A Comparative Study of Linear Programming and Nonlinear Programming Models of the Ship Speed Optimisation Problem in Maritime Transportation

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Abstract

The performance of the widely used nonlinear programming model of the ship speed problem in maritime transportation is compared with the performance of the hardly employed linear programming model, originally developed by Brown et al. (2007). The comparison basis employs a case study, namely that of the SEAFIGHTER patrol boat of the US Navy. Results obtained by the computational implementation of the linear programming and nonlinear programming models in the LINGO demonstrate that the performance of the nonlinear programming model is inferior to that of the linnear programming model.

Keywords

Maritime transportation, ship speed, linear programming, nonlinear programming, model performance, discretisation, regression

1 | Introduction

Ship speed has merited consideration for as long as ships have been employed as the vehicle for maritime transportation, in view of its bearing on fuel costs, and consequently on the competitive position of maritime tranportation vis-a-vis other modes of transportation, such as road and rail. As Lavon and Shneerson[1] have pointed out, ship speed is considered in two distinct time instants of ship lifetime: ship design and ship operation. The role of ship speed in ship design is treated analytically by Jansson and Shneerson[2] in the context of liner shipping. The concern of the work reported in this paper is confined to the role of ship speed in ship operation, and as such, the focus of this paper is on the role that ship speed plays in maritime transportation planning. As stated by Chistiansen et.al.[3], "maritime transportation planning problems can be classified in the traditional manner according to the planning horizon into strategic, tactical and operational problems". In view of the temporal variation of cargo transport demand, port bunker fuel price variation, market competition between ship operators, ship route planning, ship fleet sizing and deployment, and increasing environmental concern with regard to greenhouse gas emissions, the role of the ship speed optimisation problem in maritime transportation planning has been been greatly enhanced. This has resulted in the formulation of several ship speed optimisation problem variants[4,5], as well as the development of a varirty of solution methods[6].

In almost all work on the ship speed optimisation problem in maritime transportation, it is assumed that a ship cruises at a uniform single speed for each leg between an origin – destination port pair within the shipping network under consideration. In view of the well established nonlinear relation between ship fuel consumption rate and ship speed[5,7,8], and the fact that ship speed is the decision variable in existent models, this results in a nonlinear programming model.

A notable exception, which has been curiously ignored, apart from a passing citation by Ronen[9], is that of Brown et al.[10], whereby a ship multiple speed level mix is selected a priori, whilst defining the time spent at each selected speed level as a decision variable, resulting in set of decision variables and a linear programming model. Effectively, in this reformulation of the speed optimisation problem, a linear programming model replaces the traditional nonlinear programming model, which clearly is a significant advantage, both from a mathematical standpoint and a computational point of view. However, there does not exist a quantitative assessment of the results obtained for the same speed optimisation problem variant instance. It is precisely the objective of the work presented in this paper to provide such an assessment.

The rest of this paper is organised as follows. In Section, the ship speed optimisation problem is described. This is followed by presenting the nonlinear programming and the linear programming formulations in Sections 3 and 4, respectively. A case study is presented in Section 5, with a view to serving as a basis for a comparative quantitative assessment of the nonlinear programming and the linear programming formulations. The paper is concluded in Section 5, with a general comparative assessment of the aforesaid formulations, and recommendations for future research.

2 | Problem description

As Psaraftis[12] points out, the term "speed optimisation" depends on the context in which it is employed. In this paper, the definition that is recommended by Psaraftis[11] as the most useful for the purposes of ship optimisation, namely (bold letters are purposely used to emphasise the quotation from Psaraftis[11])

speed optimization can be defined as the selection of an appropriate speed profile so as to optimize a specific objective while meeting various requirements (or constraints) on the ship´s operation. The speeds that correspond to the chosen speed profile are called "optimal speeds",

is the defintion that is employed here, in view of the objective of this paper being a comparative assessment of nonlinear and linear programming models for ship speed optimisation.

In accordance with the classification of maritime transportation planning problems into strategic, tactical, and operational that is presented by Christiansen et al.[3], it is worth noting that ship speed pervades all three levels of planning problems. In strategic planning, ship speed size is considered in the determination of ship hauling capacity. In tactical planning, ship speed lies at the core of ship routing and scheduling. In operational planning, ship speed needs to be determined, so as to meet due weather constraints for arrival at ports as planned along ship routes. In addition to the importance of ship speed in the economics of maritime transportation, in view of the fact that ship fuel consumption rate is dependent on ship speed, environmental considerations, in the way of greenhouse gas emissions that have become an increasingly important consideration in the last few decades, have added another dimension to the role of ship speed in maritime transportation planning problems[12].

Ship speed optimisation has a number of variants, depending on the objective of the problem under consideration, such as ship scheduling, ship routing, ship fleet sizing, and ship fuel bunkering[13,14]. All ship speed optimisation problem variants require in their mathematical formulation, in the objective and or in the constraints, a relation expressing the variation of ship fuel consumption rate (per unit time) with ship speed. This relation may be determined employing the Holtrop-Menen procedure[15,16] (in two steps) to determine the relation between ship fuel consumption rate (per unit time) and ship power propulsion rate (per unit time). From data describing the relation between ship propulsion rate (per unit time) and ship speed, the required variation of ship fuel consumption rate (per unit time) with ship speed is finally obtained. This variation is nonlinear, typically in the form

$$F = a_3 v^3 + a_2 v^2 + a_1 v + a_0, \quad (1)$$

where F and v depict ship fuel consumption rate and ship speed respectively, and a_0, a_1, a_2, and a_3 depict regression parameters, which are obtained by least squares regression. In a more recent study[17], the Holtrop-Mensen procedure has been calibrated using “statistical samples from model tests, sea trials, or computational fluid dynamics results”[17]. An alternative to the data–based Holtrop-Menen method[15,16] is presented by Bialystocki and Konovessis[18], which is based on an algorithm which requires four input parameters of a given ship and voyage, namely, “ship´s draft in the suggested voyage, weather force, weather direction, and date of the fore coming voyage”[18]. For the ship-voyage pair tested, the variation of ship fuel consumption rate (per unit time) is found to be quadratic with ship speed, namely,

$$F = b_2 v^2 + b_1 v + b_0, \quad (2)$$

where b_0, b_1, and b_2 are regression parameters. Psaraftis and Kontovas[4,5] have discussed at length the nature of the variation of ship speed consumption (per unit time) with ship speed, showing its complexity in view of its dependence on many factors, and pointing out[4] that "It is known from basic naval architecture that fuel consumption depends non-linearly on both ship speed and ship payload", and concluding that[11] "the function f can be a complex function which may not even be defined in complex form". It is worth noting that assuming that this function is convex, then the optimal solution is one where ship speed is uniform[19]; however, the convexity assumption may not hold in practice, as for example when different ship fuel consumption rate – ship speed curves are used for different fuels, as is the case in the case study which is presented in Section 5. In almost all work reported on ship speed optimisation problem variants the cubic function is employed to represent the variation of ship fuel consumption rate (per unit time) with ship speed. However, there is one notable exception which has not been used in the ship speed optimisation literature, namely that of Brown et al.[10], whereby, instead of using ship speed as a decision variable, a ship speed level mix is selected a priori and time spent in each ship speed level (which is known) is defined as the decision variable. In the next two Sections, the nonlinear programming and linear programming models, respectively, are formulated for the ship speed optimisation problem variant for a ship voyage between two ports, wherby it is required to minimise ship fuel consumption and having a due arrival date at destination port.

3 | Nonlinear Programming model

The nonlinear programming (NLP) model for the ship speed optimisation problem under variant consideration may be stated as follows.

$$\text{Minimise } F(v)\, L/v\,, \quad (1)$$

$$\text{subject to } L/v \leq D \text{ , (2)}$$

$$F(v)\ L/v \leq T \text{ , (3)}$$

$$v_{min} \leq v \leq v_{max} \text{ , (4)}$$

where D, L, T, v, F(v) depict due arrival date at destination port, origin port pair-destination port pair distance, ship fuel tank capacity, ship speed, and ship fuel rate consumption rate function of ship speed, respectively, and the subscripts min and max denote minimum and maximum values, respectively.

Objective function (1) depicts ship voyage fuel consumption, which is to be minimised. Constraint (2) ensures ship arrival at destination port on or before due date. Constraint (3) ensures that ship voyage fuel consumption does not exceed ship fuel tank capacity. Constraint (4) ensures that ship speed lies between allowed minimum and maximum values, namely that it defines the domain of the decision variable v.

In view of the nonlinearity of the function F, objective function (1) is nonlinear; furthermore, constraint (3) is clearly nonlinear. As a result, the model defined by (1) - (4) is a NLP model, where v is the decision variable.

4 | Linear programming model

The linear programming (LP) model for the problem under consideration may be stated as follows[10].

$$\text{Minimise } \Sigma_{s\in S}\ F_s\ T_s \text{ , (5)}$$

subject to

$$\Sigma_{s\in S}\ v_S\ T_s \geq L \text{ , (6)}$$

$$\sum_{s\in S} T_s \leq D \ , (7)$$

$$\sum_{s\in S} F_s \, T_s \leq T \ , (8)$$

$$T_s \geq 0, \ \forall s\in S \ , (9)$$

where subscript s denotes ship speed level, S is the set of ship speed levels available for selection, F_s denotes ship fuel consumption rate at ship speed level s, and T_s denotes the time spent at ship speed level v_s.

Objective function (5) depicts ship voyage fuel consumption. Constraint (6) ensures ship transit between origin – destination port pair. Constraint (7) ensures ship arrival at destination port on or before due date. Constraint (8) ensures that ship voyage fuel consumption does not exceed ship fuel tank capacity. Constraints (9) ensure that time spent at each ship speed level is nonnegative, namely they define domain of decision variable T_s , $\forall s\in S$.

5 | Case study

In order to make a comparative assessment of the NLP and LP models, a re case study is considered, which is based on a voyage of the SEAFIGHTER patrol boat of the US Navy, and which is described in Brown et al[10]. As stated in Brown et al.[10], SEAFIGHTER employs "mixed-propulsion modes, one for slow motion keeping, and one for high-speed transits", whereby diesel engines and gas turbines are used in slow motion mode (in ship speed range 5 - 20 knots) and fast motion mode (in ship speed range 20 -55 knots), respectively; see Fig.9 in Brown et al[10].

5.1 | Fuel consumption – speed variation

Two ship fuel consumption rate – ship speed curves are obtained by least squares regression, one for slow motion mode and the other for fast motion mode, using points in Fig.9 in Brown et al[10] for discretisation as shown in Tables 1 and 2, respectively.

Table 1 Discretisation of ship speed and ship fuel consumption rate: slow motion mode

Ship speed (knots)	Ship fuel consumption rate (gal/h)
5	0.02

7	0.03
10	0.06
12	0.12
15	0.20
17	0.32
20	0.54

Table 2 Discretisation of ship speed and ship fuel consumption rate: fast motion mode

Ship speed (knots)	Ship fuel consumption rate (gal/h)
20	1.10
22	1.40
25	1.80
27	2.00
30	2.25
32	2.38
35	2.55
38	2.82
40	3.02
42	3.27
45	3.67
48	4.23
50	4.60
55	5.64

Based on data in Tables 1 and 2, least squares regression is used to obtain the slow motion mode and the fast motion mode ship fuel consumption rate – ship speed consumption rate variation curves. The slow mode and fast mode motion curves so obtained are shown in Figs. 1 and 2, respectively, where R is the statistical coefficient of regression.

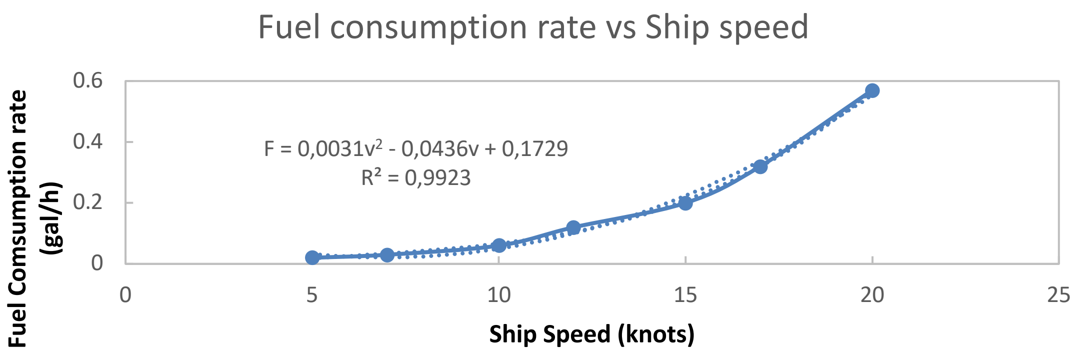

FIGURE 1 | Ship fuel consumption rate – ship speed variation: slow motion mode

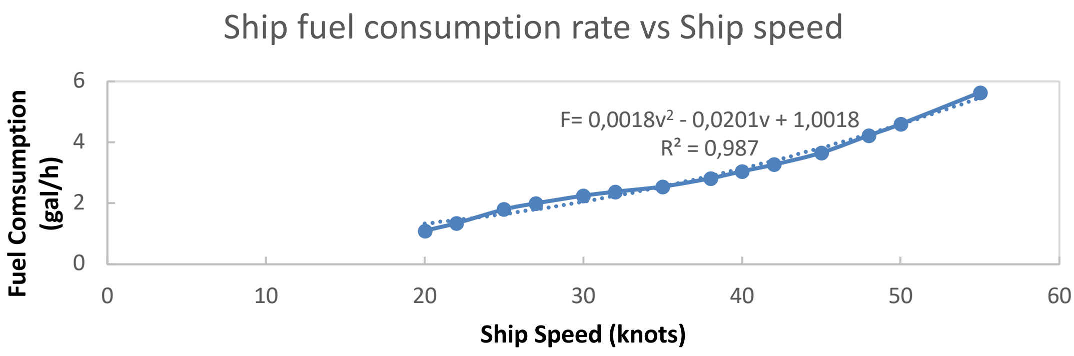

FIGURE 2 | Ship fuel consumption rate – ship speed variation: fast motion mode

5.2 | Voyage description

The voyage that is used as a basis for comparison of the performance of the nonlinear and linear programming models ispresented in Brown et al[10]. The ship is to travel between two ports which are 2,000 nautical miles (NM) apart. Maximum voyage duration allowed is 65 hours. The ship leaves the port of origin with full fuel tank capacity, namely 159.500 gallons (gal).

5.3 | Computational implementation

The NLP and LP models have been computationally implemented employing the LINGO modelling language and optimiser[20]. The results so obtained may be summarised as follows. On the one hand, the global optimal solution of the NLP model yields a ship speed value of 30.76 knots, a voyage duration of 65 hours, and a voyage fuel consumption of 135.681 gallons. On the other hand, the LP model yields a ship speed fuel mix of 20 knots for 37 hours and 45 knots for 28 hours, thus giving voyage duration time of 65 hours; furthermore, the LP model yields a voyage fuel consumption of 123.605 gallons.

5.4 | Analysis of results

From the results presented in Subsection 5.3, it may be observed that the LP model is superior to the NLP model in providing substantially lower voyage fuel consumption whilst voyage duration is the same; furthermore, the weighted ship speed given by the LP model, 30.76 knots, coincides with that given by the NLP model.

6. Conclusions

Using a speed optimisation problem variant instance of the US Navy patrol boat SEAFIGHTER, It has been demonstrated in the work reported in this paper that the linear programming formulation yields better results than the nonlinear programming formulation for the ship speed optimisation problem. Furthermore, it is shown that the uniform single speed assumption does not in fact hold unless ship fuel consumption rate is a convex function of ship speed, as assumed in most work on the ship speed optimisation problem[19].

Therefore, great care must be taken in using the uniform single speed assumption, in view of the complex nature of the functional relation between ship fuel consumption rate and ship speed, as pointed out by Psarftis and Kontovas[4].

In view of the findings summarised in the first paragraph above of this section, the linear programming formulation of the ship speed optimisation problem holds many promises that worthy of exploring in further research. These include complex functional relations between ship fuel consumption and ship speed to take into account dependence on ship payload and weather conditions encountered in ship voyages, ship routing in both liner and tramp shipping, and ship fuel bunkering.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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