Theoretical Determination of Partition of Suspended to Total Sediment Discharge

Chun-Yao Yang

Pierre Y. Julien

The analytical solution of Einstein’s suspended load function provides us insight to the partition of suspended and total sediment discharge as well as the partition of measured and total sediment discharge. Both of the ratios can be reduced to a function of the ratio of shear velocity to the fall velocity of suspended material  $u\_{\*}/ω$, flow depth $h$, the median grain size of bed material $d\_{50}$. When the suspended material is fine (such as silt or clay), the ratio of suspended to total load is further reduced to a function of the ratio of flow depth to bed material size, and the ratio of measured to total load is reduced to a function of flow depth. Under this condition, at least 80% of sediment load is in suspended when  $h/d\_{s}>10$, and 90% of sediment load can be measured when $h>1$ m. The measurements of 35 watersheds in South Korea are used to demonstrate the proposed analysis. Furthermore, we found that for the sand bed rivers in South Korea, sediment is predominantly transported in suspension. The ratio of bedload increases when the grain size increases. When the discharge is larger than mean discharge, at least 90% of sediment is transported in suspension for gravel and sand bed rivers.

# 1. Introduction

The quantification of total sediment transport rate from the combination of suspended and bed loads is still one of the difficult topics in river engineering. Bedload is naturally difficult to measure and the total sediment load is often not available (Turowski, Rickenmann, and Dadson 2010; Wohl, Lane, and Wilcox 2015). (Einstein 1950) proposed a bedload function to calculate bed material load in sand bed rivers. The method was a breakthrough in the study of sediment transport. Several methods to calculate the total sediment load were developed from it, e.g. (Colby and Hembree 1955) and (Toffaleti 1977). In this study we will focus on (Colby and Hembree 1955) and its derivatives. (Colby and Hembree 1955) compared the total sediment load computed by Einstein Procedure to the sediment measurement in Niobraba river and found that the computed size distributions of sediment discharge compared poorly to the measurement. The Einstein Procedure is not designed to apply to single cross-sections either. They modified the Einstein Procedure to compute the total sediment load based on the depth-integrated samples. The method is known as Modified Einstein Procedure (MEP). The MEP is generally applicable to different streams because depth integrating samplers measure much of the sediment discharge. However, the MEP has been subjected to several empirical adjustments over time. (Colby and Hubbell 1961) introduced four nomographs to simplify the computation of MEP. (Lara 1966) pointed out that the following assumption by Colby and Hembree was not valid: the theoretical exponent for vertical distribution of sediment (“Z” value) is not always 0.7. Lara suggested Z values determined based on the power relationship between Z and fall velocity $Z=αω^{β}$ for the size frictions having significant percentage quantities in both suspended and bed sediment materials. Two major revisions of MEP were proposed by (Burkham and Dawdy 1980): 1) they replaced the effective roughness $k\_{s}$ from $d\_{65}$ to $5.5d\_{65}$; 2) they developed a procedure to compute the bedload transport intensity $ϕ\_{\*}$ from shear intensity $ψ\_{\*}$ instead of arbitrarily dividing $ϕ\_{\*}$ by 2. The method by Burkham and Dawdy is known as the Revised Modified Einstein procedure. (Shen and Hung 1983) reaffirmed the use of least squares fit of the power relationship between $Z$ and $ω$. In addition, they proposed an optimization procedure for the calculation of suspended sediment discharge.

Computer programs have been developed to ease the MEP calculations (Burkham and Dawdy 1980; Stevens 1985; Holmquist-Johnson and Raff 2006). The latest development is the Bureau of Reclamation Automated Modified Einstein Procedure (BORAMEP)(Holmquist-Johnson and Raff 2006). (Shah-Fairbank 2006) did a thorough testing on BORAMEP. In 2009, she proposed the Series Expansion of the Modified Einstein Procedure (SEMEP) to obtain the total sediment transport rate for depth-integrating and point samplers (Shah-Fairbank 2009; Shah-Fairbank, Julien, and Baird 2011; Shah-Fairbank and Julien 2015). The SEMEP assumes a Z value evaluated by the Rouse’s equation (Rouse 1937), so there is no need for a minimum of two overlapping size fractions for both the bed material and suspended material. In addition, the bedload is calculated directly from measured load, so there is no need to arbitrarily divide the bedload transport intensity by two. Lastly, unlike MEP, the calculated total sediment discharge is always larger than the suspended sediment discharge (Julien 2010).

The objective of this study is to derive the ratio of measured to the total sediment discharge as well as the ratio of suspended to the total sediment load as functions of water depth and grain size. A case study of South Korea were be present and compared with the BORAMEP.

# 2. Method

## 2.1. Ratio of suspended to total sediment discharge

The unit suspended load $q\_{s}$ is given by (Einstein 1950):

$$q\_{s}=0.216q\_{b}\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}+J\_{2}\right\}$$

$$J\_{1}=\int\_{E}^{1}\left(\frac{1−z}{z}\right)^{Z}dz$$

$$J\_{2}=\int\_{E}^{1}lnz\left(\frac{1−z}{z}\right)^{Z}dz$$

where $Z=ω/β\_{s}κu\_{\*}$; $E=a/h$ is the ratio of the thickness of the bed layer to flow depth, and $a=2d\_{s}$ is commonly used; $d\_{s}$ is the median grain size of bed; $h$ is flow depth; vertical elevation above the channel bed; $q\_{b}$ is unit bedload.

Since the total sediment discharge $q\_{t}=q\_{b}+q\_{s}$, the ratio of suspended to total sediment discharge can be derived as follow:

$$\frac{q\_{s}}{q\_{t}}=\frac{0.216\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}+J\_{2}\right\}}{1+0.216\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}+J\_{2}\right\}}$$

As above equation shows, $Q\_{s}/Q\_{t}$, or $q\_{s}/q\_{t}$, only changes with $h/d\_{s}$ and $Z$ two variables ($E=2d\_{s}/h$):

$$q\_{s}/q\_{t}=f(h/d\_{s},Z)$$

## 2.2. Ratio of measured to total sediment discharge

The measured unit sediment discharge can be evaluated by integrating the product of flow velocity and volumetric sediment concentration from the nozzle height $d\_{n}$ to the free surface at $z=h$:

$$q\_{m}=\int\_{d\_{n}}^{h}Cvdz$$

where $q\_{m}=$ measured unit sediment discharge, and $C$ and $v$ are the sediment concentration by volume and velocity at an elevation $z$ above the bed.

The turbulent velocity profile can be expressed with the logarithmic velocity profile (Keulegan 1938):

$$v=\frac{u\_{\*}}{κ}ln\left(\frac{30z}{k\_{s}}\right)$$

The concentration profile is as defined by (Rouse 1937):

$$C=C\_{a}\left(\frac{h−z}{z}\frac{a}{h−a}\right)^{Z}$$

$$Z=\frac{ω}{β\_{s}κu\_{\*}}$$

in which $ω$ is assumed to calculate with the median particle size in suspension (Shah-Fairbank 2009),

$$ω=\frac{8ν}{d\_{50ss}}\left[\left(1+0.0139d\_{\*}^{3}\right)^{0.5}−1\right]$$

$$d\_{\*}=d\_{50ss}\left[\frac{(G−1)g}{ν^{2}}\right]^{\frac{1}{3}}$$

and $C\_{a}=q\_{b}/11.6u\_{\*}a$, according to (Einstein 1950).

By inserting Eqs. (???) and (???) into Eq. (???), the measured load is written as:

$$q\_{m}=\int\_{d\_{n}}^{h}C\_{a}\left(\frac{h−z}{z}\frac{a}{h−a}\right)^{Z}\frac{u\_{\*}}{κ}ln\left(\frac{30z}{k\_{s}}\right)dz$$

The only difference between $q\_{m}$ and $q\_{s}$ is the lower limits of the integrals. $q\_{m}$ is the integration from $A=d\_{n}/h$ to 1:

$$q\_{m}=0.216q\_{b}\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}′+J\_{2}′\right\}$$

$$J\_{1}′=\int\_{A}^{1}\left(\frac{1−z}{z}\right)^{Z}dz$$

$$J\_{2}′=\int\_{A}^{1}lnz\left(\frac{1−z}{z}\right)^{Z}dz$$

$J\_{1}′$ and $J\_{2}′$ values for the sampled zone. The integrals starts from the nozzle height $d\_{n}$ to the free surface $h$. On the other hand, $J\_{1}$ and $J\_{2}$ start from the bed layer $2d\_{s}$ to the free surface.

The unit bedload $q\_{b}$ can be directly calculated from Eq. (???) with $q\_{m}$ is known. The ratio of measured to total sediment discharge can be shown as:

$$\frac{q\_{m}}{q\_{t}}=\frac{0.216\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}′+J\_{2}′\right\}}{1+0.216\frac{E^{Z−1}}{(1−E)^{Z}}\left\{ln\left(\frac{30h}{d\_{s}}\right)J\_{1}+J\_{2}\right\}}$$

As can be seen from above equation, $q\_{m}/q\_{t}$ is a function of $Z$, $h/d\_{s}$, and $A$. But since $d\_{n}$ is fixed with the same sampler, $q\_{m}/q\_{t}$ can be shown as a function of $Z$, $h$, and $d\_{s}$.

$$q\_{m}/q\_{t}=f(Z,h,d\_{s})$$

In this study, the Einstein integrals $J\_{1}$, $J\_{2}$, $J\_{1}′$ and $J\_{2}′$ are computed using scipy.integrate.quad method in Python.

## 2.3. Case study

The daily discharge and sediment measurement in South Korea are provided by K-water. The total sediment discharge is calculated by K-water using BORAMEP, version 1.8. In this study, the total sediment load will be recalculated by SEMEP and compared with the result of BORAMEP.

We analyzed the daily discharges and sediment measurements of 35 gaging stations in South Korea, including seven gaging stations on the Han River, fourteen on the Nakdong River, five on the Geum River, five on the Yeongsan River, and four on the Seomjin River, respectively (Fig. 1). The total area of the studied watersheds covered 47,000 km2. Among 2084 sediment samples, 2036 were measured by depth-integrating using D-74, the rest with a P-61 or a grab sampler. Only the samples of depth-integrating sampler were used in this study. The grain size distribution of bed material and suspended material were provided at all stations. Bed materials were sampled using the US BM-54 bed material sampler, 60L Van Veen Grab sampler or by grid sampling. The grain size distributions of suspended material were determined by laser diffraction. The lengths of record are summarized in Fig. 2, and a detailed data description and analysis is available in (Julien et al. 2017).

# 3. Results and Discussions

## 3.1. Ratios to total sediment discharge

As Eq. (???) shown that the ratio of suspended to total sediment discharge $Q\_{s}/Q\_{t}$ is only a function of $h/d\_{s}$ and $Z$. The analytical solution of Eq. (???) is plotted in Fig. ???. Fig. ??? shows the ratio $Q\_{s}/Q\_{t}$ at constant values of $Z$ while varying the value of $h/d\_{s}$. The ratio of $Q\_{s}/Q\_{t}$ increases when the values of $Z$ decrease.

Fig. ??? plots values of the ratio $Q\_{m}/Q\_{t}$ by varying the value of water depth as a function of $d\_{s}$. When $Z$ is close to 0, $d\_{s}$ becomes insignificant to $Q\_{m}/Q\_{t}$, and the measured sediment discharge is more than 90% of the total sediment discharge when depth $h>1$ meter.

## 3.2. Case study in South Korea

We first compared the total sediment discharge calculation by MEP and SEMEP. The total sediment load of 1962 out of 2036 suspended load samples was calculated by SEMEP. The uncalculated samples are due to measured flow depths lower than the nozzle height. In this case, the total load is calculated as the product of discharge and measured concentration. Using MEP, 1808 samples were calculated. Figure ??? presents the ratio of measured load $Q\_{m}$ to the total load $Q\_{t}$ for all samples computed by MEP and SEMEP. In Figure ???(a), the predicted total sediment load is compared to the measured load. The predictions from SEMEP are close to measured, while the predictions from MEP tended to be slightly higher on average, with a scatter larger than 2 orders of magnitude. Figure ???(b) shows that the values of $u\_{\*}/ω$ range from 15 to 1825. The $Q\_{m}/Q\_{t}$ of MEP range from $8×10^{−8}$ to $26$. The $Q\_{m}/Q\_{t}$ of SEMEP range from 0.5 to 0.995. According to Julien (2010), the primary mode of transport should be suspended load when $u\_{\*}/ω>5$. Therefore, $Q\_{m}/Q\_{t}$ is expected to be close to 1. This is true for sand-bed rivers, but deviations are noticeable for cobble and gravel-bed streams. Also $Q\_{m}/Q\_{t}$ should always be lower than 1 because the total load cannot be less than measured load. The result of MEP do not always satisfy this requirement. 29 samples out of 1808 resulted in $Q\_{m}>Q\_{t}$ when using the MEP. It is physically impossible for the total load to be smaller than the measured load. Other known issues with MEP are reported by (Shah-Fairbank 2009). For instance, MEP requires at least two overlapping bins between suspended material and bed material to determine $Z$. In addition, $Z$ for the remaining bins are determined by regression analysis when overlapping bins exist, and sometimes a negative $Z$ exponent can be generated, which erroneously implies that the sediment concentration increases towards the free surface. Therefore, SEMEP is considered more accurate and is used for the rest of analysis.

As shown in Eq. ???, the ratio $Q\_{s}/Q\_{t}$ only changes with $Z$ and $h/d\_{s}$. Due to the materials in suspension are fine, the values of $Z$ are small ($Z<0.16$) and therefore the change of $Z$ only results in little change in $Q\_{s}/Q\_{t}$. As shown in Fig. ???, $Q\_{s}/Q\_{t}$ becomes only a function of $h/d\_{50}$. Eq. ??? shows that $Q\_{m}/Q\_{t}$ is a function of $u\_{\*}/ω$, $h/d\_{s}$, and $d\_{n}/h$. When $Z$ and $d\_{s}$ are small and the nozzle height is fix, $Q\_{m}/Q\_{t}$ becomes only a function of water depth $h$ as shown in Fig. ???(f). The deviations are due to the values of $Z$ are not constant. $Q\_{m}/Q\_{t}$ decreases when $Z$ increases.

Further, we investigated how the ratio $Q\_{s}/Q\_{t}$ relates to (a) $u\_{\*}/ω$, (b) sediment concentration $C$, (c) flow discharge $Q$, (d) the ratio of flow discharge to mean annual discharge $Q/\overline{Q}$ (Fig. ???). Due to the median grain sizes of suspended material are silt at all the stations (average $d\_{50ss}$ = 0.023 mm), we found that $u\_{\*}/ω$ are generally high. $Q\_{s}/Q\_{t}$ is close to 1 and averages 0.99 in sand bed rivers (Fig. ???(a)). For gravel bed and cobble bed rivers, $Q\_{s}/Q\_{t}$ increases as $Q$, $C$, or $Q/\overline{Q}$ increases (Fig. ???(b), (c), and (d)). For a given discharge, $u\_{\*}/ω$, or concentration, $Q\_{s}/Q\_{t}$ reduces when the grain size is larger. For gravel bed rivers, $Q\_{s}/Q\_{t}$ varies from 0.871 to 0.999; for cobble bed rivers, $Q\_{s}/Q\_{t}$ ranges from 0.232 to 0.971. Figure ???(d) shows that during high flows ($Q/\overline{Q}>1$), over 90% of the sediment is transported in suspension for gravel bed and sand bed rivers. This analysis clearly indicates that the predominant mode of sediment transport in Korean rivers is in suspension.

In Fig. ???, we investigated the relationships between the ratio of the measured to total sediment discharge $Q\_{m}/Q\_{t}$ and the same variables present in Fig.???. In Fig. ???(a), (b), (c), and (d), the difference among various bed materials became subtle. Overall, the measurement contains over 80% of the sediment load (1936 out of 1962 samples). Most of the sediment is measured during floods in South Korean rivers (Fig. ???(d)).

# 4. Conclusions

The analytical solutions of $Q\_{m}/Q\_{t}$ and $Q\_{s}/Q\_{t}$ bring insights for practical applications. First, for rivers with fine suspended materials, such as the streams in South Korea, the ratio $Q\_{m}/Q\_{t}$ dependents only on the water depth. When the depth is higher than 1 meter, the depth-integrating sampler measure over 80% of the total sediment discharge. Second, with low values of $Z$, $Q\_{s}/Q\_{t}$ becomes a function of $h/d\_{s}$.

This study compares the total sediment transport rates calculated by MEP and SEMEP with the sediment samples collected by the depth-integrating sampler D-74 in South Korea. We conclude that SEMEP outperformed MEP in terms of stability, consistency and accuracy. SEMEP calculated the total sediment load from 1962 samples, while MEP calculated 1808 samples. The original method of MEP requires at least two overlapping bins between suspended materials and bed materials. Errors sometimes occurred when creating the power relationship between the Rouse number and fall velocity. Instead of using the overlapping bins, the Rouse number of SEMEP is determined by the median grain size of the suspended material.The ratios of the suspended load to total load calculated by MEP vary from $10^{−7}$ to 20. The large variation in total sediment load calculation with MEP occur due to the determination of Z and the threshold grain size of suspended and bed materials. Additionally, the ratio should never be greater than 1, and this raises the suspicion regarding the accuracy of the MEP method. The ratios between suspended load and total load range from 0.2 to 1, and 97% of them are greater than 0.9. For this reason, the SEMEP calculations are considered better and more accurate.

The SEMEP shows that the depth-integrating samples measured over 80% of the sediment load when the flow is larger than the mean discharge, and over 90% when the flow is 10 times larger than the mean. For the sand bed rivers in South Korea, sediment is predominantly transported in suspension. The ratio of bedload increases when the grain size increases. When the discharge is larger than mean discharge, at least 90% of sediment is transported in suspension for gravel and sand bed rivers.

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