Material Sensitivity to Mean Stress: A Review of the Walker Fatigue Life Prediction Parameter and a Modified SWT Fatigue Life Prediction Parameter

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Abstract

Accurate fatigue life predictions are important when designing components subject to repeated loading cycles. The Walker model is a popular choice for predicting fatigue life during cyclic loading with non-zero mean stress effects. The Walker model suggests that sensitivity to non-zero mean stress is dependent on material. This work serves as an overview of the Walker fatigue life model, for which a life prediction parameter is derived from the Walker equivalent stress. In addition, this work analyzes an alternative fatigue life prediction model that considers material sensitivity to mean stress addition: a modified SWT parameter proposed by Lv *et al*. Both the Walker model and the modified SWT model were utilized for existing data and compared to the original SWT model. Both the Walker model and the modified SWT model provided equivalent or better life predictions than the SWT model for non-zero mean stress fatigue data analyzed.

Introduction

Engineering components are commonly subjected to cyclic loadings that induce stresses with a mean offset, as shown in Fig 1.

Fatigue life is known to be dependent on stress amplitude, σ_a, and the ratio of minimum to maximum stress, $R = \sigma_{min}/\sigma_{max}$. Several models exist for making fatigue life predictions with non-zero mean stress including Goodman, Morrow, SWT, Manson-Halford, Soderberg, and Gerber. These models assume that for a given cyclic loading with $R \neq -1$ (non-zero mean stress), there exists an equivalent fully reversed stress amplitude, σ_{eq} ($R = -1$), that results in the same life. For instance:

$$Goodman : \sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} \tag{1}$$

$$Morrow : \sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}} \tag{2}$$

$$SWT : \sigma_{ar} = \sqrt{\sigma_a \sigma_{max}} \tag{3}$$

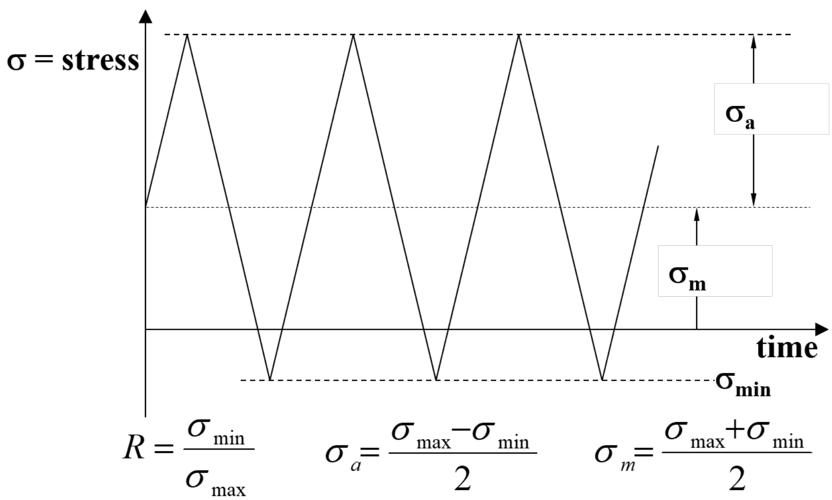

Figure 1: Mean Stress [1]

These equivalent fully reversed stresses can be used in determining fatigue life prediction parameters.

The Walker Equivalent Stress

Walker postulated the relationship

$$\sigma_{eq} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \tag{4}$$

where the Walker parameter, γ, is a material property describing the mean stress sensitivity. γ ranges from 0 to 1 with lower values corresponding to high sensitivity to mean stress and larger values of γ corresponding to low sensitivity to mean stress. It will be seen that higher reltative ultimate strength leads to a reduction in γ. For $\gamma = 0.5$, the model reduces to the familiar Smith-Watson-Topper relationship, $\sigma_{eq} = \sigma_a^{0.5}\sigma_{max}^{0.5}$. A constant life plot of σ_a vs σ_m is shown in Fig 2. From the constant life plots, it is observed that materials with higher values of γ can endure greater stress amplitudes at higher mean stresses than materials with lower values of γ.

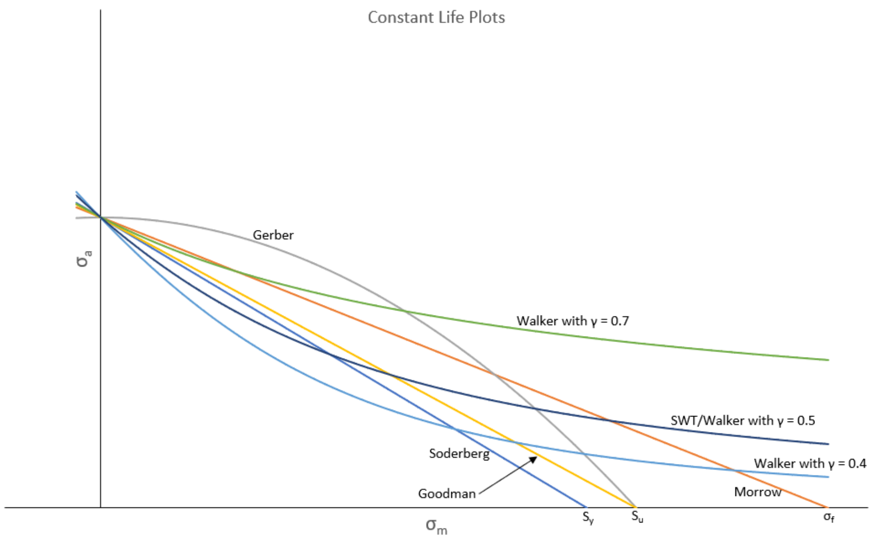

Figure 2: Constant life predictions for various mean stress models.

Whereas the previous models discussed are typically fit using only data from zero mean stress tests, the Walker parameter is typically fit using zero mean stress and non-zero mean stress data. While this requires additional testing, the resulting model may in theory yield more accurate predictions for non-zero mean stress fatigue life predictions.

Derivation of Walker Strain-Life Parameter

In order to make fatigue life predictions for nonzero mean stress applications using the Walker model, a strain life parameter must be derived. Dowling details this derivation for the Walker model, as well as derivation of parameters for other models [2]. For the purpose of seeing where the life prediction life prediction parameter comes from, as well as for a means of understanding how to derive strain life parameters for other models, the derivation is included here.

The Manson Coffin equation describes strain life behavior for zero mean stress cyclic loading

$$\epsilon_a = \frac{\sigma'_f}{E}(2N)^b + \epsilon'_f(2N)^c \tag{5}$$

where σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ϵ_f is the fatigue ductility coefficient, c is the fatigue ductility exponent, and E is the Modulus of Elasticity.

Since this equation applies to the strain life behavior of zero mean stress fatigue loading, a form that accounts for mean stress must be formulated. In the above equation, the elastic strain comes from the first term on the right-hand side. This means that the (fully reversed) stress amplitude can be written as

$$\sigma_{ar} = \sigma'_f(2N)^b \tag{6}$$

In addition to the form given in earlier, the Walker equivalent stress can also be written as

$$\sigma_{eq} = \sigma_{ar} = \sigma_a(\frac{2}{1-R})^{1-\gamma} \tag{7}$$

Next, the right-hand side of Eq.(6) can be equated to the right-hand side of Eq.(7), solved for σ_a, and an equivalent life N^* can be defined

$$\sigma_a = {\sigma'}_f(2N)^b(\frac{1-R}{2})^{1-\gamma} = {\sigma'}_f(2N^*)^b \tag{8}$$

Solving for N^* gives

$$N^* = N(\frac{1-R}{2})^{(1-\gamma)/b} \tag{9}$$

Substituting N^* into Eq.(5) results in the Walker strain life parameter that can be used to make life predictions.

$$\epsilon_a = \frac{{\sigma'}_f}{E}(\frac{1-R}{2})^{1-\gamma}(2N)^b + {\epsilon'}_f(\frac{1-R}{2})^{c(1-\gamma)/b}(2N)^c \tag{10}$$

This same derivation technique can be used for other models as well. Doing so gives the familiar Morrow and Manson-Halford strain life parameters.

Predicting the Walker parameter

Because non-mean stress fatigue data is frequently unavailable, attempts to approximate the Walker parameter have been studied.[3] presents a method for approximating γ for steels

$$\gamma = -0.0002\sigma_u + 0.8818 \tag{11}$$

Dowling ran a number of non-zero mean stress fatigue tests for several different types of steel specimens. He plotted the experimental lives against predicted lives he determined using the Walker strain life parameter. For each type of steel tested, γ was adjusted so that the predictions best matched the experimental lives. These different γ values were plotted against the ultimate strengths of the different steels they corresponded to, and Dowling performed linear regression to obtain Eq.(11)

Although γ can be predicted from Eq.(11), it is worth noting that Dowling suggests simultaneous optimization of γ, b, ${\sigma'}_f$, ${\epsilon'}_f$, and c.[3] summarizes this optimization procedure and reports corresponding γ, ${\sigma'}_{fw}$, b_w, ${\epsilon'}_{fw}$, and c_w material properties for several metals.

When available for a given material, these 'corrected' material properties can be used with the Walker strain life equation to, in theory, increase the accuracy of predictions. Comparison between Walker life predictions with and without these modified material constants is presented in the analysis section.

The Modified SWT Parameter

A group of researchers suggested an alternative method to account for material sensitivity to mean stress in fatigue life predictions [4]. These researchers also defined a parameter, γ, that is used in their suggested modified SWT strain life parameter model. The researchers referred to this γ as the same Walker parameter discussed above. However, review of their work showed that the Walker equivalent stress is never actually considered in determining γ or the strain life parameter. Thus, the γ suggested by Lv *et al* is fundamentally different than the Walker parameter discussed above. As such, the parameter will from here on be referred to as γ_{Lv} to differentiate from the Walker parameter γ. It is important to note that both parameters are similar in that they are both material properties that describe a material's sensitivity to mean stress, and both range from 0 to 1.

The researchers looked at the familiar SWT strain life parameter.

$$\sigma_{max}\epsilon_a = \frac{{\sigma'_f}^2}{E}(2N)^{2b} + \epsilon'_f(2N)^{b+c} \tag{12}$$

The researchers modified the SWT parameter by multiplying the left-hand side by $2\gamma_{Lv}$. For life predictions in the elastic range

$$2\gamma_{Lv}\sigma_{max}\epsilon_a = \frac{{\sigma'_f}^2}{E}(2N)^{2b} + \epsilon'_f(2N)^{b+c} \tag{13}$$

The Lv equation for equivalent stress can determined by comparison to SWT, who proposed that $\sigma_{eq} = \sqrt{\sigma_{max} * \epsilon_a * E}$ [5]. For strains proportional to stresses (elastic region), the model simplifies to the familiar $\sigma_{eq} = \sqrt{\sigma_{max} * \sigma_a}$ Eq.(3). Noting the similarity between Eq. (12) and (13), the Lv equivalent stress function is determined to be $\sigma_{eq} = \sqrt{2 * \gamma_{Lv} * \sigma_{max} * \epsilon_a * E}$ or for elastic strains

$$\sigma_{eq} = \sqrt{2 * \gamma_{Lv} * \sigma_{max} * \sigma_a} \tag{14}$$

Sensitivity of the Lv model to mean stress is demonstrated in the constant life plot in Fig 3. Note that the Lv predicted equivalent stresses do not intersect the expected σ_a for zero mean stress, but instead are offset by a factor of $\sqrt{2\gamma}$. In fact, life predictions are seen to be identical to SWT predictions multiplied by $\sqrt{2\gamma}$ for any mean stress. Thus, Lv essentially serves as a correction to SWT where $\gamma > 0.5$ for non-conservative SWT predictions, and $\gamma < 0.5$ for conservative SWT life predictions.

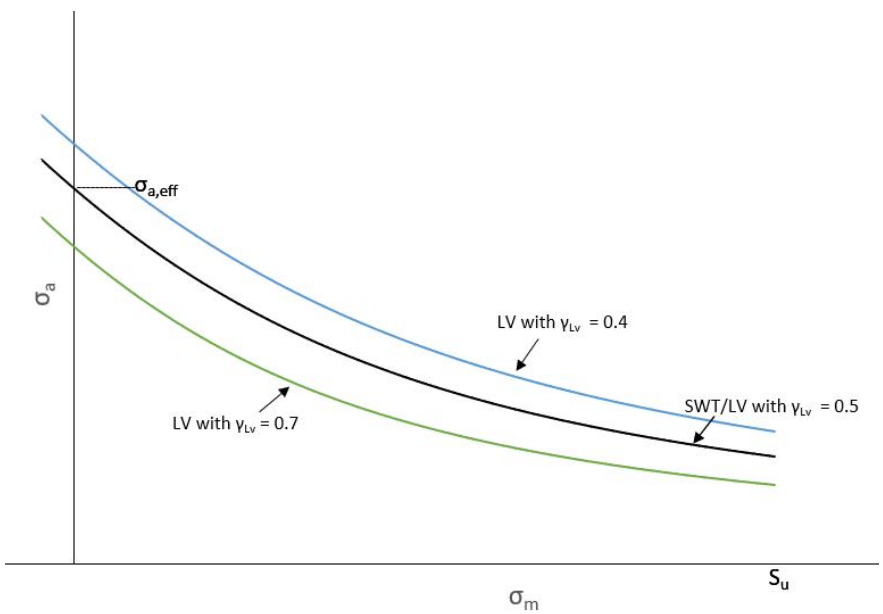

Figure 3: Lv Constant life plot. Sensitivity to mean stress increases with increasing γ_{Lv}

For a materials with lower γ_{Lv} values, the predicted life will increase, while materials with a higher γ_{Lv} values will have lower life predictions. When γ_{Lv} is 0.5, the original SWT parameter will remain. The researchers applied this modified strain life parameter to a number of existing data sets and fit γ_{Lv} values to best fit predicted lives to the experimental lives. Using the γ_{Lv} values from these fits and the material properties for the metals that were analyzed, Lv *et al* proposed the following relationship to predict γ_{Lv}

$$\gamma_{Lv} = 0.5 \pm \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y} \tag{15}$$

The authors did not give a well-defined rule for whether to use + or - in the $\pm$ part of the equation, but they explained that subtraction should be used for materials with higher (relative for a material) ultimate strengths. Analysis of more concrete guidelines for addition or subtraction is included later.

Analysis of Methods

In 1991, Fatemi and Wehner [6] looked at the fatigue behavior of hardened carbon steel with non-zero mean stress. A strain life approach was used to calculate life predictions for the data from this Fatemi paper. These predictions can be compared against the actual test life results. The data that Fatemi gathered are shown in the appendix.

For the Smith-Watson-Topper approach, the strain life parameter Eq.(12) proposed in the original SWT paper was used. Fatemi provided the material properties in his paper, which can be seen in Table 1. Eq. (12) can be solved iteratively for N. Doing so yields the SWT life prediction.

Table 1: Fatemi Data Material Properties

Rockwell C Hardness	HRC	55
Yield Strength	σ_y (MPa)	1731
Ultimate Tensile Strength	σ_u (MPa)	2165
Fatigue Strength Coefficient	σ'_f (MPa)	3372
Fatigue Strength Exponent	b	-0.103
Fatigue Ductility Coefficient	ϵ'_f	0.038
Fatigue Ductility Exponent	c	-0.47
Cyclic Modulus	E (MPa)	200,000

The Walker model was also applied to this data. γ was predicted using the equation that Dowling proposed Eq.(11) with the ultimate strength provided by Fatemi, which gave γ = 0.4488. With γ and the cyclic material properties known, the lives were predicted for the different tests using the Walker strain life parameter derived by Dowling Eq.(10). Dowling suggested simultaneous optimization of cyclic material properties along with γ, presented in Table 2. Predictions for SWT, Walker (with the predicted γ), and optimized-parameter Walker methods are presented in Fig 4.

Table 2: Optimized material properties

σ_{fw}' (MPa)	3762
b_w	-0.1147
ϵ'_{fw}	0.0759
c_w	-0.5951
γ_w	0.4286

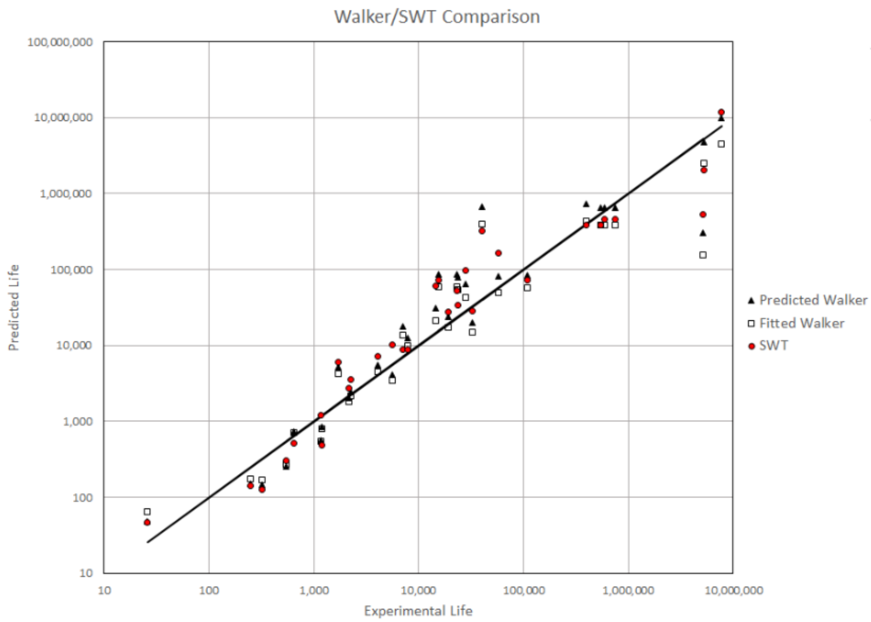

Figure 4: Fatemi data: Walker (Dowling) vs SWT. $\gamma_{predicted} = 0.4488,\ \gamma_{fit} = 0.4286$

Fig 4 shows that the three different predictions are quite similar. Comparing the errors (logarthmically) of the three predictions showed that both Walker predictions performed slightly better than SWT. The fact that they are not substantially different may be explained by the fact that the Walker parameter is close to a value of 0.5, which is the special case of Walker that returns SWT. It is likely that for materials with predicted γ values that are farther from 0.5, the Walker and SWT predictions would vary more from one another.

The optimized Walker prediction tends to be more conservative than the predicted Walker prediction. However, the optimized predictions are not noticeably better than the predictions that utilized the original material properties and the predicted γ. This indicates that using the optimized properties is not critical, and it should not be seen as a concern if the optimized properties are not available for a material for which it is desired to make life predictions using the Walker model.

In addition to applying the Walker model to Fatemi's non-zero mean stress data, the modified SWT parameter proposed by Lv *et al* was applied to Fatemi's data. In attempt to understand the $\pm$ part of the proposed γ_{Lv} prediction equation, predictions were made using both the addition and subtraction γ_{Lv} values.

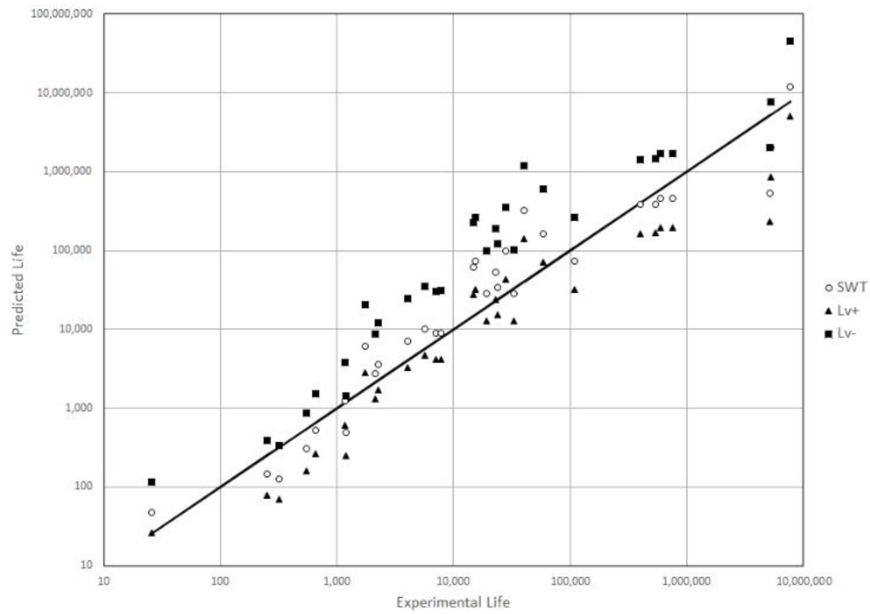

Figure 5: Fatemi Data: Lv vs SWT. $\gamma_{Lv+} = 0.5 + \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y} = 0.6114$, $\gamma_{Lv-} = 0.5 - \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y} = 0.3886$

Fig. 5 shows that an increase in γ_{Lv} causes a downward shift in predicted life. This means that using subtraction in the predictive equation is the conservative option when using the modified SWT model in question. Using addition to predict γ_{Lv} resulted in an over-prediction for nearly all data points in the Fatemi data. Comparing the error for these predictions showed that Lv (addition) performed better than SWT.

For comparison purposes, the data for the Walker prediction and the conservative Lv prediction are plotted on the same axes, as seen in Fig. 6. Both performed better than SWT, but Lv is the more conservative of the two.

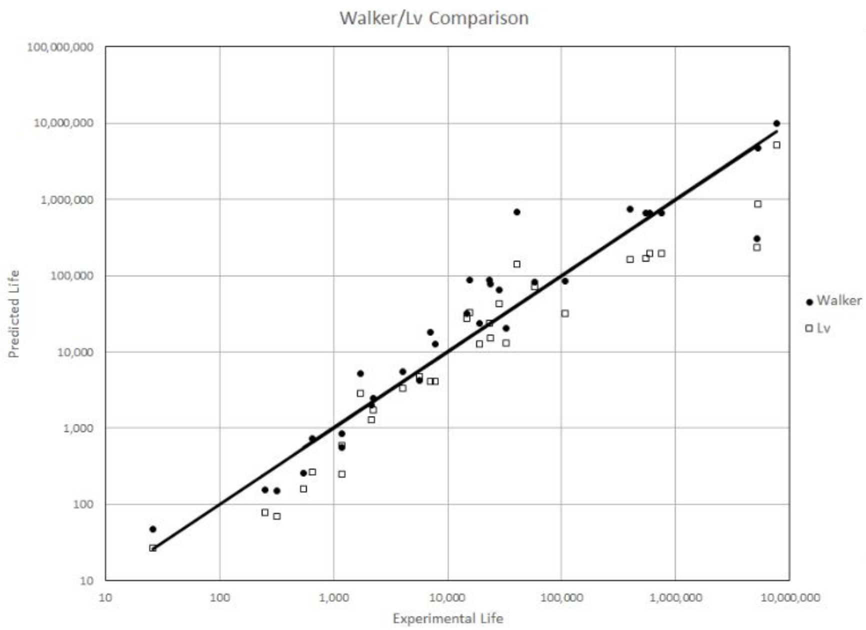

Figure 6: Fatemi Data: Predicted Walker vs Lv+

Lv *et al* also analyzed a number of other non-zero mean stress data sets to develop their model. Their γ_{Lv} predictive equation and modified SWT paramater were applied to these data again, as well as SWT for comparison purposes. The materials analyzed are a steel alloy, a carbon steel, and aluminum. The results are shown in Figs 7, 8, and 9. Note that in Fig 8 the Walker prediction is included as well. This could be done since Dowling's predictive γ equation holds for steels. It does not hold for alloys or aluminum, so only the modified SWT parameter was considered for the other two materials.

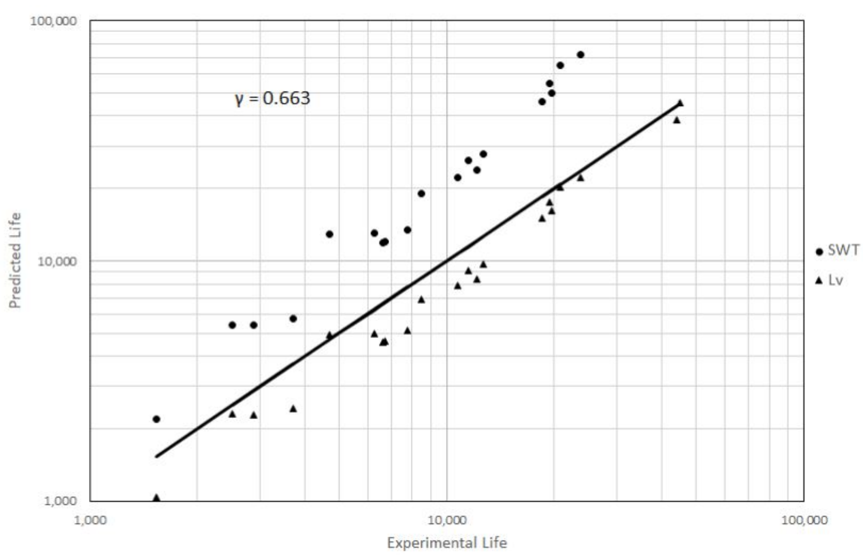

Figure 7: Super Alloy GH4133 (Lv 2016) : SWT vs Lv+

For the GH4133 alloy, the modified SWT predictions performed better than the original SWT parameter predictions. SWT over-predicted for every test, while the modified SWT parameter was conservative for most data points (but much the predictions were much closer to the actual life).

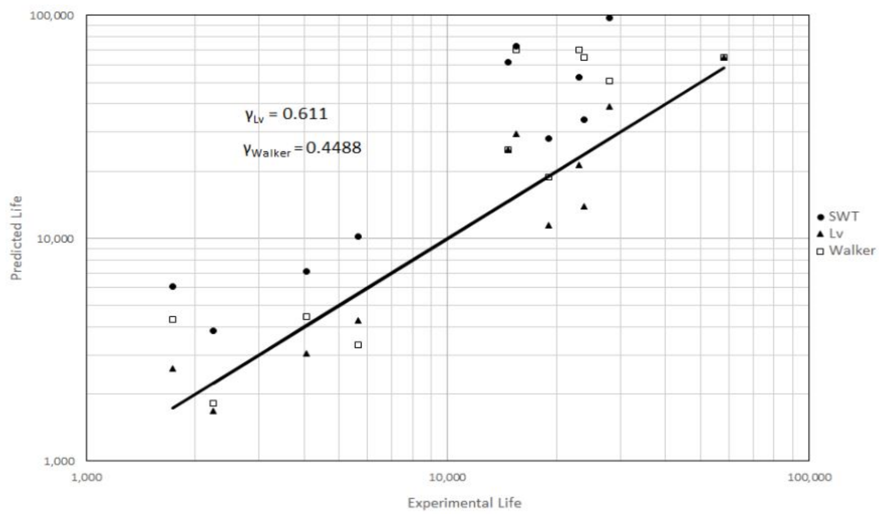

Figure 8: Carbon Steel : Predicted Walker vs Lv+

Again, the Lv approach tended to be the most conservative prediction. There was more scatter in this data, so all three models had predictions that did not compare well to the experimental life for certain data points. Comparing the error showed that the modified SWT approach gave the closest predictions, overall. As noted earlier, the Walker γ and γ_{Lv} are different parameters. This is demonstrated by the fact that the two values vary for the two models.

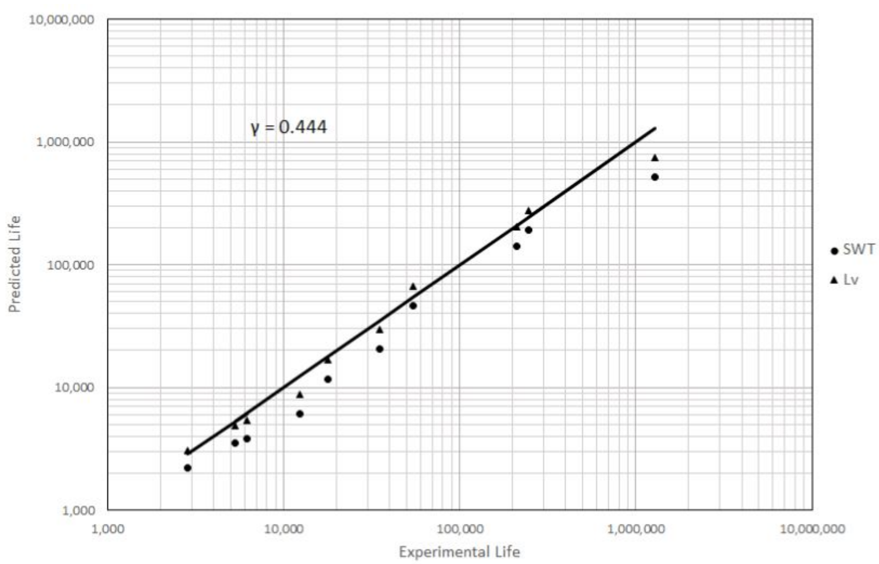

Figure 9: Aluminum: Lv- vs SMT

For the aluminum data, both the SWT predictions and the modified SWT predictions are close to the experimental life values. Yet again, the modified SWT approach predictions were closer to the actual life than the original SWT parameter predictions. However, in this case, the Lv approach was less conservative than SWT, which differs from the other data sets analyzed. This can be explained by the fact that subtraction was used in the Lv prediction equation, as lower values of γ_{Lv} result in higher life predictions.

Lv *et al* applied their suggested model to a number of data sets. In addition to developing a relationship to predict γ_{Lv}, the researchers also fit γ_{Lv} to these data sets. In Table 3, their fitted values are compared to the values predicted by their suggested equation. This was done in an attempt to come up with some threshold that can indicate whether to use the addition or subtraction in their equation.

Table 3: Analysis of $\pm$ relationship for Lv *et al* (σ in MPa)

Materials			σ_y	σ_u	σ'_f	Test fit γ_{Lv}	Calculated γ_{Lv}	+-	% diff
SAE	1015 St		228	415	726	0.7352	0.7908	1	-7.57
GSMnNi63	St		312	501	846	0.8113	0.7325	1	9.72
Ck45	St		531	790	1271	0.6949	0.6961	1	-0.17
SAE	4130 St	Norm	647	799	1144	0.6903	0.6051	1	12.34
CC	450 SS	H1150	678	1015	1360	0.6253	0.6991	1	-11.80
SAE	4130 St	Hard	1200	1241	1586	0.5457	0.5168	1	5.30
CC	450 SS	H900	1354	1405	1750	0.4758	0.4815	-1	-1.20
PH13-8Mo	SS,H1000		1358	1413	1758	0.5969	0.5198	1	12.91
300M	St		1634	1958	2303	0.4157	0.4098	-1	1.42
SAE	1045 St	705HB	1827	2082	2131	0.4839	0.4348	-1	10.15
SAE	1045 St	55HRC	1731	2165	2690	0.4286	0.3886	-1	9.33
6061-T6	Al		276	310	420	0.633	0.5580	1	11.85
Al	Mg4.5Mn	Cld Rl	298	363	476	0.6681	0.5983	1	10.44
2014-T6	Al		438	494	581	0.4803	0.4399	-1	8.41
7075-T6	Al		489	567	729	0.415	0.4261	-1	-2.68
7075-T6	Al		521	572	736	0.4774	0.4533	-1	5.04
Ti-6Al-4V			1006	1034	1271	0.5431	0.5137	1	5.41

As Lv *et al* indicated in their paper, subtraction tends to be used for metals with higher (relative) ultimate strengths. For steels, the switch between using addition or subtraction appears to occur at $\sigma_u \approx 1300$ MPa. However, for predictions that give a value near 0.5, whether addition or subtraction is used is less critical. This is seen for PH13-8Mo stainless steel, for which addition was used despite its higher ultimate strength. One potential advantage to the Lv approach over the Walker approach is that the predictive material sensitivity equation suggested by Dowling for Walker is limited to steels, while there is no such stipulation for Lv.

Precautions

Examination of constant life plots Fig 2 for σ_a vs σ_{mean} shows unreasonably high values of σ_a for mean stresses above σ_y. As with SWT, caution should be used when applying the Walker life predictions in instances with substantial mean stresses.

Conclusions

Walker and Lv fatigue life predictions are presented. Life estimates are compared to existing fatigue life data. Walker performed equal to or better than SWT for both predicted and fit values of γ. Optimization of cyclic material properties produced little difference in Walker life predictions. A modified Smith Watson Topper equation proposed by Lv was discussed and its constant life equation derived. While Lv performed well on the data sets examined, caution is advised when using the Lv model at mean stresses near zero. A threshold for the $\pm$ condition is estimated for steels based on ultimate strength from existing data, with $\gamma_{Lv} = 0.5 + \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y}$ for $\sigma_u < 1300$ MPa and $\gamma_{Lv} = 0.5 - \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y}$ for $\sigma_u > 1300$. γ_{Lv} predictions fit experimental data for different material types, while the γ Walker prediction presented is specific to steels. For materials that behave according to SWT, $\gamma_{Lv} = 0.5$ and $\gamma = 0.5$ (Walker) is expected.

Appendix

Table 4: Fatemi Mean Stress Fatigue Data

ϵ_a (%)	R	σ_a MPa)	σ_m (MPa)	N_f (Cycles)
0.562	-4.12	1111	-677	5,233,360
0.5	-1.94	1056	-337	40,230
0.5	-1.98	1034	-341	399,610
0.6	-1.64	1278	-308	23,860
1	-1.21	1732	-165	650
0.3	-1	561	0	7,707,000
0.4	-1	862	0	542,069
0.4	-1	834	0	590,050
0.4	-1	834	0	750,791
0.5	-1	1058	0	23,133
0.5	-1	986	0	15,492
0.5	-0.99	983	4.6	107,740
0.6	-1	1310	0	7,040
0.6	-0.87	1225	83	7,870
0.7	-1	1226	0	1,727
0.9	-1	1764	0	1,192
1.2	-1	1924	0	320
1.2	-1.01	1866	-12	250
1.5	-1	2083	0	26
0.35	-0.02	677	645	28,090
0.4	-0.07	816	703	19,030
0.4	-0.04	788	724	32,610
0.5	-0.20	989	664	4,070
0.6	-0.33	1151	578	2,150
1	-0.82	1640	165	550
0.2	0.50	404	1205	5,111,600
0.25	0.41	488	1171	58,040
0.3	0.32	581	1123	14,670
0.4	0.22	739	1165	5,640
0.5	-0.03	995	942	2,240
0.6	-0.05	1100	1000	1,180

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