Scientific Paper

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Example $2.23$ The desired final orientation of the hand of a Cartesian-Euler robot is given. Find the necessary Euler angles.

$$\begin{matrix}^{R}T\_{H}=\left[\begin{matrix}n\_{x}&o\_{x}&a\_{x}&p\_{x}\\n\_{y}&o\_{y}&a\_{y}&p\_{y}\\n\_{z}&o\_{z}&a\_{z}&p\_{z}\\0&0&0&1\end{matrix}\right]=\left[\begin{matrix}−0.167&−0.773&−0.612&7\\0.932&0.078&−0.354&2\\0.321&−0.630&0.707&6\\0&0&0&1\end{matrix}\right]\end{matrix}$$

Solution: From these equations, we find:

$$\begin{matrix}ϕ=ATAN2\left(a\_{y},a\_{x}\right)=ATAN2(−0.354,−0.612)=30^{∘} or 210^{∘}\end{matrix}$$

Realizing that both the sines and cosines of $30^{∘}$ and $210^{∘}$ can be used for the remainder,

$$\begin{matrix}\begin{matrix}ψ=ATAN2\left(−n\_{x}Sϕ+n\_{y}Cϕ,−o\_{x}Sϕ+o\_{y}Cϕ\right)=(0.891,0.454)=63^{∘} or 243^{∘}\\θ=ATAN2\left(a\_{x}Cϕ+a\_{y}Sϕ,a\_{z}\right)=ATAN2(−.707,0.707)=−45^{∘} or 135^{∘}\end{matrix}\end{matrix}$$

Unlike Example $2.22$, for a robot of Euler + cylindrical configuration, it will not be easy to find the angles because there are two rotations about the $a$-axis. The angles have to be decoupled individually. In this case, it is probably better to use the Denavit-Hartenberg technique presented in Section $2.12$.

2.10.3 Articulated Joints Articulated joints consist of three rotations other than RPY or Euler. Similar to section $2.9.4$., we develop the matrix representing articulated joints in Section 2.12, when we discuss the Denavit-Hartenberg representation. 2.11 Forward and Inverse Kinematic Equations: Position and Orientation The matrix representing the final location and orientation of the robot is a combination of a positioning choice and an orientation choice. For example, if the robot is Cartesian and RPY, the final position and orientation of the end frame are:

$$\begin{matrix}^{R}T\_{H}=T\_{cart }\left(p\_{x},p\_{y},p\_{z}\right)×RPY\left(ϕ\_{a},ϕ\_{o},ϕ\_{n}\right)\end{matrix}$$

Similarly, for a spherical and Euler robot, we get:

$$\begin{matrix}^{R}T\_{H}=T\_{sph }(r,β,γ)×Euler(ϕ,θ,ψ)\end{matrix}$$

The forward and inverse kinematic solutions for these cases are not developed here, since many different combinations are possible. Instead, in complicated designs, the Denavit-Hartenberg representation is recommended. We will discuss this next. 2.12 Denavit-Hartenberg Representation of Forward Kinematic Equations of Robots In 1955, Denavit and Hartenberg [4] published a paper in the ASME Journal of Applied Mechanics that was later used to represent and model robots and to derive their equations of motion. This technique has become a standard way of representing robots and modeling their motions and, therefore, is essential to learn. Additionally, the method by which frames are assigned and handled can be used in countless other applications.

The Denavit-Hartenberg (D-H) method of representation is a very simple way of modeling robot links and joints of any configuration, regardless of the sequence or complexity. It can also be used to represent transformations in any coordinates we have already discussed, such as Cartesian, cylindrical, spherical, Euler, and RPY. Additionally, it can be used for representation of all-revolute articulated robots, SCARA robots, or any possible combinations of joints and links. Although the direct modeling of robots with the previous techniques is faster and more straight forward, the D-H representation has an added benefit; as we see later, analyses of differential motions and Jacobians, dynamic analysis, force analysis, and others are based on the results obtained from D-H representation [5-9].

Robots may be made of a succession of joints and links in any order. The joints may be either prismatic (linear) or revolute (rotational), move in different planes, and have offsets. The links may also be of any length, including zero, may be twisted and bent, and may be in any plane. We need to be able to model and analyze any robot configuration, whether or not it follows any of the preceding specific coordinates.

To do this, we assign a reference frame to each joint and use a general procedure to transform from one joint to the next (one frame to the next). If we combine all the transformations from the base to the first joint, from the first joint to the second joint, etc., until we get to the last joint, we will have the robot’s total transformation matrix. In the following sections, we define a general procedure based on the D-H representation to assign reference frames to each joint. Then we define how a transformation between any two successive frames may be accomplished. Finally, we write the total transformation matrix for the robot. A robot may be made of a series of links and joints in any form. Figure $2.27$ represents three successive joints and two links between them. Although these joints and links are not necessarily similar to any real robot joint or link, they are very general and can easily represent any joint or link in real robots. The joints may be revolute or prismatic, or both. Although in real robots it is customary to only have 1-DOF joints, the joints in Figure $2.27$ represent 1- or 2-DOF joints.

In Figure $2.27$ a, we assign joint number $n$ to the first joint, $n+1$ to the second joint, and $n+2$ to the third joint. There may be other joints before or after these. Each link is also assigned a link number as shown. Link $n$ will be between joints $n$ and $n+1$, and link $n+1$ is between joints $n+1$ and $n+2$. Although this seems tedious, in practice it is very intuitive and simple.

To model the robot with the D-H representation, we first assign a local reference frame to each and every joint. To each joint, we assign a local $z$-axis and an $x$-axis (although for moving frames, we have used the $n−,o−$, and $a$ - designations, for simplicity we use $x,y$, and $z$ here). We normally do not need to assign a $y$-axis, since we always know that $y$-axes are mutually perpendicular to both $x$ - and $z$-axes. In addition, the Denavit Hartenberg representation does not use the $y$-axis at all. The following is the procedure for assigning a local reference frame to each joint: - All joints, without exception, are represented by a $z$-axis. If the joint is revolute, the $z$-axis is in the direction of rotation as followed by the right-hand rule for rotations. If the joint is prismatic, the $z$ axis is along the direction of the linear movement. In each case, the index number for the $z$-axis of joint $n$ is $n−1$. For example, the $z$-axis representing motions about joint number $n+1$ is $z\_{n}$. These simple rules allow us to quickly assign $z$-axes to all joints. For revolute joints, the rotation about the $z$ axis represented by $(θ)$ is the joint variable. For prismatic joints, the length of the link along the $z$-axis represented by $d$ is the joint variable. - As shown in Figure $2.27$ a, in general, joints may not necessarily be parallel or intersecting. As a result, the $z$-axes may be skew lines. There is always one line mutually perpendicular to any two skew lines, called the common normal, which is the shortest distance between them. We always assign the $x$-axis of the local reference frame in the direction of the common normal between the previous and current axes. Therefore, if $a\_{n}$ represents the common normal between $z\_{n−1}$ and $z\_{n}$, the direction of $x\_{n}$ is along $a\_{n}$. Similarly, if the common normal between $z\_{n}$ and $z\_{n+1}$ is $a\_{n+1}$, the direction of $x\_{n+1}$ will be along $a\_{n+1}$. The common normal lines between successive joints are not necessarily intersecting or collinear. As a result, the origins of two successive frames may also not be at the same location. Based on the above, we can assign coordinate frames to all joints, with the following special cases:

Figure 2.27 The Denavit-Hartenberg representation of a general purpose joint-link combination.

$R\_{p}=\left[\begin{matrix}Cγ&−Sγ&0&0\\Sγ&Cγ&0&0\\0&0&1&0\\0&0&0&1\end{matrix}\right]×\left[\begin{matrix}Cβ&0&Sβ&0\\0&1&0&0\\−Sβ&0&Cβ&0\\0&0&0&1\end{matrix}\right]×\left[\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&r\\0&0&0&1\end{matrix}\right]$ $^{R}T\_{P}=T\_{sph}(r,β,γ)=\left[\begin{matrix}CβCγ&−Sγ&SβCγ&rSβCγ\\CβSγ&Cγ&SβSγ&rSβSγ\\−Sβ&0&Cβ&rCβ\\0&0&0&1\end{matrix}\right]$