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PHYSICS

E-gravity theory as a Yang-Mills theory

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Yang-Mills theories are a very fruitful concept in quantum field theory. Fundamental interactions and its unifications can be described with Yang-Mills theory. However, gravity is still not modeled in the framework of Yang-Mills theory. It is modeled in terms of the Einstein-Hilbert action in the case of semiclassical field theory, but ordinary quantization of the spacetime field fails due to UV divergences. A possible approach to quantum gravity called E-gravity theory avoids UV-divergences. Primary, this theory is based on a spacetime discretization and the assignment of a curvature measure to discretized spacetime. This paper shows that this approach is also a special case of Yang-Mills theory.

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DATE RECEIVED:
June 06, 2016

DOI:
10.15200/winn.146522.21148

ARCHIVED:
June 06, 2016

KEYWORDS:
quantum gravity, Yang-Mills theory, Finite element methods, gauge theory

CITATION:
Patrick Linker, E-gravity theory as a Yang-Mills theory, *The Winnower* 3:e146522.21148 , 2016 , DOI: 10.15200/winn.146522.21148

INTRODUCTION

The most plausible approach for making a model for fundamental elementary particle interaction is the Yang-Mills theory. In this theoretical framework, a local invariance of a physical action under a gauge group action is assumed. This invariance imposes the existence of a gauge connection that is also called “gauge boson field”. Electromagnetic, electroweak and strong interaction can be described very well in terms of Yang-Mills theory. Electroweak interaction is generated by imposing a $U(1) \times SU(2)$ gauge group invariance and strong interaction can be obtained if the action functional is invariant under $SU(3)$ group transformations. The Standard model of particle physics is based on the gauge group $U(1) \times SU(2) \times SU(3)$. May be A^a_μ a gauge connection in spacetime direction μ with generator index a, f^{abc} the structure constants of the gauge group, g the coupling constant and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ the associated field strength. Then the Yang-Mills Lagrangian density L has the form:

$$L = \frac{1}{2} F^a_{\mu\nu} F^{a\mu\nu} + L_{FP}. \qquad (1)$$

The additional Lagrangian density L_{FP} is arising from the measure factor of the quantum path integral and contains the Faddeev-Popov ghost fields c^a that are Grassmannian variables. It holds:

$$L_{FP} = c^{a\dagger}\left(\delta^{ab} \partial_\mu \partial^\mu + g f^{abd} \partial_\mu A^{d\mu}\right) c^b \qquad (2)$$

However, gravity, also another fundamental force, cannot be described in terms of Yang-Mills theory. The only well-known consistent way for describing gravity is General Relativity, which is a classical theory of gravity. An open question is whether gravity can be formulated as a Yang-Mills theory.

Several research papers show that there is existing a relationship between gravity and Yang-Mills theory (Bern, Z. et al. 1998) (Hsu, J.P. 2006).

There are existing theories that quantize gravity, e.g. Loop Quantum Gravity. The derivation of Loop Quantum Gravity is based on a reformulation of General Relativity by introducing $SU(2)$ gauge fields (Ashtekar,A. 1985). However, Loop Quantum Gravity does not coincide with Yang-Mills theory, e.g. due to the use of constraint terms and the quantization in terms of Wilson loops. Another recent approach to quantum gravity is E-gravity theory (Linker, P. 2016). This theory provides a discretization of spacetime with simplices. Simplices can have two or more simplex corners that are identified (in mathematical terms: there exists a nontrivial equaliser). Because of this additional property, a curvature measure can be assigned to each simplex. General Relativity is rewritten in this discrete form and a path integral over all possible spacetime geometries is performed.
In this research paper, an alternative derivation of E-gravity theory is shown. At first, spacetime is discretized by finite elements which are the simplices. Finite element methods are typically used for engineering problems, but are also applied to quantum field theories (Sopuerta, Carlos F. et al. 2005). Instead of using assumptions about the discretization of the Einstein-Hilbert action, a Yang-Mills theory with the diffeomorphism operator as the gauge group is worked out. Also, the field strength tensor, which is different from the smooth fiber bundle curvature form, is derived. With proper projection of the Yang-Mills theory to physical states, E-gravity theory is recovered.

THE DERIVATION OF E-GRAVITY THEORY BY YANG-MILLS THEORY

The theory of gravity is assumed to be a gauge theory with respect to the Poincaré group. If General Relativity is transformed locally by the Poincaré group, the action remains the same. Generators of the Poincaré group are local spacetime translations and local Lorentz transformations. Since the translation generators $t_{\vec{a}} = -ia_\mu \partial^\mu$ corresponding to a 4-vector $\vec{a}$ are operators and not finite-dimensional matrices, Yang-Mills theory in the form (1) cannot be applied. The goal is to find a matrix-like gauge group for gravity. The first step to find a proper gauge group is to use a finite element discretization of spacetime. In general, a physical field $\phi(\vec{x})$ (can be scalar, spinor, vector or tensor field) on a spacetime point $\vec{x}$ is discretized as follows:

$$\phi(\vec{x}) = \sum_{k=0}^{4} \phi_k \, \sigma_k(\vec{x}). \qquad (3)$$

Here, ϕ_k are the field values at simplex corner (or node) k and $\sigma_k(\vec{x})$ the corresponding form functions that have the value 1 at simplex point k and the value 0 elsewhere. These form functions generate a discretized spacetime without self-intersection of simplices. Additionally, if two simplex nodes are identified (a key feature of E-gravity theory), the sum in equation (3) runs from 0 to $4-M$ with the number of identified nodes M. Denotes Λ the set of all possible form functions that discretize the spacetime and $\aleph$ the set of all nodes in the spacetime, one obtains the path integral:

$$Z = \int d[\Lambda] \prod_{k\in\aleph} d[\phi_k] e^{iS(\{\phi_k | k\in\aleph\},\Lambda)}. \qquad (4)$$

Clearly, the set Λ is quite different from the simplex node equaliser set. However, the path integral (4) can be reduced to E-gravity theory by an appropiate gauge group. May the gauge operator χ acting on a function $f(\vec{x})$ with the entire spacetime region Ω be defined as follows:

$$\chi f(\vec{x}) = \int_\Omega d^4y \chi(\vec{x},\vec{y}) f(\vec{y}). \qquad (5)$$

Because the operator (5) modifies the physical fields globally, the field χ is a global gauge transformation. The operator (5) can be turned into a local gauge transformation by replacing $\chi \mapsto \chi_{\vec{x}}, \chi(\vec{x},\vec{y}) \mapsto \chi_{\vec{x}}(\vec{x},\vec{y})$.
For a plausible model of gravity, the invariance of the action under the modified Poincaré group $MP(1,3)$ with group elements χ acting on physical states in 4-dimensional spacetime is imposed. This group is called “modified”, because all elements of the group acting like the convolution operator (5); these groups are “infinite-dimensional matrices”. Moreover it holds $\chi^\dagger \chi = 1$, i.e. $\chi \in MP(1,3)$ is an unitary transformation. Due to linearity of χ and the invariance of the action $S(\{\phi_k | k \in \aleph\}, \Lambda)$ under group transformations in $MP(1,3)$, the choice of form functions is arbitrary. If the deformation of simplices by χ is diffeomorphic, one can set without loss of generality $\chi(\vec{x}) = \chi_{\vec{x}}(\vec{x})$, i.e. local and global gauge transformations are the same. This can be easily seen if a differential operator ∇ with respect to $\vec{x}$ is performed: By chain rule,

$$\nabla \chi(\vec{x}) = \nabla \chi_{\vec{x}}(\vec{x}) = \nabla \chi_{\vec{x}}(\vec{x})|_{\text{subscript } \vec{x} \text{ fixed}} + \nabla \chi_{\vec{x}}(\vec{x})|_{\text{argument } \vec{x} \text{ fixed}} \qquad (6)$$

and equation (6) is only true if and only if $\nabla \chi_{\vec{x}}(\vec{x})|_{\text{argument } \vec{x} \text{ fixed}} = 0$. Hence, the gauge connection vanishes in case of any diffeomorphic deformations.
If two or more simplex nodes are identified, one performs a non-diffeomorphic deformation. The left hand side of equation (6) does not exist in general. By switching the gauge transformation from a global to a local transformation, one can generalize the gauge group $\chi \in MP(1,3)$ to a non-diffeomorphic gauge group. This implies the existence of a non-smooth (!) gauge connection Ξ_μ.

Gauge group χ ensures that physical laws are unchanged not only if local diffeomorphic deformations are present as it is the case in General Relativity. Local non-diffeomorphic deformations also let physical laws be the same. Diffeomorphic spacetime deformations of General Relativity can be assumed as a very dilute collection of non-diffeomorphic deformations on Planck scales in average. An important condition for this gauge group is that given the map $\chi: \sigma_k(\vec{x}) \mapsto \sigma'_k(\vec{x})$ between form functions $\sigma_k(\vec{x}), \sigma'_k(\vec{x})$ for arbitrary k, the resulting form function $\sigma'_k(\vec{x})$ generates also a simplex element.

Clearly, a gauge connection which is not differentiable, makes not sense in a physical field theory. If $\Xi_\mu(\vec{p}) \neq 0$ on a certain spacetime point $\vec{p}$, then $\Xi_\mu(\vec{p})$ is singular, since Ξ_μ is necessary only for non-diffeomorphic deformations. As a consequence, physical states can only be observed in regions with $\Xi_\mu = 0$ for all spacetime directions μ. Field strengths $F_{\mu\nu}$ are not vanishing necessarily, because derivatives of Ξ_μ are not zero in general even if $\Xi_\mu = 0$.

The path integral (4) can be reduced with the assumptions made above. Introducing a parameter $\zeta_r \in (0,1)$, where r denotes the index of the topological simplex structure (i.e. it denotes which nodes are identified). This parameter runs over all different states of a simplex with same topology and therefore it is called the "diffeomorphic deformation parameter"; in diffeomorphic deformation this parameter changes. For non-diffeomorphic deformations, the gauge connection Ξ_μ is changing. Therefore:

$$\int d[\Lambda] = \sum_{r \in T} d[\zeta_r] \int d[\Xi_\mu|_{gauged}\ ]. \quad (7)$$

The set T is the set of all possible simplex topologies, i.e. the set of all identified simplex nodes. Physical action does not change under diffeomorphic deformations (including Lorentz transformations) and hence, the integral over ζ_r for each $r \in T$ is constant. The integration over all gauge connections makes only sense for $\Xi_\mu \mapsto 0$. All contributions with $\Xi_\mu \neq 0$ can be neglected; only physical states are taken into account, while the number of unphysical states is vanishing small. Hence, the Faddeev-Popov factor in the path integral factorizes in the path integral and can be omitted. One is left with the following path integral:

$$Z = \sum_{r \in T} \prod_{k \in N} \int d[\phi_k] e^{(iS_{matter} + \frac{i}{2} tr(F_{\mu\nu}F^{\mu\nu})|_{\Xi_\mu = 0})_r}. \qquad (8)$$

The path integral (8) is the path integral of E-gravity theory if the field strength term $\frac{1}{2} tr(F_{\mu\nu}F^{\mu\nu})|_{\Xi_\mu=0}$ corresponds to the Einstein-Hilbert action in vacuum. It holds $F = d\Xi$ for a 1-form gauge connection $\Xi = \Xi_\mu dx^\mu$ and a 2-form field strength tensor F for $\Xi = 0$. Using theorem of Stokes over a 2-dimensional area A, one obtains $\int_A F = \int_A d\Xi = \int_{\partial A} \Xi$. If A is sufficiently small, i.e. $|A| \mapsto 0$ ($|\ldots|$ denotes the measure of a set), one can define the field strength corresponding to the area A:

$$F_A := \frac{1}{|A|} \int_A F = \frac{1}{|A|} \int_{\partial A} \Xi. \quad (9)$$

From (9) it follows that if $\Xi \mapsto 0$ and the path integration is not performed around a singularity, it holds $F_A = 0$. This is the case if there are no identified simplex nodes in a simplex element. Every node which is identified with another node causes a nonvanishing value of Ξ due to the change of spacetime structure by making topological "shortcuts" in it. The statement $\delta^2 I(s_0 \ldots s_4) = 0$ with coboundary map δ, E-semigroup elements $s_0, \ldots, s_4$ and indicator function I (that is 1 if there are no identified simplex nodes and 0 otherwise) where the $s_i, i \in \{0, \ldots, 4\}$ are pairwise different is proven. If there are 2 or more identified simplex nodes (these induce singularities), one can choose closed paths in arbitrary spacetime directions such that all 3 or more singularities are contained in the path. Since A is small, one has $\Xi \approx const.$ everywhere on ∂A and in this case $F_A = 0$.

It has to be shown that if exactly one simplex node s_k is identified with $s_j, j \neq k$, it holds $R = 0$ if k is even and j is odd or j is even and k is odd, $R = -K$ if k and j are odd and $E = K$ if k and j are even; the value K is a specific gravitational constant. At first, the following decomposition of the gauge field action (which is known from electromagnetic theory) can be performed:

$$S_{bos} = \frac{1}{2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)|_{\Xi_\mu=0} = \frac{1}{2}\mathrm{tr}\left(F_{0i}F_{0i} - F_{ij}F_{ij}\right)|_{\Xi_\mu=0}. \quad (10)$$

The indices i, j are running from 1 to 3 (all space directions). The term F_{0i} is the “gravito-electric” field strength and the term F_{ij} is the “gravito-magnetic” field strength. By Lorentz transformations, the time direction can be replaced by a direction pointing anywhere in spacetime, where all other indices denote all other directions perpendicular to the direction with index 0; action functional does not change after any Lorentz transformation.

Simplex elements have to be oriented properly while discretizing spacetime. Non-diffeomorphic deformations have singular values of the connection across the 0-direction, if the simplex node generators s_1 or s_3 are identified with another simplex node (by convention of the coboundary map, all nodes with odd index are negatively oriented), and have singular values of the connection perpendicular to this 0-direction, if all other simplex node generators s_0, s_2 and s_4 are identified with another simplex node (by convention of the coboundary map, all nodes with even index are positively oriented). Every singularity has the same magnitude of the value Ξ given by K'. This assumption relies on the choice of infinite hypercomplex numbers $(\alpha, \alpha+1, \alpha+2, \ldots), (\beta, \beta+1, \beta+2, \ldots), \alpha \neq \beta$ with ordinary finite complex numbers α, β for the gauge connection field; indeed, it holds $(\alpha, \alpha+1, \alpha+2, \ldots) = (\beta, \beta+1, \beta+2, \ldots) \mapsto \infty$ by nonstandard analysis. If k is even and j is odd or j is even and k is odd, the path integration passing through 0-direction encounters one singularity and the path integration passing through directions only perpendicular to 0-direction encounters also one singularity; hence $\sqrt{F_{0i}F_{0i}} = \sqrt{F_{ij}F_{ij}} = K'$ and $S_{bos} = 0$.

If k and j are odd, the path integration in arbitrary 0-direction encounters two singularities of Ξ with equal magnitude, while path integration in directions perpendicular to the 0-direction have no singularities; hence: $\sqrt{F_{0i}F_{0i}} = 2K'$, $F_{ij} = 0$ and $S_{bos} = cK'^2$ with a constant c. Finally, if k and j are even, the path integration in directions perpendicular to 0-direction encounters two singularities of Ξ with equal magnitude, while path integration in directions in 0-direction have no singularities; hence: $\sqrt{F_{ij}F_{ij}} = 2K'$, $F_{0i} = 0$ and $S_{bos} = -cK'^2$ with a constant c. The values of S_{bos} are also infinite values, but with renormalization of the Yang-Mills coupling constant g this infinity can be removed.

DISCUSSION

In four spacetime dimensions, the Ricci scalar defined by $R = \delta^2 I(s_0 \ldots s_4)$ (averaged over a volume) can be also recovered by a Yang-Mills theory. Some idealizations like the equal values of a topological singularity are assumed, which might lead to small inaccuracies if the field strength is computed by more advanced concepts (not treated in this research paper). The equivalence between Yang-Mills theory and discretized Einstein-Hilbert action with curvature measure R is only valid for 3+1-dimensional spacetime (one direction generated by 2 simplex nodes makes the pure "electric" field strength and all three other directions generated by all other 3 simplex nodes make the pure "magnetic" field strength). Extra dimensions can play a role if E-gravity theory is embed in a certain plausible framework of superstring theory or M-theory. However, there is still not found a way to do this. However, there is evidence that the Standard model of particle physics with gravity is recovered if superstring theories are compactified on the Calabi-Yau manifolds (Candelas et al. 1985). Getting the Standard model combined with E-gravity theory from superstring theory or M-theory by suitable compactification is still not clear even since the gauge group $\mathrm{MP}(1,3)$ has different structure compared with gauge groups that impose all other fundamental interactions.

Inhomogenity in spacetime, which is also a feature of E-gravity theory, is caused exactly by the field Ξ which is nonvanishing between the simplex elements, where physical states can occur. This gauge connection separates simplex elements which are different. E-gravity theory in terms of a Yang-Mills theory would conserve energy, momentum and spin-angular momentum, but if the 1-form field Ξ is "cut out", energy and momentum has to be created and annihilated to include effects performed by the gauge connection.
Gravitational interactions are able to materialize multiple particles. From the Yang-Mills theory, gravity couples on the currents (here, the fermionic case with fermion fields Ψ and coupling constant g)

$$J_{\mu}^{a} = g\psi^{\dagger}\gamma_{\mu}T_{x\bar{x}}^{a}\psi, \qquad (11)$$

where $T_{x\bar{x}}^{a}$ is the generator of diffeomorphic deformations. It is a convolution operator and from (11) it holds that gravity is induced by correlations of particles in the entire spacetime. A simple example is the correlation of an apple that correlates with the earth; the apple falls towards earth because gravity is triggered by the apple-earth correlation. In systems where quantum entanglement can occur, physical states can correlate; therefore gravity is also produced in entangled systems. Assuming that the generators $T_{x\bar{x}}^{a}$ of the simplex modification are acting only on a small, bounded region, the correlation current (11) involves only correlations of states that are very close to each other. Moreover, due to the very small support of these generators, one can assume that the commutators of these are neglectible; an abelian gauge theory arises. Therefore, gravity has an "electromagnetic" character and the force is transmitted with speed of light.

The fact that entangled states produce gravitational energy can be understood as follows: If two particles are interacting with each other, the structure of quantum uncertainty between these particles is modified by gravity. Two different independent random motions that are interacting with each other, will combine to one correlated random motion. But such a process would lead to decreasing entropy . Gravity that is caused by correlation avoids a violation of the second law of Thermodynamics, it increases entropy by ΔS and therefore, the energy that is produced by gravity has to be of order $T\Delta S$ for a given temperature T. Energy has to be bounded by Planck energy which implies that the entropy change due to gravitational interaction ΔS has to be finite.

Quantum Gravity theories are still not proven by experiments. Some other research was performed to discuss effects predicted by quantum gravity (Takeuchi et al. 2016). However, experiments for effective gravity theories (e.g. General Relativity) were performed. A recent experimental verification of General Relativity (Abbott 2016) is the detection of gravitational waves from binary Black Holes. In terms of E-gravity theory, two Black Holes are correlating with each other and therefore these would induce a gravitational current given by (11). By using the Yang-Mills form of E-gravity theory one would obtain equations similar to Maxwell's equations to electromagnetism with a correlation current instead of a electric current. The fields of the gauge connection have large values on small length and time scales. If the action of E-gravity theory is treated as a mean-field theory for large length and time scales, General Relativity can be recovered; moreover, since the regions of spacetime where large values of the nonvanishing gauge connection occur are very little, one can treat these gauge connections also as small values on large length and time scales. This implies that E-gravity theory predicts also waves induced by gravitational correlations. The largest amplitudes of gravitational waves occurring when the two Black Holes are collided; the Black Holes have the largest correlation in this case.

Another experimental result, where predictions of E-gravity theory agree, is the entropy of a Black Hole. Black Hole entropy computed with E-gravity theory is proportional to the event horizon area of the Black Hole (Linker 2016).

Gravity is a very weak force in comparison with all other fundamental forces of nature. In E-gravity theories the value of the curvature measure vanishes at the most of the possible simplex configurations; making gravitational effects nearly irrelevant for particle scattering at low energies and low particle densities. Therefore, E-gravity provides an explaination of the weakness of gravity in comparison with all other fundamental forces.

CONCLUSIONS

An interesting correspondence between Yang-Mills theory and General Relativity is shown in this research paper. Yang-Mills theory with another version of the Poincaré group, called $\mathrm{MP}(1,3)$ with special assumptions about simplex topology turns out to coincide with the Einstein-Hilbert action that uses a generalized curvature measure based on chain complexes of simplices. Formulating fundamental interactions in terms of Yang-Mills theory makes it easier to find unified descriptions of fundamental forces. Indeed, the gauge group of Standard model of particle physics combined with E-gravity theory can be regarded as the gauge group $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3) \times \mathrm{MP}(1,3)$ that looks like a unified description of fundamental forces. Unfortunately, a plausible Grand Unified Theory is still not found; only experimentally unverified models are proposed. However, E-gravity theory also applies to effects, where not all other fundamental forces are unified.

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