MATH207 - Ordinary Differential Equations

Manas Sambare

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## Question 1

$\frac{dS}{dt}=−rSI+μ $

$\frac{dI}{dt}=rSI−kI $

**a) Do a qualitative analysis of the model given there and show that the solutions are not periodic.**

**b) Modify the equations to incorporate the changes as prescribed. Do a qualitative analysis to show that solutions are cyclic.**

Answer:

$\frac{dS}{dt}=−rSI+Sμ $

$\frac{dI}{dt}=rSI−kI $

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## Question 2

Let  $x=x\left(t\right)$ be the number of thousands of animal species *A* at time *t.*

Let  $y=y\left(t\right)$ be the number of thousands of animal species *B* at time *t.*$$

$$Suppose\left\{\begin{matrix}\frac{dx}{dt}=x−0.5xy\\\frac{dy}{dt}=y−0.5xy\end{matrix}\right.$$

**(a) Is the interaction between species** ***A*** **and** ***B*** **symbiotic, competitive, or a predator-prey relationship?**

Answer: Competitive

**(b) What are the equilibrium populations?**

Answer:

$\frac{dx}{dt}=0 when x=0 and y=2$ $$

$\frac{dy}{dt}=0 when x=2 and y=0$

Coordinates:

$\frac{dx}{dt}= \frac{dy}{dt}=0$

 $when:  x=0  \rightarrow  \left(0,0\right)         y=2 \rightarrow  \left(2,2\right)$

$∴ equilibrium populations \left\{\begin{matrix}x=0 and y=0\\x=2 and y=2\end{matrix}\right.$

**(e) If**$x=0$, **what happens to**$y\left(t\right)$**? How is this indicated in the phase-plane? If**$y=0$**, what happens to**$x\left(t\right)$**? How is this indicated in the phase-plane?**

Answer:

As $x=0$, $\frac{dy}{dt}$ increases, which means that $y$ increases.

As $y=0$, $\frac{dx}{dt}$ increases, which means that $x$ increases.

**(g) For each of the initial conditions given below, describe how the number of species of *A*and *B*change with time and what the situations will look like in the long run.**

**i)**$x(0)=2 and y(0)=1.8$

Answer:  Population of *A* will increase over time and the population of *B*will decrease since it is a competitive environment. In the long run, the population of *B* would tend to zero.

**ii)**$x(0)=2 and y(0)=2.3$

Answer:  Population of *A* will decrease over time and the population of *B*will increase since it is a competitive environment. In the long run, the population of *A* would tend to zero.

**iii)**$x(0)=2.2 and y(0)=2 $

Answer:  Population of *B* will decrease over time and the population of *A*will increase since it is a competitive environment. In the long run, the population of *B* would tend to zero.

## Question 3

$$Suppose\left\{\begin{matrix}\frac{dx}{dt}=0.03x−0.001x^{2}−0.01xy\\\frac{dy}{dt}=0.05y−0.001y^{2}−0.01xy\end{matrix}\right.$$

Answer:

$\frac{dx}{dt}=0 when x=0 and y=3−x$

$\frac{dy}{dt}=0 when x=5−y and y=0$

The population will tend to the equilibrium point $\left(0,5\right)$, this means that in the long run the population of $A$ will tend to zero and the population of $B$ will tend to $5000$.