than 1 m above the ground is assumed to be constant and equal to 30 ˚C. The atmospheric pressure is assumed to be normal. The refractive index, n, is such that $n - 1$ is proportional to the density of the air. Discuss the accuracy of your result.

Figure 3: http://ipho.org/problems-and-solutions/1984/15th_IPhO_1984(pdf).pdf

From the XXI International Physics Olympiad (Groningen, Netherlands 1990)

Theoretical Problem 1

Question 1. X-ray Diffraction from a crystal.

We wish to study X-ray diffraction by a cubic crystal lattice. To do this we start with the diffraction of a plane, monochromatic wave that falls perpendicularly on a 2-dimensional grid that consists of N_1 x N_2 slits with separations d_1 and d_2. The diffraction pattern is observed on a screen at a distance L from the grid. The screen is parallel to the grid and L is much larger than d_1 and d_2.

a - Determine the positions and widths of the principal maximum on the screen. The width is defined as the distance between the minima on either side of the maxima.

We consider now a cubic crystal, with lattice spacing a and size $N_0.a$ x $N_0.a$ x $N_1.a$. N_1 is much smaller than N_0. The crystal is placed in a parallel X-ray beam along the z-axis at an angle Θ (see Fig. 1). The diffraction pattern is again observed on a screen at a great distance from the crystal.

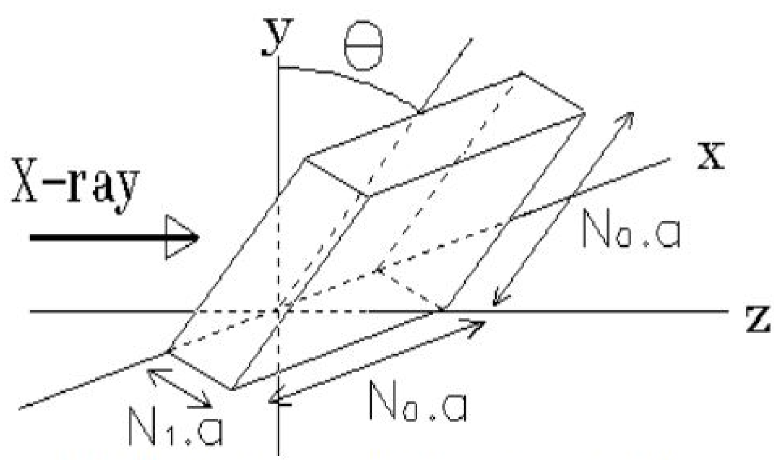


Figure 1 Diffraction of a parallel X-ray beam along the z-axis. The angle between the crystal and the y-axis is Θ.

Figure 4: http://ipho.org/problems-and-solutions/1990/21st_IPhO_1990%20Problems%20and%20Solutions.pdf

b - Calculate the position and width of the maxima as a function of the angle Θ (for small Θ).

- What in particular are the consequences of the fact that $N_1 << N_0$.

The diffraction pattern can also be derived by means of Bragg's theory, in which it is assumed that the X-rays are reflected from atomic planes in the lattice. The diffraction pattern then arises from interference of these reflected rays with each other.

c - Show that this so-called Bragg reflection yields the same conditions for the maxima as those that you found in b.

Figure 5: http://ipho.org/problems-and-solutions/1990/21st_IPhO_1990%20Problems%20and%20Solutions.pdf

In some measurements the so-called powder method is employed. A beam of X-rays is scattered by a powder of very many, small crystals. (Of course the sizes of the crystals are much larger than the lattice spacing, a).
Scattering of X-rays of wavelength 0.15 nm by Potassium Chloride [KCl] (which has a cubic lattice, see Fig.2) results in the production of concentric dark circles on a photographic plate. The distance between the crystals and the plate is 0.10 m, and the radius of the smallest circle is 0.053 m (see Fig. 3). K^+ and Cl^- ions have almost the same size, and they may be treated as identical scattering centres.

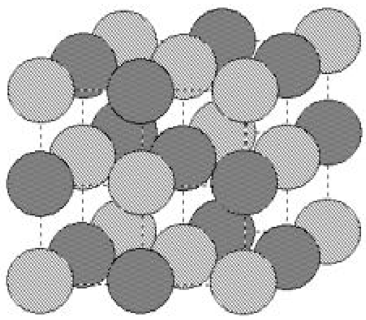


Figure 2. The cubic latice of Potassium Chloride in which the K^+ and Cl^-wons have almost the same size.

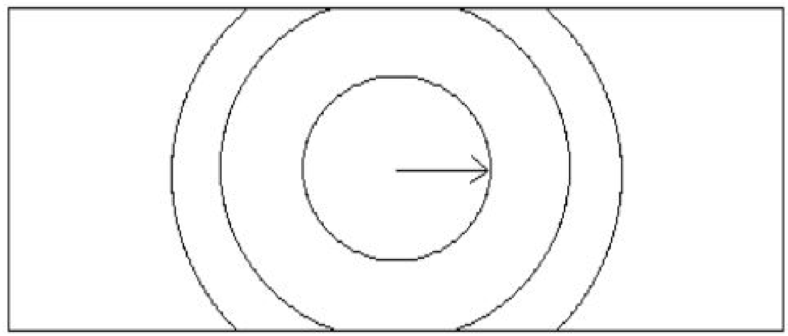


Figure 3. Scattering of X-rays by a powder of KCl crystals results in the production of concentric dark circles on a photographic plate.

d - Calculate the distance between two neighbouring K ions in the crystal.

Figure 6: http://ipho.org/problems-and-solutions/1990/21st_IPhO_1990%20Problems%20and%20Solutions.pdf

From the XXXIV International Physics Olympiad (Taipei, Taiwan 2003)

Theoretical Problem 3 (Part B)

Part B

Light Levitation

A transparent glass hemisphere with radius R and mass m has an index of refraction n. In the medium outside the hemisphere, the index of refraction is equal to one. A parallel beam of monochromatic laser light is incident uniformly and normally onto the central portion of its planar surface, as shown in Figure 3. The acceleration of gravity $\vec{g}$ is vertically downwards. The radius δ of the circular cross-section of the laser beam is much smaller than R. Both the glass hemisphere and the laser beam are axially symmetric with respect to the z-axis.

The glass hemisphere does not absorb any laser light. Its surface has been coated with a thin layer of transparent material so that reflections are negligible when light enters and leaves the glass hemisphere. The optical path traversed by laser light passing through the non-reflecting surface layer is also negligible.

(b) Neglecting terms of the order $(\delta/R)^3$ or higher, find the laser power P needed to balance the weight of the glass hemisphere. (4.0 points)

Hint: $\cos\theta \approx 1-\theta^2/2$ when θ is much smaller than one.

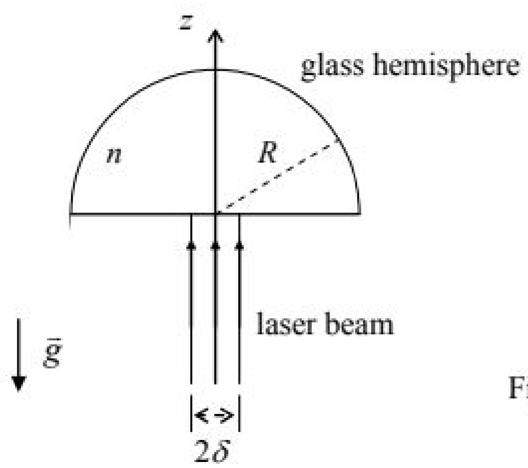


Figure 3

Figure 7: http://ipho.org/problems-and-solutions/2003/IPhO_2003_Theoretical%20Question%203.pdf

From the XXIV International Physics Olympiad (Williamsburg, Virginia, USA 1993)

Theoretical Problem 2

LASER FORCES ON A TRANSPARENT PRISM

By means of refraction a strong laser beam can exert appreciable forces on small transparent objects. To see that this is so, consider a small glass triangular prism with an apex angle $A = \pi - 2\alpha$, a base of length $2h$ and a width w. The prism has an index of refraction n and a mass density ρ.

Suppose that this prism is placed in a laser beam travelling horizontally in the x direction. (Throughout this problem assume that the prism does not rotate, i.e., its apex always points opposite to the direction of the laser beam, its triangular faces are parallel to the xy plane, and its base is parallel to the yz plane, as shown in Fig. 1.) Take the index of refraction of the surrounding air to be $n_{air} = 1$. Assume that the faces of the prism are coated with an anti-reflection coating so that no reflection occurs.

The laser beam has an intensity that is uniform across its width in the z direction but falls off linearly with distance y from the x axis such that it has a maximum value of I_0 at $y = 0$ and falls to zero at $y = \pm 4h$ (Fig. 2). [Intensity is power per unit area, e.g. expressed in W m^{-2}.]

Fig. 1.

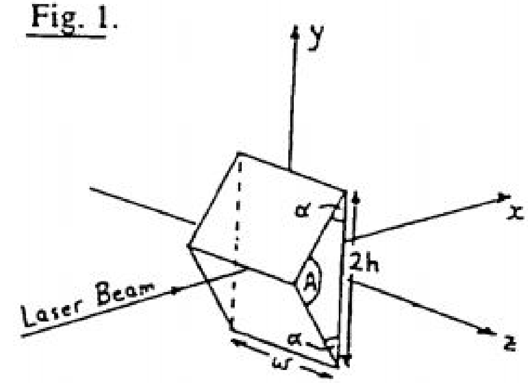


Fig. 2.

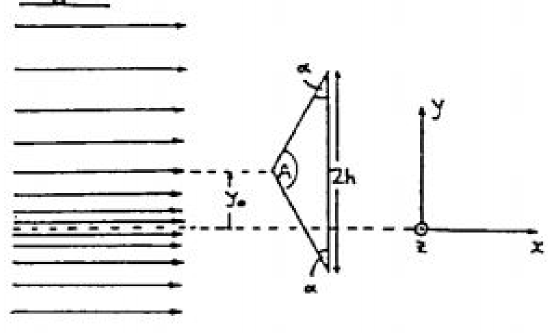


Figure 8: http://ipho.org/problems-and-solutions/1993/24th_IPhO_1993.pdf

1) Write equations from which the angle θ (see Fig. 3) may be determined (in terms of α and n) in the case when laser light strikes the upper face of the prism.

Fig. 3.

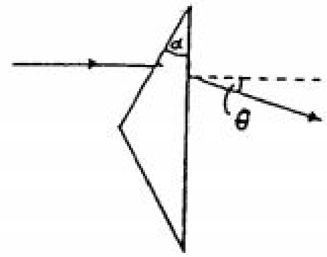


Figure 9: http://ipho.org/problems-and-solutions/1993/24th_IPhO_1993.pdf

2) Express, in terms of I_0, θ, h, w and y_0, the x and y components of the net force exerted on the prism by the laser light when the apex of the prism is displaced a distance y_0 from the x axis where $|y_0| \leq 3h$.
Plot graphs of the values of the horizontal and vertical components of force as functions of vertical displacement y_0.

3) Suppose that the laser beam is 1 mm wide in the z direction and 80 µm thick (in the y direction). The prism has $\alpha = 30^o$, $h = 10$ µm, $n = 1.5$, $w = 1$ mm and $\rho = 2.5$ g cm^{-3}. How many watts of laser power would be required to balance this prism against the pull of gravity (in the $-y$ direction) when the apex of the prism is at a distance $y_0 = -h/2$ ($= -5$ µm) below the axis of the laser beam?

4) Suppose that this experiment is done in the absence of gravity with the same prism and a laser beam with the same dimensions as in (3), but with $I_0 = 10^8$ W m^{-2}.
What would be the period of oscillations that occur when the prism is displaced and released a distance $y = h/20$ from the center line of the laser beam?

Figure 10: http://ipho.org/problems-and-solutions/1993/24th_IPhO_1993.pdf

From the III Estonian-Finnish Olympiad in Physics (2005)

Problem 5

5. Anemometer (6 points)

Anemometer is a device measuring flow rate of a gas or a fluid. Let us look the construction of a simple laser-anemometer. In a rectangular pipe with thin glass walls flows a fluid (refractive index $n = 1{,}3$), which contains light dissipating particles. Two coherent plane waves with wavelength $\lambda = 515\,\mathrm{nm}$ and angle $\alpha = 4°$ between their wave vectors, are incident on a plate so that *(a)* angle bisector of the angle between wave vectors is normal to one wall of the pipe and *(b)* pipe is parallel to the plane defined by wave vectors. Behind the pipe is a photodetector, that measures the frequency of changes in dissipated light intensity.

Figure 11: http://www.ioc.ee/~kalda/ipho/es/E_S_2005_e.pdf

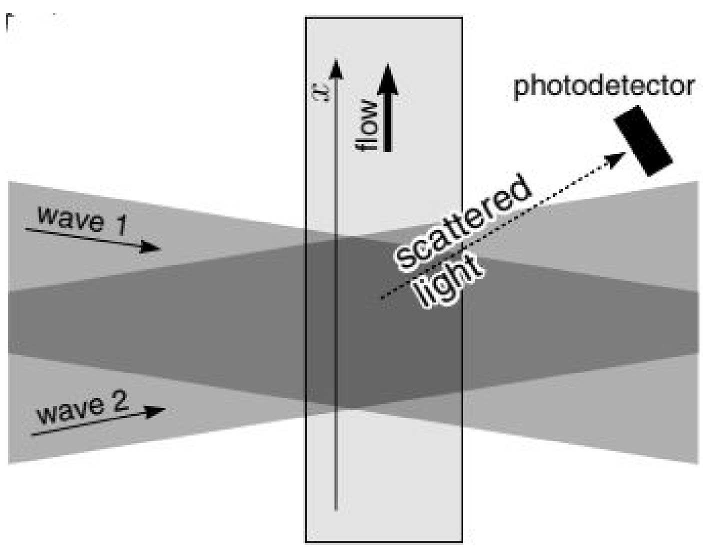


Figure 12: http://www.ioc.ee/~kalda/ipho/es/E_S_2005_e.pdf

1) How long is the (spatial) period Δ of the interference pattern created along x-axis (see Figure; 2 pts)?

2) Let the oscillation frequency of the photometer signal be $\nu = 50\,\text{kHz}$. How large is the fluid's speed v? What can be said about the direction of the fluid flow (2 pts)?

3) Let us consider a situation, when the wavelengths of the plane waves differ by $\delta\lambda = 4{,}4\,\text{fm}$ ($1\,\text{fm} = 10^{-15}\,\text{m}$). What is the frequency of signal oscillations now (fluid's speed is the same as in previous section)? Is it possible to determine the flow direction with such a device (2 pts)?

Figure 13: http://www.ioc.ee/~kalda/ipho/es/E_S_2005_e.pdf

From the XI Estonian-Finnish Olympiad in Physics (2013)

Problem 2

2. CELLPHONE CAMERA (6 points) A photographer focussed his camera to distance L and took a photo. On the photo, all farther objects (up to infinity) turned also out to be sharp. Additionally, all closer objects down to distance s were sharp.

i) *(4 points)* What is the minimum possible L?

ii) *(2 points)* Find the corresponding s.

Background. We consider the image of a pointlike object to be sharp if its image is smaller than one pixel on the sensor. Otherwise the image is blurry. The lens of the camera may be viewed as a convex lens. The camera is focussed by changing the distance between the sensor and the lens.

Parameters. Calculate the answer for a cellphone made by a well-known company.

Figure 14: http://www.ioc.ee/~kalda/ipho/es/es2013_eng.pdf

The focal length of its camera $f = 4.3\,\text{mm}$ and the diameter of the lens $D = 1.8\,\text{mm}$. The sensor is $w = 4.6\,\text{mm}$ wide corresponding to $N = 3264$ pixels.

Figure 15: http://www.ioc.ee/~kalda/ipho/es/es2013_eng.pdf

From the XIII Open Estonian-Finnish Olympiad in Physics (2015)

Problem 2

2. HOLOGRAPHIC LENS (7 points) *(by J. Kalda)* Monochromatic light can be focused into a point using an holographic lens (better known as a *Fresnel zone plate*). This is a thin film which has a system of opaque and transparent concentric rings: the opaque rings obstruct those light waves which would arrive at the focus in the opposite phase (as compared with the light from the transparent rings). In what follows, let us assume that the diameter and focal length of such a lens $d = f = 10\,\text{cm}$; the wavelength of the monochromatic incident light radiation is

Figure 16: http://www.ioc.ee/~kalda/ipho/es/e-s-2015-eng.pdf

$\lambda = 5 \times 10^{-7}\,\text{m}$.

i) *(1.5 points)* Starting enumeration from the (opaque) centre of the lens, what is the radius of the m-th transparent ring (more precisely, the circle at the middle of the ring)?

ii) *(1.5 points)* Let us consider also a glass lens of the same focal length f and diameter d, such that a parallel incident beam is collected perfectly into the focus (has aspheric surfaces). What is the minimal possible thickness of such a lens if the coefficient of refraction of the glass is $n = 1.5$?

iii) *(2 points)* If a short light pulse falls onto a lens, the behaviour of the holographic lens is very different from that of the glass lens. Indeed, if we ignore the dispersion of the glass (the dependence of n on the wavelength), the pulse duration at the focus of the glass lens is the same as that of

Figure 17: http://www.ioc.ee/~kalda/ipho/es/e-s-2015-eng.pdf

the incident beam. Meanwhile, in the case of a short pulse ($\tau = 3 \times 10^{-14}$ s), the pulse duration at the focus of the holographic lens becomes significantly dilated. Sketch qualitatively the light intensity I at the focus of the holographic lens as a function of time. A scale is required for time t, but not for I. The speed of light $c = 3 \times 10^8$ m/s.

iv) *(1 point)* Even if the light source is an ideal laser providing a perfectly monochromatic radiation, the short pulse is no longer strictly monochromatic. Estimate the width of the range of wavelengths in the pulse of duration τ.

v) *(1 point)* Estimate the duration of this light pulse in the focus of the glass lens. Assume that in the glass, the light wave group velocity v_g depends on the wavelength

Figure 18: http://www.ioc.ee/~kalda/ipho/es/e-s-2015-eng.pdf

at the rate $\frac{\mathrm{d}v_g}{\mathrm{d}\lambda} = 0.02 \times \frac{v_g}{\lambda}$ and that for the wavelength λ, group speed is equal to the phase speed.

Figure 19: http://www.ioc.ee/~kalda/ipho/es/e-s-2015-eng.pdf

From the XXVI International Physics Olympiad (Canberra, Australia 1995)

Theoretical Problem 2

Theoretical Question 2

Sound Propagation

Introduction

The speed of propagation of sound in the ocean varies with depth, temperature and salinity. Figure 1(a) below shows the variation of sound speed c with depth z for a case where a minimum speed value c_0 occurs midway betweeen the ocean surface and the sea bed. Note that for convenience $z = 0$ at the depth of this sound speed minimum, $z = z_S$ at the surface and $z = -z_b$ at the sea bed. Above $z = 0$, c is given by

$$c = c_0 + bz \quad .$$

Below $z = 0$, c is given by

$$c = c_0 - bz \quad .$$

In each case $b = \left|\dfrac{dc}{dz}\right|$, that is, b is the magnitude of the sound speed gradient with depth; b is assumed constant.

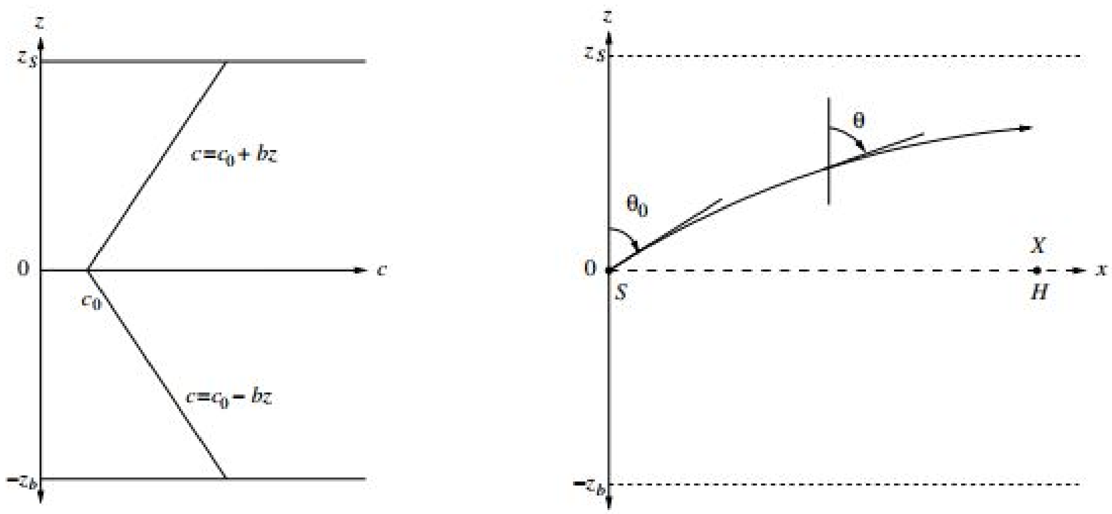


Figure 1 (a)

Figure 1 (b)

Figure 20: http://ipho.org/problems-and-solutions/1995/IPhO_1995_Theoretical%20Questions.pdf

Figure 1(b) shows a section of the z–x plane through the ocean, where x is a horizontal direction. The variation of c with respect to z is shown in figure 1(a). At the position $z = 0$, $x = 0$, a sound source S is located. A 'sound ray' is emitted from S at an angle θ_0 as shown. Because of the variation of c with z, the ray will be refracted.

(a) (6 marks)
Show that the trajectory of the ray, leaving the source S and constrained to the z–x plane forms an arc of a circle with radius R where

$$R = \frac{c_0}{b \sin\theta_0} \quad \text{for} \quad 0 \leq \theta_0 < \frac{\pi}{2} \ .$$

(b) (3 marks)
Derive an expression involving z_S, c_0 and b to give the smallest value of the angle θ_0 for upwardly directed rays which can be transmitted without the sound wave reflecting from the sea surface.

(c) (4 marks)
Figure 1(b) shows the position of a sound receiver H which is located at the position $z = 0$, $x = X$. Derive an expression involving b, X and c_0 to give the series of angles θ_0 required for the sound ray emerging from S to reach the receiver H. Assume that z_S and z_b are sufficiently large to remove the possibility of reflection from sea surface or sea bed.

Figure 21: http://ipho.org/problems-and-solutions/1995/IPhO_1995_Theoretical%20Questions.pdf

(d) (2 marks)
Calculate the smallest four values of θ_0 for refracted rays from S to reach H when

- $X = 10000$ m
- $c_0 = 1500$ ms^{-1}
- $b = 0.02000$ s^{-1}

(e) (5 marks)
Derive an expression to give the time taken for sound to travel from S to H following the ray path associated with the **smallest** value of angle θ_0, as determined in part (c). Calculate the value of this transit time for the conditions given in part (d). The following result may be of assistance:

$$\int \frac{dx}{\sin x} = \ln \tan\left(\frac{x}{2}\right)$$

Calculate the time taken for the direct ray to travel from S to H along $z = 0$. Which of the two rys will arrive first, the ray for which $\theta_0 = \pi/2$, or the ray with the smallest value of θ_0 as calculated for part (d)?

Figure 22: http://ipho.org/problems-and-solutions/1995/IPhO_1995_Theoretical%20Questions.pdf