Title

Math Solutions Consulting

4.

a.

$2log\_{5}(x+1)−log\_{5}(x−1)=1$
$−\frac{log(x−1)−2log(x+1)}{log(5)}=1$
$log(x−1)−2log(x+1)=−log(5)$
$log(x−1)−2log(x+1)=log(x−1)+log\left(\frac{1}{(x+1)^{2}}\right)=log\left(\frac{x−1}{(x+1)^{2}}\right)$
$log\left(\frac{x−1}{(x+1)^{2}}\right)=−log(5)$
$log\left(\frac{x−1}{(x+1)^{2}}\right)=log\left(\frac{1}{5}\right)$
$log\left(\frac{x−1}{(x+1)^{2}}\right)=log\left(\frac{1}{5}\right)$
$5(x−1)=(x+1)^{2}$
$5x−5=(x+1)^{2}$
$5x−5=x^{2}+2x+1$
$−x^{2}+3x−6=0$
$x^{2}−3x+6=0$
$x^{2}−3x=−6$
Sumar $\frac{9}{4}$ en ambos lados: $x^{2}−3x+\frac{9}{4}=−\frac{15}{4}$
$\left(x−\frac{3}{2}\right)^{2}=−\frac{15}{4}$
$x−\frac{3}{2}=\frac{i\sqrt{15}}{2}$ or $x−\frac{3}{2}=−\frac{i\sqrt{15}}{2}$
$x=\frac{3}{2}+\frac{i\sqrt{15}}{2}$ or $x=\frac{3}{2}−\frac{i\sqrt{15}}{2}$

b.

$log\_{2}(x)+log\_{4}(x)+log\_{16}(x)=7$
$\frac{log(x)}{log(2)}+\frac{log(x)}{log(4)}+\frac{log(x)}{log(16)}=7$
$\frac{(log(2)log(4)+log(2)log(16)+log(4)log(16))log(x)}{log(2)log(4)log(16)}=7$
$log(x)=\frac{7log(2)log(4)log(16)}{log(2)log(4)+log(2)log(16)+log(4)log(16)}$
$x=exp\left(\frac{7log(2)log(4)log(16)}{log(2)log(4)+log(2)log(16)+log(4)log(16)}\right)$

c.

$3log(x)−log(x)−log(9)=0$
$2log(x)−log(9)=0$
$2log(x)=log(9)$
$log(x)=\frac{log(9)}{2}$
Dado que $\frac{log(9)}{2}=log(\sqrt{9})=log(3)$
$log(x)=log(3)$
$x=3$

d.

$log\_{6}2x−log\_{6}(x+1)=0$
$\frac{log(2x)}{log(6)}−\frac{log(x+1)}{log(6)}=0$
$\frac{log(2x)−log(x+1)}{log(6)}=0$
$log(2x)−log(x+1)=0$
$log\left(\frac{2x}{x+1}\right)=0$
$\frac{2x}{x+1}=1$
$2x=x+1$
$x=1$

5.

a.

$6^{2x−1}=10$
$2x−1=\frac{log(10)}{log(6)}$
$2x=1+\frac{log(10)}{log(6)}$
$x=\frac{1}{2}+\frac{log(10)}{2log(6)}$
$x≈1.1425$

c.

$2^{(}x+1)+2^{(}x+2)+2^{(}x+3)+2^{(}x+4)=1$
$\begin{matrix} Simplificando y sustituyendo y&=2^{x}\\2^{x+1}+2^{x+2}+2^{x+3}+2^{x+4}&=30×2^{x}\\&=30y\\30y=1\end{matrix}$
$y=\frac{1}{30}$
$2^{x}=\frac{1}{30}$
$x=−\frac{log(30)}{log(2)}$
$x≈−4.9069$

8-10.

a.

$lim\_{x\rightarrow \infty }\frac{3x^{5}+5x^{3}+12}{x^{5}−8x^{4}+6x}$
Dividendo los sumandos de $\frac{3x^{5}+5x^{3}+12}{x^{5}−8x^{4}+6x}$ por $x^{5}$.
$lim\_{x\rightarrow \infty }\frac{3+\frac{5}{x^{2}}+\frac{12}{x^{5}}}{1−\frac{8}{x}+\frac{6}{x^{4}}}$
Las expresiones $\frac{12}{x^{5}},\frac{5}{x^{2}},\frac{6}{x^{4}}$ y $−\frac{8}{x}$ tienden a cero cuando $x$ tiende a infinito.

b.

$lim\_{x\rightarrow 1}\frac{2x^{2}−x−1}{x^{2}+2x−3}$
$lim\_{x\rightarrow 1}\frac{(x−1)(2x+1)}{(x−1)(x+3)}$
$lim\_{x\rightarrow 1}\frac{2x+1}{x+3}$
$lim\_{x\rightarrow 1}\frac{2x+1}{x+3}=\frac{2⋅1+1}{1+3}=\frac{3}{4}$

$lim\_{x\rightarrow 0}\frac{tanx}{x}$
$lim\_{x\rightarrow 0}\frac{sin(x)}{xcos(x)}$
$lim\_{x\rightarrow 0}\frac{sin(x)}{xcos(x)}=\left(lim\_{x\rightarrow 0}\frac{sin(x)}{x}\right)\left(lim\_{x\rightarrow 0}\frac{1}{cos(x)}\right)$
$lim\_{x\rightarrow 0}\frac{1}{cos(x)}=\frac{1}{cos(0)}=1$ $lim\_{x\rightarrow 0}\frac{sin(x)}{x}$
Aplicando regla de L’Hôpital obtenemos:
$lim\_{x\rightarrow 0}\frac{sin(x)}{x}=lim\_{x\rightarrow 0}\frac{\frac{d}{dx}sin(x)}{\frac{dx}{dx}}$
$=lim\_{x\rightarrow 0}\frac{cos(x)}{1}$
$=lim\_{x\rightarrow 0}cos(x)$
$lim\_{x\rightarrow 0}cos(x)=cos(0)=1$