# p4

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q 3

$Q2Ax(n) \leftrightarrow Form(n)\ \&\ (\exists p, q, r \leq n)(Var(r)\ \&\ (\forall s \leq l(p))\overline{Free(s, r, p)}\ \&\ n = cond(gen(r, cond(p, q)), cond(p, gen(r, q))))$

$N7Ax(n) \leftrightarrow Form(n)\ \&\ (\exists p, q, r \leq n)(Var(p)\ \&\ \overline{r = 0}\ \&\ Free(r, p, q)\ \&\ n = cond(neg(cond(sub(q, p, 0), neg(gen(p, cond(q, s$

q 4
4a

Will show by giving informal derivations of forward and reverse cases (using previously proven lemmas, TF as rule of inference, definition of $\leq$.
The forward case:
1. $Sy = S(x + z) \supset y = x + z$ by axiom.
2. $S(x + z) = Sx + z$ by axiom.
3. $Sx + z = S(x + z)$ by symmetry.
4. $S(x + z) = Sx + z \supset (Sy = S(x + z) \supset Sy = Sx + z)$ by axiom.
5. $Sx + z = S(x + z) \supset (Sy = Sx + z \supset Sy = S(x + z))$ by axiom.
6. $Sy = S(x + z) \leftrightarrow Sy = Sx + z$ by MP and TF logic.
7. $Sy = Sx + z \supset y = x + z$ by TF logic.
8. $x' \leq y' \leftrightarrow \exists z'(y' = x' + z')$ by definition.
9. $y = x + z \leftrightarrow x \leq y$ by definition.
10. $Sy = Sx + z \leftrightarrow Sx \leq Sy$ by definition.
11. $Sx \leq Sy \supset x \leq y$ by TF logic.

The reverse case:
1. $y = x + z \supset (Sy = Sy \supset Sy = S(x + z))$ by axiom.
2. $y = x + z \supset (Sy = Sy \supset Sy = Sx + z)$ by TF since $S(x + z) = Sx + Z$ by axiom.
3. $x \leq y \supset (Sy = Sy \supset Sx \leq Sy)$ by TF given definition of $\leq$ .
4. $x \leq y \supset (Sy = Sy \supset Sx \leq Sy) \supset ((x \leq y \supset Sy = Sy) \supset (x \leq y \supset Sx \leq Sy))$ by axiom.
5. $(x \leq y \supset Sy = Sy) \supset (x \leq y \supset Sx \leq Sy)$ by MP.

6. $Sy \le Sy \supset (x \le y \supset Sy \le Sy)$ by axiom.
7. $Sy \le Sy \leftrightarrow \exists z(Sy = Sy + z)$ by definition.
8. $Sy = Sy + 0$ by axiom.
9. $Sy = Sy$ by axiom.
10.$Sy \le Sy \leftrightarrow Sy = Sy$ by TF.
11. $Sy \le Sy$ by TF.
12. $x \le y \supset Sy \le Sy$ by MP.

Therefore $x \le y \leftrightarrow Sx \le Sy$ and since this is TF valid and PA is TF complete, then also $\vdash x \le y \leftrightarrow Sx \le Sy$.

4b
For any two numbers $x, y,\ \ x = y$ or $x \neq y$.
If $x = y$ then $x + 0 = y$ so $\exists z(y = x + z)$. So $x \le y$.
If $x \neq y$ then $\exists z'(y = x + z')$ or $\overline{\exists z'(y = x + z')}$ . So $x \le y$ or $\overline{x \le y}$.

For any instances of x and y in PA, call then a and b, then $x \le y \supset a \le b$ and $\overline{x \le y} \supset \overline{a \le b}$. Since $x \le y \vee \overline{x \le y}$ is TF valid and PA is TF complete, then $\vdash x \le y \vee \overline{x \le y}$. By instantiation $\vdash a \le b \vee \overline{a \le b}$. Then $x \le y \supset \vdash a \le b$ and $\overline{x \le y} \supset \vdash \overline{a \le b}$.

q 1

1a holds for $1 < i \le l(j)$
1b holds for $i = 10$
1c holds for no $i$
1d holds for $i = 7$ and $i = 12$.

q 2

$occ(0, 19, m) = 8$
$occ(1, 19, m) = 4$
$1 < j \to occ(j, 19, m) = 0$
$subst(m, 19, p, 1) = \gamma(\text{`('}^\frown\text{`z'}^\frown\text{`='}^\frown\text{`y'}^\frown\text{`} \supset \text{'}^\frown z^\frown\text{`='}^\frown\text{`S'}^\frown\text{`S'}^\frown\text{`0'}^\frown\text{`)'})$
$subst(m, 19, p, 2) = \gamma(\text{`('}^\frown\text{`z'}^\frown\text{`='}^\frown\text{`S'}^\frown\text{`S'}^\frown\text{`0'}^\frown\text{`} \supset \text{'}^\frown z^\frown\text{`='}^\frown\text{`S'}^\frown\text{`S'}^\frown\text{`0'}^\frown\text{`)'})$