HW 03-b

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2. NOTE: latex floors are not working in my document application, so **Floor** = F(x) and **Ceiling** = C(x) for all parts of question #2

a.

For Evens:

Assume that n = 2m, m being an integer

F(2m + 1)/2) = C(2m/2)

F((2m/2) + (1/2)) = C(m)

C(m) = m

F(m + (1/2)) = m

m = m

For Odds:

Assume that 2 = (2m + 1), m being an integer

F(((2m + 1) + 1) / 2) = C(2m + 1) / 2))

F((2m + 2)/2) = C((2m/2) + (1/2))

F((2m/2) + (2/2)) = C(m + (1/2))

F(m+1) = C(m + (1/2))

F(m + 1) = (m + 1)

C(m + (1/2)) = (m + 1)

(m + 1) = (m + 1)

b.

For Evens:

Assume n = 2m, m being an integer

f(2m/2) + 1 = C((2m + 1) /2)

F(m) + 1 = C((2m/2) + (1/2))

F(m) = m

m + 1 = C(m + (1/2))

m + 1 = m + 1

For Odds:

Assume n = 2m + 1, m being an integer

F((2m + 1)/2) + 1 = C(((2m + 1) + 1) /2)

F((2m/2) + (1/2) + 1) = C((2m/2) + (2/2))

F(m + (1/2)) + 1 = C(m + 1) = m + 1

m + 1 = m + 1

c.

D(n) = T(n + 1) - T(n)

prove D(1) = 2:

D(1) = T(2) - T(1)

= T(F(1)) + T(C(1)) + 2 - 0
= T(1) + T(1) + 2
= 0 + 0 + 2
= 2

prove D(n) = T(n + 1) - T(n)

= T(F(n+1)/2) + T(C((n + 1)/2)) + n + 1 - (T(F(n/2)) + T(C(n/2)) + n)
= T(F ((n+1)/2))+T(C[(n+1)/2])+n+1- T(F[n/2])-T(C[n/2])-n
= T(F((n/2) + 1)/2)) + T(C((n+1)/2)) - T(F(n/2)) - T(C(n/2)) + 1

D(F(n/2)) + 1 = T(F(n/2) + 1) - T(F(n/2)) + 1

T(F(n/2) + 1) - T(F(n/2)) + 1 = T(F((n+1)/2)) + T(C((n+1)/2)) - T(C(n/2))

= T(F((n+1)/2)) - T(C(n/2))
= T(F((n+1)/2)) = T(C(n/2)
= T(F((n+1)/2)) = T(F((n+1)/2))

d.

D(n) = D(F(n/2)) + 1

$= D(F(n/4)) + 1 + 1$
$= D(F(n/8)) + 1 + 1 + 1$
$= D(F(n/2^k)) + k$
let $n = 2^k$ and $k = F(\lg(n))$
$= D(1) + F(\lg(n))$
$= F(\lg(n)) + 2$

e.

$T(n) - T(1) = \Sigma_{k=1}^{n-1}(F(\lg(k))$

$T(n) = D(1) + D(2) + D(3) + \ldots + D(n-1)$

$T(n) = T(2) - T(1) + T(3) - T(2) + T(4) - T(3) + \ldots + T(n) + T(n-1)$

$T(n) - T(1) = \Sigma_{k=1}^{n} - \Sigma_{k=1}^{n-1}$

$T(n) = T(n)$

because we know that $T(n) = \Sigma_{k=1}^{n-1}D(k)$, as well as $D(n) = F(\lg(n)) + 2$,

$T(n) = \Sigma_{k=1}^{n-1}F(\lg(n)) + 2$

f.

$T(n) = \Sigma_{k=1}^{n-1}F(\lg(n)) + \Sigma_{k=1}^{n-1}(2)$

$= \Sigma_{k=1}^{n-1}F(\lg(n)) + 2(n - 1)$
$= \Sigma_{k=1}^{n-1}F(\lg(n)) + 2n - 2$
$= nF(\lg(n)) - 2^{F(\lg(n))+1} + F(\lg(n)) + 2$
$= 2n + (n-1)\ F(\lg(n-1)) - 2^{F(\lg(n-1))}\ 2 + F(\lg(n-1))$
$= 2n + n\lg(n-1) - 2^{F(\lg(n-1))} * 2$
if we make this big O, nlg(n) will dominate the terms, so we get O(nlg(n))