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PHYSICS

Topological Dipole Field Theory

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ABSTRACT

Quantum theory has found that elementary particles in addition to the classic field quantity have also quantum-mechanical degree of freedom. This research paper defines another hypothetical intrinsic degree of freedom which has a topological nature. A topological quantum field theory is constructed to this hypothetical degree of freedom.

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INTRODUCTION

A well-known topological quantum field theory is the Chern-Simons theory which is strongly related to knot theory (Chern et al. 1974). This theory is applied in various disciplines of theoretical physics. The governing field of the theory is the non-abelian 1-form geometric connection A where the action of the Chern-Simons theory does not change when it is varied by A. Therefore the field theory is topological. Observables of this theory are given by knot invariants. Chern-Simons theory is a Schwarz-type topological quantum field theory in which the whole action is independent on variations in geometric quantities. There were postulated a couple of other topological quantum field theories in literature (Atiyah 1989).

In physics, the occurrence of topological defects are also well-known. As an example, ordered media can have topological defects (Mermin 1979). Topological defects can affect electromagnetic interactions taking place in a physical system. Electromagnetic systems with topological defects are also studied in research literature (Bakke et al. 2010).

This research paper shows how it is possible to generalize the concept of topological defects to elementary particles. Since quantum physics has found out that elementary particles have properties that are not predicted by classical physics

(e.g. the spin of a particle) it can be assumed that some other microscopic properties of particles are present but not predicted yet. The main purpose of this paper is to show that charged elementary particles like electrons can possess additional internal degree of freedom. This is performed by regarding a topological quantum field theory which is able to take topological defects into account. Primarily, particles with electric charge are described by Quantum electrodynamics. Quantum electrodynamics is a physical theory with a very high agreement with experiments. However, there can be a difference between the real behavior of charged particles and the predictions of Quantum electrodynamics. Some more detailed experimental tests for Quantum electrodynamics are the measurement of the anomalous dipole moment of the muon (Hagiwara et al. 2007).

For the derivation of a quantum field theory which includes topological corrections to ordinary quantum electrodynamics a Witten-type topological quantum field theory is proposed (Witten 1988). The basic quantum field is assumed as a dipole field strength tensor that arises from topological defects. This field is regarded as the geometric quantity of the theory. An additional dipole field will generate a proper generalization of the electromagnetic field strength tensor in quantum electrodynamics. It is shown, how quantum observables will be independent on the dipole field strength tensor. This ensures that the quantum field theory is a topological quantum field theory.

THEORY

Witten-type topological quantum field theories are based on cohomology theories. The action S of such theories must contain a symmetry. More precisely, there must exist a differential operator δ such that $\delta S = 0$. This differential operator is similar to a Lie derivative and must satisfy the exactness condition

$$\delta^2 = \delta\delta = 0. \quad (1)$$

The theory treated in this paper is assumed to be a 4-dimensional theory like ordinary quantum electrodynamics. It is assumed that the operator δ is the Čech coboundary map. Assuming that geometric fields are defined on a set $U = \cap_{i=1}^{n} A_i$ with arbitrary sets A_i and the condition that U is a point in spacetime. Then it is easy to compute the action of the Čech coboundary operator on a function f:

$$\delta f(U) = \sum_{i=1}^{n} (-1)^i f(A_1 \cap \ldots \cap \hat{A}_i \cap \ldots \cap A_n). \quad (2)$$

Here, the hat denotes that the set is omitted. It is easy to show that the definition (2) satisfies the exactness condition (1). The number of intersecting sets n where the intersection of these sets generates the spacetime point can be chosen arbitrary. A simple case is given by the choice $n = 3$. Let be B a 2-form field which is the dipole field strength tensor of the charged elementary particle. Because this field is induced by hypothetical topological defects in the elementary particle, this field must be the geometrical quantity of the action. When assuming that the generalized electromagnetic field strength tensor is given by

$$F = dA + B' \quad (3)$$

with the electromagnetic 1-form gauge connection A, the field B' can be interpreted as the intrinsic curvature. In the model treated in this paper quantum electrodynamics is replaced by the generalized field strength tensor (3) where the field of the intrinsic degree of freedom B' induces additional topological interactions. The fields B' are the observables of the topological quantum field theory. Therefore, B' must lie in the cohomology classes of the Čech cohomology, i.e. $\delta B' = 0$ but $B \neq \delta X$ with an arbitrary 2-form field X.

A suitable action for the generalized quantum electrodynamics has the form:

$$S = S_{QED}(\psi, A, B') + S_{top}(B). \quad (4)$$

The field ψ is the fermion field. Assuming that A and ψ always satisfying the Čech cocycle condition $\delta A = 0, \delta\, \psi = 0$, the action of δ on the first term of (4) vanishes. Considering the action functional existing on the spacetime manifold M

$$S_{top}(B) = \int_M B \wedge \delta B \quad (5)$$

then it can be easily shown that $\delta S_{top}(B) = 0$ if it holds $\delta B \wedge \delta B = 0$ for all $U \subset M$.

When the auxiliary condition

$$\delta B \wedge \delta B = 0 \quad (6)$$

is imposed, it is straightforward to show that the only tensor invariant of the antisymmetric tensor $(\delta B)_{\mu\nu}$ that is not vanishing is given by $I =* (\delta B \wedge * \delta B)$.

To show that above assumptions construct a topological quantum field theory, the action (5) must be varied by the field B. If all quantum fields vanish on the boundary of M it holds $\int_M \delta\Lambda = 0$ for arbitrary 4-form field Λ. For the variation operator $\Delta_{\mu\nu}$ it holds the functional derivative rule $\Delta S_{top}(B,\delta B) := \frac{\partial S_{top}}{\partial B_{\mu\nu}} - \delta(\frac{\partial S_{top}}{\partial \delta B_{\mu\nu}})$. From

$* (\frac{\partial B}{\partial B_{\mu\nu}} \wedge \frac{\partial B}{\partial B_{\mu\nu}}) = \epsilon^{\alpha\beta\gamma\eta}\delta_{\alpha\mu}\delta_{\beta\nu}\delta_{\gamma\mu}\delta_{\eta\nu} = 0$ and (6) it follows $\frac{\partial B}{\partial B_{\mu\nu}} \wedge \delta B = -\delta B \wedge \frac{\partial B}{\partial B_{\mu\nu}}$.

Hence:

$$\Delta_{\mu\nu} S_{top} = 2\frac{\partial B}{\partial B_{\mu\nu}} \wedge \delta B := \delta \Xi^{\mu\nu}. \qquad (7)$$

The condition (7) is also a condition for a Witten-type topological quantum field theory. It ensures that the every observable O that lies in a Čech cohomology class has an expectation value

$$< O >= \int d[\psi]d[A]d[B]Oe^{iS} \qquad (8)$$

is independent on variations in the geometrical quantity B. If the auxiliary field λ is introduced which is also involved in the functional integration, i.e. $d[\psi]d[A]d[B] \mapsto d[\psi]d[A]d[B]d[\lambda]$, the action S can be extended by a term that describes the auxiliary condition in the following manner:

$$S \mapsto S + \int_M \lambda\delta B \wedge \delta B. \qquad (9)$$

The action in the general form (9) describes the complete quantum field theory.

CONCLUSIONS

This generalization of quantum electrodynamics by topological quantum fields is a model for describing a hypothetical intrinsic dipole moment of a charged elementary particle. Above considerations show that the topological term of the action is similar to the abelian Chern-Simons theory. The photon field strength tensor of quantum electrodynamics is coupled to an additional dipole moment which is of purely topological nature.

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