Problema sobre columnas

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The formula for the critical load of a column was derived in 1757 by Leonhard Euler, the great Swiss mathematician. Euler's analysis was based on the differential equation of the elastic curve:

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

Figure 1. This is a caption

Find the solution to this equation and apply the following conditions to obtain the values for the constants of integration:

$$v|_{x=0} = 0$$
$$v|_{x=L} = 0$$

Figure 2. This is a caption

Finally explain how to get the following result:

$$P = n^2 \frac{\pi^2 EI}{L^2}$$

Figure 3. This is a caption

$d^2v =$ dos veces la derivada de la función

$m = A \ \sin \ \gamma x + B \ \cos \ \gamma x$

$m' = A \ \gamma \cos \ \gamma x \ - B \ \gamma \ \sin \ \gamma x$

$m" = A\gamma^2 \sin \gamma x \ - B\gamma^2 \cos \gamma x$

$-A\gamma^2 \sin \gamma x - B\gamma^2 \cos \gamma x + \left(\frac{P}{EI}\right)(A \ \sin \ \gamma x + B \cos \gamma x) = 0$

Factorizando $: \ -A\gamma^2 \sin \gamma x - B\gamma^2 \cos \gamma x + \left(\frac{P}{EI}\right) A \ \sin \ \gamma x + \left(\frac{P}{EI}\right) B \ \cos \gamma x = 0$

$A \ \sin \ \gamma x \left(-\gamma^2 + \frac{P}{EI}\right) + B \ \cos \gamma x \left(-\gamma^2 + \frac{P}{EI}\right) = 0$

Para despejar: $-\gamma^2 + \frac{P}{EI} = 0$

$\frac{P}{EI} = \gamma^2$ Por lo tanto $\gamma^2 = \frac{P}{EI} \quad \gamma = \sqrt{\frac{P}{EI}}$

Entonces:

$m = A \ \sin \ \gamma x + B \cos \gamma x$

$m = A \ \sin \ \sqrt{\frac{P}{EI}} + B \ \cos \sqrt{\frac{P}{EI}} x$

Las condiciones son:

$$v|_{x=0} = 0$$
$$v|_{x=L} = 0$$

Figure 4. This is a caption

$m = A \ \sin \ \sqrt{\frac{P}{EI}(0)} + B \ \cos \sqrt{\frac{P}{EI}(0)}$

Por lo tanto $B = 0$

$A \ \sin \ \sqrt{\frac{P}{EI} \cdot L} + B \ \cos \sqrt{\frac{P}{EI} \cdot L}$

$\sin \sqrt{\frac{P}{EI} \cdot L} = 0$

$\sqrt{\frac{P}{EI} \cdot L} = n\pi$

n=1,2,3,...

Para despejar

$\sqrt{\frac{P}{EI} \cdot L} = n\pi$

$\frac{P}{EI} \cdot L^2 = n^2\pi^2$ por lo tanto $P = \frac{n^2\pi^2(EI)}{L^2}$

Cuando n=1

$Pcr = \frac{\pi^2(EI)}{L}$