Title

Math Solutions Consulting

1. Solución:

$\frac{dy(x)}{dx}+3x^{2}y(x)=x^{2}$
$\frac{dy(x)}{dx}=x^{2}−3x^{2}y(x)$
$\frac{dy(x)}{dx}=x^{2}(−3y(x)+1)$
$\frac{\frac{dy(x)}{dx}}{−3y(x)+1}=x^{2}$
$∫\frac{\frac{dy(x)}{dx}}{−3y(x)+1}dx=∫x^{2}dx$
$−\frac{1}{3}log(−3y(x)+1)=\frac{x^{3}}{3}+c\_{1}$
$y(x)=−\frac{1}{3}e^{−x^{3}−3c\_{1}}+\frac{1}{3}$
$y(x)=c\_{1}e^{−x^{3}}+\frac{1}{3}$

2. Solución:

$x\frac{dy(x)}{dx}−y(x)=x^{2}sin(x)$
$\frac{dy(x)}{dx}−\frac{y(x)}{x}=xsin(x)$
Sea $μ(x)=e^{∫−1/xdx}=\frac{1}{x}$
Multiplicando ambos lados por $μ(x)$ : $\frac{dy(x)}{dx}−\frac{y(x)}{x^{2}}=sin(x)$
Sustituyendo $−\frac{1}{x^{2}}=\frac{d}{dx}\left(\frac{1}{x}\right)$: $\frac{\frac{dy(x)}{dx}}{x}+\frac{d}{dx}\left(\frac{1}{x}\right)y(x)=sin(x)$
Aplicando la regla del producto de forma inversa $f\frac{dg}{dx}+g\frac{df}{dx}=\frac{d}{dx}(fg)$ en el lado izquierdo:
$\frac{d}{dx}\left(\frac{y(x)}{x}\right)=sin(x)$
$∫\frac{d}{dx}\left(\frac{y(x)}{x}\right)dx=∫sin(x)dx$
$\frac{y(x)}{x}=−cos(x)+c\_{1}$
$y(x)=x\left(−cos(x)+c\_{1}\right)$

3. Solución:

$x^{2}\frac{dy(x)}{dx}+x(x+2)y(x)=e^{x}$
$\frac{dy(x)}{dx}+\frac{(x+2)y(x)}{x}=\frac{e^{x}}{x^{2}}$
Sea $μ(x)=e^{∫(x+2)/xdx}=e^{x}x^{2}$
Multiplicando ambos lados por $μ(x)$ : $\left(e^{x}x^{2}\right)\frac{dy(x)}{dx}+\left(e^{x}x(x+2)\right)y(x)=e^{2x}$
Sustituyendo $e^{x}x(x+2)=\frac{d}{dx}\left(e^{x}x^{2}\right)$ $\left(e^{x}x^{2}\right)\frac{dy(x)}{dx}+\frac{d}{dx}\left(e^{x}x^{2}\right)y(x)=e^{2x}$
$\frac{d}{dx}\left(\left(e^{x}x^{2}\right)y(x)\right)=e^{2x}$
$∫\frac{d}{dx}\left(\left(e^{x}x^{2}\right)y(x)\right)dx=∫e^{2x}dx$
$\left(e^{x}x^{2}\right)y(x)=\frac{e^{2x}}{2}+c\_{1}$
$y(x)=\frac{e^{x}+2c\_{1}e^{−x}}{2x^{2}}$
$y(x)=\frac{e^{x}}{2x^{2}}+\frac{c\_{1}e^{−x}}{x^{2}}$

4. Solución:

$x\frac{dy(x)}{dx}+(x+1)y(x)=e^{−x}sin(2x)$
$\frac{dy(x)}{dx}+\frac{(x+1)y(x)}{x}=\frac{e^{−x}sin(2x)}{x}$
Sea $μ(x)=e^{∫(x+1)/xdx}=e^{x}x$
Multiplicando ambos lados por $μ(x)$ : $\left(e^{x}x\right)\frac{dy(x)}{dx}+\left(e^{x}(x+1)\right)y(x)=sin(2x)$
Sustituyendo $e^{x}(x+1)=\frac{d}{dx}\left(e^{x}x\right)$ $\left(e^{x}x\right)\frac{dy(x)}{dx}+\frac{d}{dx}\left(e^{x}x\right)y(x)=sin(2x)$
$\frac{d}{dx}\left(\left(e^{x}x\right)y(x)\right)=sin(2x)$
$∫\frac{d}{dx}\left(\left(e^{x}x\right)y(x)\right)dx=∫sin(2x)dx$
$\left(e^{x}x\right)y(x)=−\frac{1}{2}cos(2x)+c\_{1}$
$y(x)=\frac{e^{−x}\left(−\frac{1}{2}cos(2x)+c\_{1}\right)}{x}$

5. Solución:

$cos(x)\frac{dy(x)}{dx}+sin(x)y(x)=1$
$\frac{dy(x)}{dx}+tan(x)y(x)=sec(x)$
Sea $μ(x)=e^{∫tan(x)dx}=sec(x)$
Multiplicando ambos lados por $μ(x)$ $sec(x)\frac{dy(x)}{dx}+(sec(x)tan(x))y(x)=sec^{2}(x)$
Sustituyendo $tan(x)sec(x)=\frac{dsec(x)}{dx}$ $sec(x)\frac{dy(x)}{dx}+\frac{dsec(x)}{dx}y(x)=sec^{2}(x)$ $\frac{d}{dx}(sec(x)y(x))=sec^{2}(x)$
$∫\frac{d}{dx}(sec(x)y(x))dx=∫sec^{2}(x)dx$
$sec(x)y(x)=c\_{1}+tan(x)$
$y(x)=sin(x)+c\_{1}cos(x)$

6. Solución:

$(x+1)\frac{dy}{dx}+(x+2)y=2xe^{−x}$
$\frac{dy(x)}{dx}(x+1)+(x+2)y(x)=2e^{−x}x$
$\frac{dy(x)}{dx}+\frac{(x+2)y(x)}{x+1}=\frac{2e^{−x}x}{x+1}$
Let $μ(x)=e^{∫\frac{(x+2)}{(x+1)}dx}=e^{x}(x+1)$
Multiplicando ambos lados por $μ(x)$ $\left(e^{x}(x+1)\right)\frac{dy(x)}{dx}+\left(e^{x}(x+2)\right)y(x)=2x$
Sustituyendo $e^{x}(x+2)=\frac{d}{dx}\left(e^{x}(x+1)\right)$ $\left(e^{x}(x+1)\right)\frac{dy(x)}{dx}+\frac{d}{dx}\left(e^{x}(x+1)\right)y(x)=2x$
$\frac{d}{dx}\left(\left(e^{x}(x+1)\right)y(x)\right)=2x$
$∫\frac{d}{dx}\left(\left(e^{x}(x+1)\right)y(x)\right)dx=∫2xdx$
$\left(e^{x}(x+1)\right)y(x)=x^{2}+c\_{1}$
$y(x)=\frac{e^{−x}\left(x^{2}+c\_{1}\right)}{x+1}$

7. Solución:

$cos^{2}(x)\frac{dy(x)}{dx}sin(x)+cos^{3}(x)y(x)=1$
$\frac{dy(x)}{dx}+cot(x)y(x)=csc(x)sec^{2}(x)$
Sea $μ(x)=e^{∫cot(x)dx}=sin(x)$
Multiplicando ambos lados por $μ(x)$ $sin(x)\frac{dy(x)}{dx}+cos(x)y(x)=sec^{2}(x)$
Sustituyendo $cos(x)=\frac{dsin(x)}{dx}$ $sin(x)\frac{dy(x)}{dx}+\frac{dsin(x)}{dx}y(x)=sec^{2}(x)$
$\frac{d}{dx}(sin(x)y(x))=sec^{2}(x)$
$∫\frac{d}{dx}(sin(x)y(x))dx=∫sec^{2}(x)dx$
$sin(x)y(x)=c\_{1}+tan(x)$
$y(x)=csc(x)\left(c\_{1}+tan(x)\right)$

8. Solución:

$(x+2)^{2}\frac{dy(x)}{dx}=−8y(x)−4xy(x)+5$
$\frac{dy(x)}{dx}+\frac{4y(x)}{x+2}=\frac{5}{(x+2)^{2}}$
Sea $μ(x)=e^{∫4/(x+2)dx}=(x+2)^{4}$
Multiplicando ambos lados por $μ(x)$ : $(x+2)^{4}\frac{dy(x)}{dx}+\left(4(x+2)^{3}\right)y(x)=5(x+2)^{2}$
Sustituyendo $4(x+2)^{3}=\frac{d}{dx}\left((x+2)^{4}\right)$ $(x+2)^{4}\frac{dy(x)}{dx}+\frac{d}{dx}\left((x+2)^{4}\right)y(x)=5(x+2)^{2}$
$\frac{d}{dx}\left((x+2)^{4}y(x)\right)=5(x+2)^{2}$
$∫\frac{d}{dx}\left((x+2)^{4}y(x)\right)dx=∫5(x+2)^{2}dx$
$(x+2)^{4}y(x)=\frac{5x^{3}}{3}+10x^{2}+20x+c\_{1}$
$y(x)=\frac{\frac{5x^{3}}{3}+10x^{2}+20x+c\_{1}}{(x+2)^{4}}$

9. Solución:

$\frac{dy(x)}{dx}\left(x^{2}−1\right)+2y(x)=(x+1)^{2}$
$\frac{dy(x)}{dx}+\frac{2y(x)}{x^{2}−1}=\frac{(x+1)^{2}}{x^{2}−1}$
Sea $μ(x)=e^{∫2/\left(x^{2}−1\right)dx}=\frac{−x+1}{x+1}$
Multiplicando ambos lados por $μ(x)$ : $\frac{(−x+1)\frac{dy(x)}{dx}}{x+1}+\frac{(2(−x+1))y(x)}{(x+1)\left(x^{2}−1\right)}=\frac{(−x+1)(x+1)}{x^{2}−1}$
Sustituyendo $\frac{2(−x+1)}{(x+1)\left(x^{2}−1\right)}=\frac{d}{dx}\left(\frac{−x+1}{x+1}\right)$ $\frac{(−x+1)\frac{dy(x)}{dx}}{x+1}+\frac{d}{dx}\left(\frac{−x+1}{x+1}\right)y(x)=\frac{(−x+1)(x+1)}{x^{2}−1}$
$\frac{d}{dx}\left(\frac{(−x+1)y(x)}{x+1}\right)=\frac{(−x+1)(x+1)}{x^{2}−1}$
$∫\frac{d}{dx}\left(\frac{(−x+1)y(x)}{x+1}\right)dx=∫\frac{(−x+1)(x+1)}{x^{2}−1}dx$
$\frac{(−x+1)y(x)}{x+1}=−x+c\_{1}$
$y(x)=\frac{(x+1)\left(x−c\_{1}\right)}{x−1}$
$y(x)=\frac{(x+1)\left(x+c\_{1}\right)}{x−1}$

10. Solución:

$\frac{dp(t)}{dt}+2tp(t)=4t+p(t)−2$
$\frac{dp(t)}{dt}=4t+p(t)−2tp(t)−2$
$\frac{dp(t)}{dt}=(2t−1)(−p(t)+2)$
$\frac{\frac{dp(t)}{dt}}{−p(t)+2}=2t−1$
$∫\frac{\frac{dpt}{dt}}{−p(t)+2}dt=∫(2t−1)dt$
$−log(−p(t)+2)=t^{2}−t+c\_{1}$
$p(t)=−e^{−t^{2}+t−c\_{1}}+2$
$p(t)=c\_{1}e^{−t^{2}+t}+2$