# An Innovative Iterative Method to Derive Forced Convection Correlations from the Lowest Number of Generated Data

Ramin Jalali[1] and Masoud Darbandi[2]
*Sharif University of Technology, P.O. Box 11365-8639, Tehran, Iran*

The turbulent flow incorporated with forced convection heat transfer is considered as a complex phenomenon and it is hard to predict analytically. Evidently, empirical correlations and numerical simulations regard as the most suitable approaches to estimate the turbulent flow integrated with forced convection heat transfer.

The main objective of this study is to derive the DittusBoelter equation (an equation used to find the heat transfer coefficient for turbulent flow through pipes) unexperimentally using the minimum number of numerical trial. This paper uses the numerical simulation data and generate novel random data to reach the dittusBoelter relation; generating minimum data.

**Nomenclature**

| | | |
|---|---|---|
| $u_i$ | = | velocity components |
| $Nu$ | = | Nusselt number |
| Re | = | Reynolds number |
| Pr | = | Prandtl number |
| $h$ | = | convective heat transfer coefficient |
| $D$ | = | internal pipe diameter |
| $k$ | = | thermal conduction coefficient |
| $\rho$ | = | density |
| $\mu$ | = | viscousity |

## 1. Numerical approach and assumptions

In order to find the convective heat transfer coefficient, some assumptions applied for the energy balance which is outlined as below 1- Mass flow rate is constant. 2- No shaft work is done by the fluid. 3- Constant heat flux is applied at the surface.

Applying the 1st law of thermodynamics to the system, the energy equation can be written as below:

$$Q = mC_P(T_{out} - T_{in}) \tag{1}$$

[1] Ph.D. Student, Department of Aerospace Engineering.
[2] Professor, Department of Aerospace Engineering, Center of Excellence in Aerospace Systems.

Where; Q is the thermal power exchanged by convection from the wall to air as the fliud inside the pipe(W). $T_{out}$ and $T_{in}$ represent the centerline temperature of the outlet and inlet air (K). m is the mass flow rate of the internal air flow (kg/s); and $C_p$ is specific heat of air determined as a function of average temperature (J/kg.K), provided $C_p$ is reasonably constant over the length of domain.

A traditional expression for calculation of heat transfer in fully developed turbulent flow in smooth tubes is that recommended by Dittus and Boelter[1]:

$$Nu = 0.023 \mathrm{Re}_d^{0.8} \mathrm{Pr}^n \tag{2}$$

Where; Nu, Re and Pr are dimensionless numbers and defined as followings:

$$Nu = \frac{hD}{k} \tag{3}$$

$$\mathrm{Re} = \frac{\rho U D}{\mu} \tag{4}$$

$$\mathrm{Pr} = \frac{C_P \mu}{k} \tag{5}$$

Where; h is convective transfer coefficient (W/m$^2$.K), D is internal diameter of tube (m), U is average velocity of air (m/s), k is thermal conductivity of air (W/m.K), ρ is density of air (kg/m$^3$) and μ is viscousity of air (Pa.s).

The properties in equation 2 are evaluated at the average fluid bulk temperature, and the exponent n has the following value of n=0.4 for heating of the fluid and n=0.3 for cooling of the fluid.

n=0.4 for heating of the fluid
n=0.3 for cooling of the fluid (6)

Equation 2 is valid for fully developed turbulent flow in smooth tubes for fluids with prandtl numbers ranging from 0.6 to 160 and with moderate temperature differences between wall and fluid conditions. More recent information by Gnielinski[2] suggests that better results for turbulent flow in smooth tubes may be obtained from the following:

$$Nu = 0.0214(\mathrm{Re}_d^{0.8} - 100)\mathrm{Pr}^{0.4} \tag{7}$$

for $0.5<\mathrm{Pr}<1.5$ and $10^4<\mathrm{Re}<5\times10^6$ or

$$Nu = 0.012(\mathrm{Re}_d^{0.87} - 280)\mathrm{Pr}^{0.4} \tag{8}$$

for $1.5<\mathrm{Pr}<500$ and $3000<\mathrm{Re}<10^6$.

Also, to take into account the property variations of fluid in different temperatures, Sieder and Tate[3] recommended a new correlation. Also, Nusselt[4] recommended an equation for not fully developed flow in the entrance of the pipe. Despite all the researches have done to evaluate the heat transfer in a turbulated pipe flow, we choose the traditional Dittus Boelter equation in our work.

## 2. Numerical Simulations

The working conditions employed in the numerical simulation are given in Table 1. As it is seen in this table, there are five Pr and Re numbers, so we have only 25 different conditions for our numerical simulations. Also, there are some intential gaps between the operating Reynolds numbers. These gaps are considered because to generate random results in order to find the correlation between Re, Pr and Nu.

**Table 1. Working Conditions**

| Pr | Re |
|---|---|
| 70 | 12000 |
| 80 | 13000 |
| 90 | 14000 |
| 100 | 20000 |
| 110 | 25000 |

The numerical simulations were carried out using commercial CFD software package ANSYS FLUENT. The mathematical modeling involves the prediction of flow and heat transfer behavior. The flow through the pipe is considered as turbulent and incompressible with constant properties. The flow also assumed to be steadyand the working fluid is air. To solve partial governing equations, finite volume methods are employed. The three-dimensional equations of continuity, momentum, energy and several additional transport equations (depending on turbulence model) are used in this study. This equations in the fluid region can be expressed as follows:

Continuity equation:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{9}$$

Momentum Equation:

$$\frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}[\mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k})] + \frac{\partial}{\partial x_j}(-\rho\overline{u_i' u_j'}) \tag{10}$$

Energy Equation:

$$\frac{\partial}{\partial x_i}[u_i(\rho E + p)] = \frac{\partial}{\partial x_j}(k_{eff}\frac{\partial T}{\partial x_j})$$
$$E = h - \frac{P}{\rho} + \frac{u^2}{2} \tag{11}$$

A pipe which is heated along its circumference with constant flux is used. Air is blown and heated through the pipe according to Figure 1. The edge meshing technique with high mesh concentration where the gradients are larger, has been used in this study. The boundary conditions are specified in Table 2. Beside it is clear that the system is symmetric, hence a symmetrical boundry condition is employe to shorten the computational time.

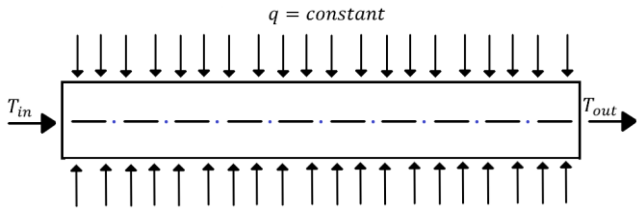


**Figure 1. Constant heat flux on pipe**

**Table 2. Boundary Types**

| Boundaries | Types |
|---|---|
| Inlet | Velocity Inlet |
| Outlet | Pressure Oulet |
| Center Line | Axis |
| Wall | Wall |

**Table 3. Calculated Nusselt number for different working conditions using CFD**

| # | Nu | Pr | Re | # | Nu | Pr | Re |
|---|---|---|---|---|---|---|---|
| **1** | 240.23 | 70 | 12000 | **14** | 403.10 | 90 | 20000 |
| **2** | 256.34 | 70 | 13000 | **15** | 483.07 | 90 | 25000 |
| **3** | 272.22 | 70 | 14000 | **16** | 278.16 | 100 | 12000 |
| **4** | 363.54 | 70 | 20000 | **17** | 296.82 | 100 | 13000 |
| **5** | 435.66 | 70 | 25000 | **18** | 315.20 | 100 | 14000 |
| **6** | 253.78 | 80 | 12000 | **19** | 420.94 | 100 | 20000 |
| **7** | 270.80 | 80 | 13000 | **20** | 504.45 | 100 | 25000 |
| **8** | 287.58 | 80 | 14000 | **21** | 289.27 | 110 | 12000 |
| **9** | 384.05 | 80 | 20000 | **22** | 308.67 | 110 | 13000 |
| **10** | 460.24 | 80 | 25000 | **23** | 327.80 | 110 | 14000 |
| **11** | 266.37 | 90 | 12000 | **24** | 437.76 | 110 | 20000 |
| **12** | 284.24 | 90 | 13000 | **25** | 524.61 | 110 | 25000 |
| **13** | 301.84 | 90 | 14000 | | | | |

A coupled solver with axsymmetric space and k-epsilon (2 eqns) for turbulent modelling is used for CFD simulation and the results are presented in table 3.

Figure 2 compares the numerical simulation results with the ones calculated from Dittus-Boelter equation in different working conditions. As it is shown in these figures, there is an error below 5% between these results. These errors came from the numberical simulation errors, also there is an uncertainity in Dittus-Boelter equation which is discussed in many researches[5-11]. Therefore, our CFD results are used for verification.

The correlation between Pr, Re and Nu using the numerical simulation results is shown below:

$$Nu = 0.001339\,\mathrm{Re}_d^{0.94}\,\mathrm{Pr}^{4.3} \qquad (12)$$

The differences between the correlation presented in equation 12 and the CFD results are shown in Figure 3. As it can be seen from this figure, there are large differences in some working conditions up to 28%. This difference occurs due to the lack of data inside the working conditions. As it was discussed above, 25 different working conditions are used to find the correlation. So, in the next step more data are generated to find a more accurate correlation between the flow parameters.

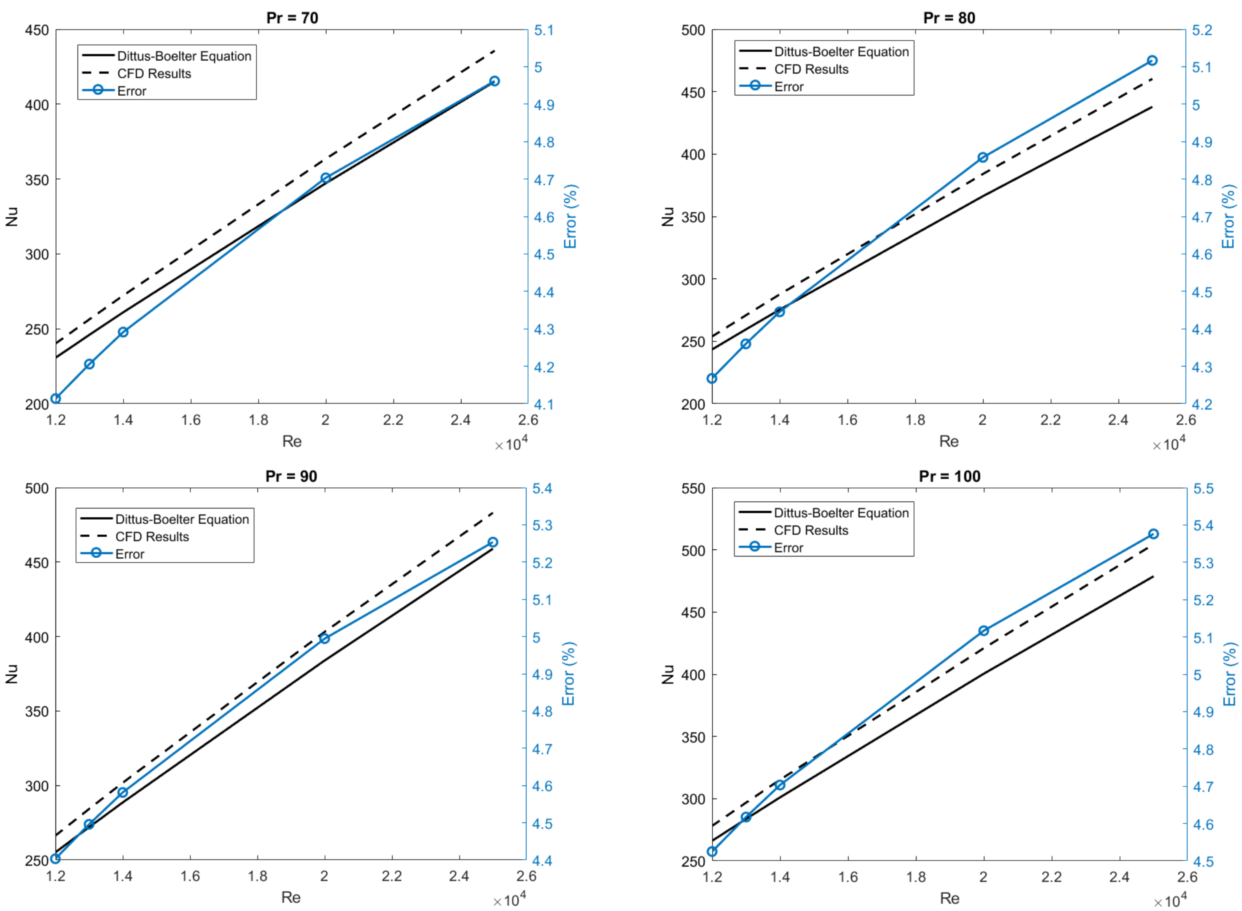


**Figure 2. Comparision between CFD results and Dittus-Boelter equation in different working conditions**

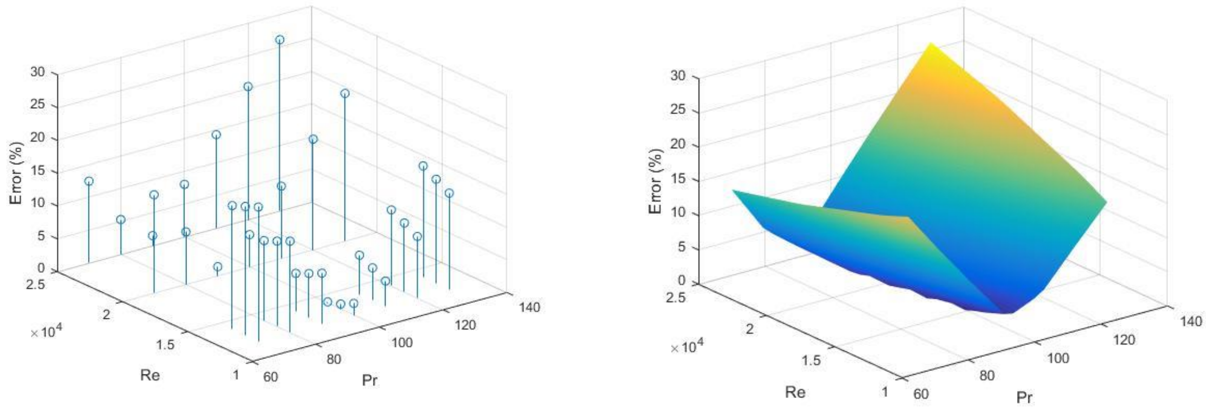


**Figure 3. Difference between calculated Nu using CFD and the correlation in equation 12**

## 3. Random Data Generation

In this section, some new set of data is generated to develope the correlation between the parameters in a more precise and systematic way. In this regard we use spline in each Pr to find the nusselt number for different Re numbers. Figure 4 shows this spline for Pr=70. As it was discussed in the previous section, there are some gaps in Reynolds numbers so in this matter we could fill out these gaps.

A set of random numbers at these points is generated and a random value is given to the Nusselt number in each of the working conditions. In order to generate this random values, a normal distribution is considered. In this normal distribution, the mean value is the calculated Nusselt number and the variance is 10% of the considered mean value. In this study, the random distribution function is extracted from the nature randomness of the turbulent flow. This method is an iterative algorithm, so the variance of the distribution fuction decreases as the iteration number increases. Figure 5 shows the allowed range for generating the random value for Nusselt number in a specified working condition.

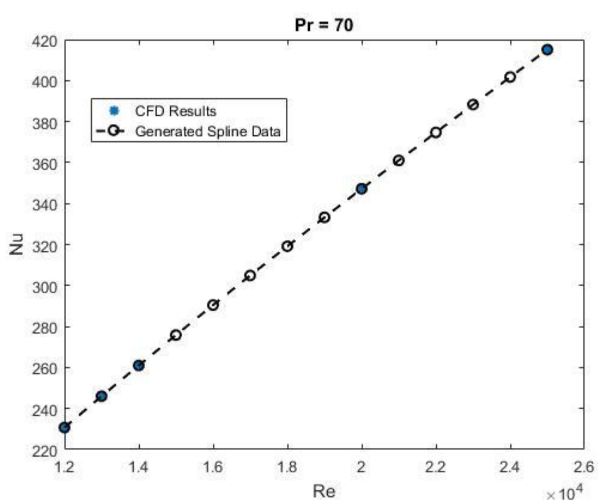


**Figure 4. Generating new data in all working conditions with spline**

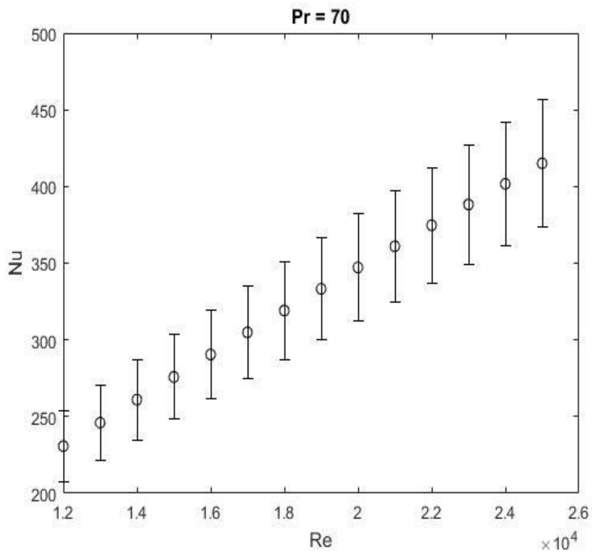


**Figure 5. Allowable range for generating random value using normal distribution function in Pr=70**

## 4. Results and Discussion

Figure 6 shows the difference between calculated Nu and the random value in the first iteration. Thiswork is done for all working conditions and the results are used as the initial condition for the next iteration. This method is used about 2723 times to reduce the error down to 5%. Figure 7 shows the convergancy plot. The error in this figure is the maximum difference between calculated Nu and the random value for all working conditions.

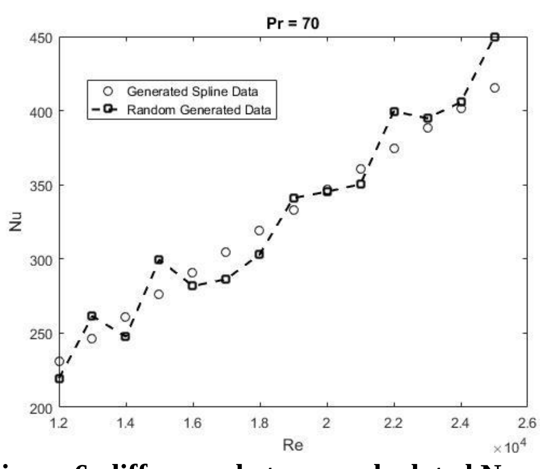


**Figure 6. difference between calculated Nu and the random value for the 1st iteration in Pr=70**

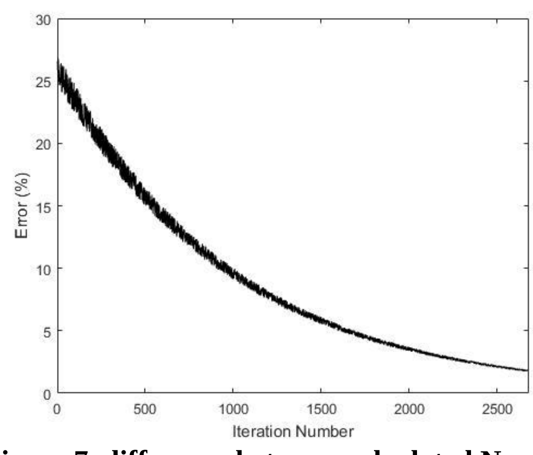


**Figure 7. difference between calculated Nu and the random value in each iteration for all working conditions**

The calculated correlation with the above method is given as below:

$$Nu = 0.025 \mathrm{Re}_d^{0.82} \mathrm{Pr}^{4.1} \tag{13}$$

Figure 8 shows the difference between the CFD results and the correlation presented in equation 13. As it can be seen from this figure, there is an error less than 5.5% for all working conditions which is an acceptable error for the considered estimation.

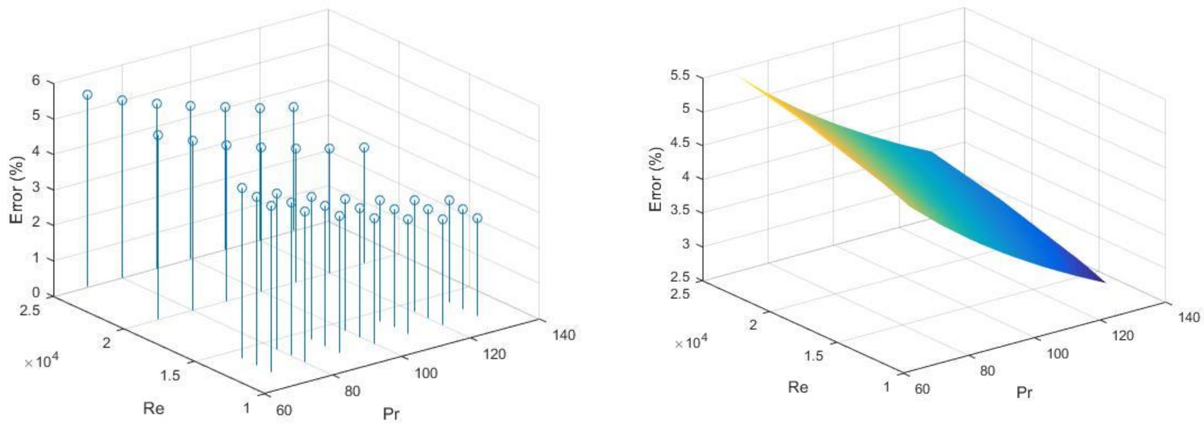


**Figure 8. difference between calculated Nu using CFD and this work correlation**

## 5. Comparision of the Methods with Artifial Neural Network

One of the most powerful tools for finding correlation between several parameters is neural network. Therefore, the results presented in the previous section are compared with those of an artificial neural network here. A dataset has been generated using a logical range for Prandtl, Reynolds and Nusselt parameter using the numerical simulation results with the working conditions of Table 1. Also, a neural network with 2 inputs and 1 output is trained using an in-house MATLAB code. A schematic of this network is shown in Figure 9.

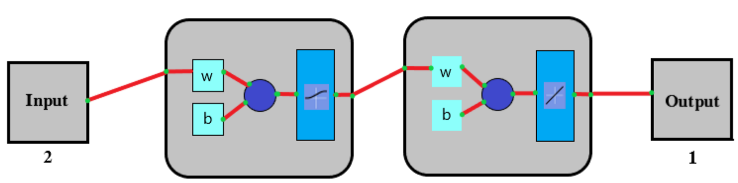


**Figure 9. A schematic of the neural network trained in an in-house MATLAB code**

The results show a great trained ANN for Nusselt number. The performance of the neural network along with the status of the solver and also the regression is shown in figure 10 and 11.

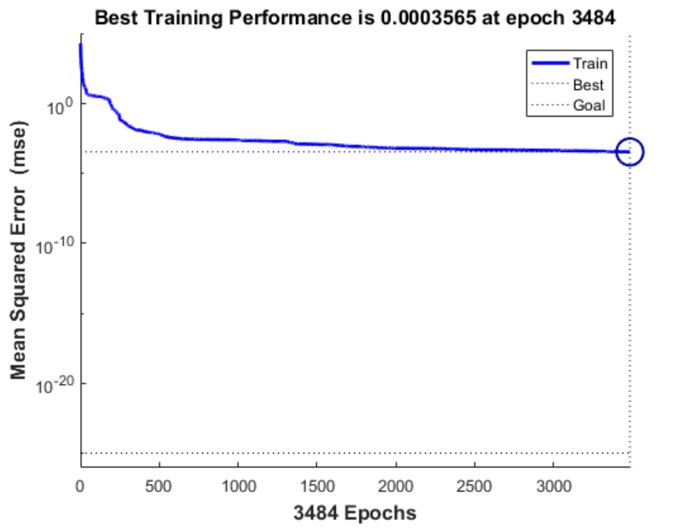


Figure 10. Training Performance of the neural network

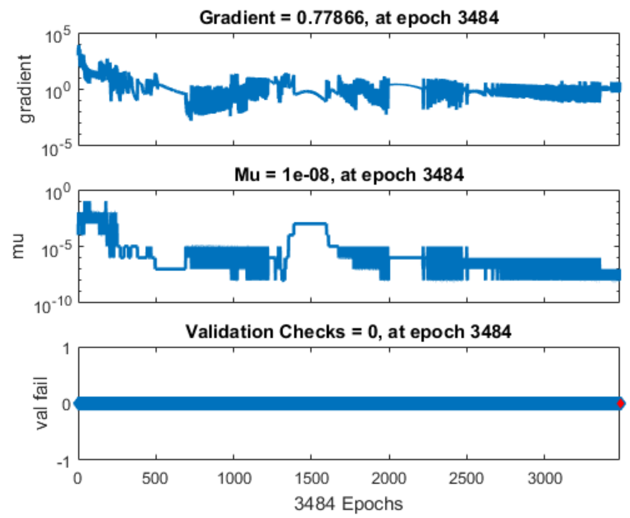


Figure 11. Training status in different epochs

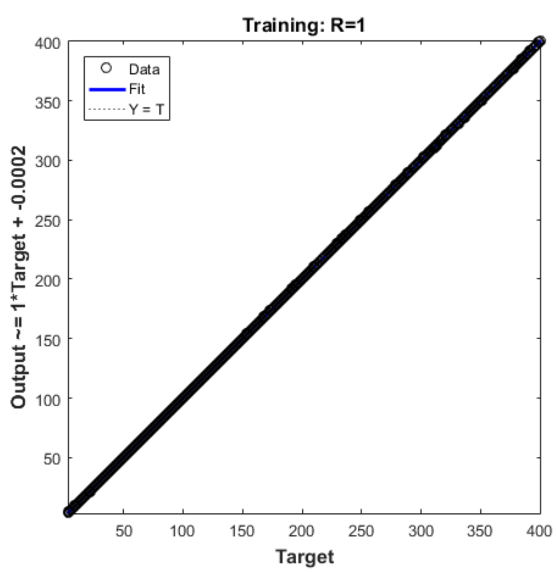


**Figure 12. The regression line of the trained neural network**

Figure 13 shows the difference between the CFD results and the correlation presented by the innovative iterative method in equation 13 within the difference between the CFD results and the results of the artificial neural network. As it is seen in this figure, there is an error less than 4.5% for all working conditions between the CFD results and the neural network results which is about 1% more precise than the innovative iterative method. It should be noted, the computational cost of the neural network is about 100 times higher than the innovative method. Therefore, in relatively simple problems with two inputs and one output, the proposed method is more efficient than neutal network.

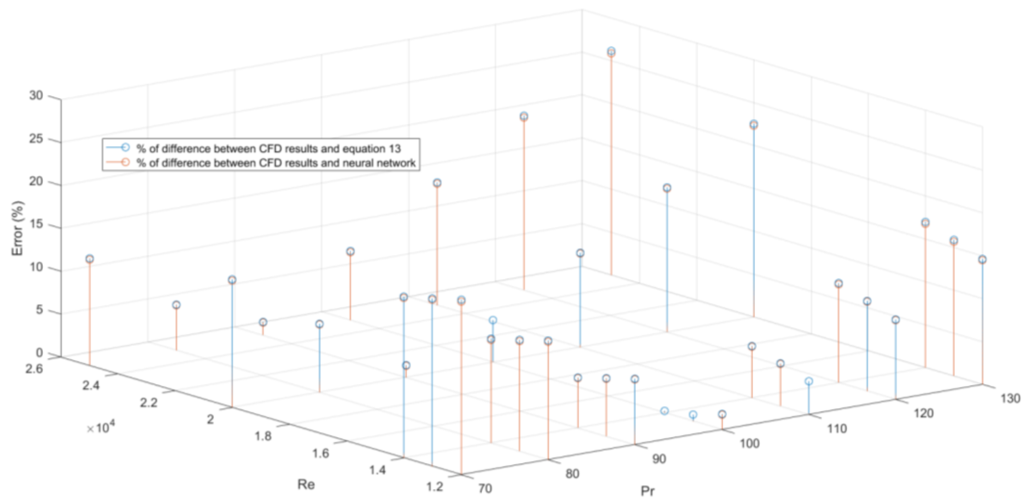


**Figure 12. Comparision of the innovative method and neural network results with CFD results**

## 6. Conclusion

In this article, we used an innovative method to find the correlation between Pr, Re and Nu in a pipe flow. The turbulated flow inside the pipe is heated. First the numerical simulation used to find the Nusselt number in some working conditions. Then, these results were validated against the Dittus-Boelter equation. After that, an iterative method was developed to generate some new random values for Nu in order to achieve a more accurate correlation. The random values were generated using a normal distribution with the mean value of the calculated Nu. The results have good agreement with those of CFD. Also these results have good agreement with those of artificial neural network as a powerful method, moreover, the presented method is about 100 times faster than neural network which is very impressive in relatively simple problems with two inputs and one output.

## 7. Acknowledgments


The authors would like to thank the financial support received from the Deputy of Research and Technology in Sharif University of Technology. Their financial support and help are greatly acknowledged.


## 8. References


[1]Dittus, F. W., and L. M. K. Boelter. *Univ. Calif. (Berkeley) Pub. Eng.*, vol. 2, p. 443, 1930.

[2]Gnielinski, V. "New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow," *Int. Chem. Engng.*, vol. 16, pp. 359-368, 1976.

[3]Sieder, E. N., and C. E. Tate. "Heat Transfer and Pressure Drop of Liquids in Tubes," *Ind. Eng. Chem.*, vol. 28, p. 1429, 1936.

[4]Nusselt, W. "Der Warmeausausch zwischen Wand und Wasser im Rohr," *Forsch. Geb. Ingenieurwes.*, vol. 2, p. 309, 1931.

[5]Gamrat, G., Faver-Marinet, M., Asendrych, D., "Conduction and entrance effects on laminar liquid flow and heat transfer in rectangular micro-channels," *Int.J.Heat and Mass Transfer*, vol. 48, pp.2943-2954, 2005.

[6]Ma, J., Li, L., Huang, Y., Liu, x., “Experimental studies on single-phase flow and heat transfer in a narrow rectangular channel,” *Nuclear Engineering and Design* 241, pp. 2865-2873, 2011.

[7]Colburn, A.P., “A method of correlating forced convection heat transfer data and a comparison with fluid friction,” *Trans. AIChE J.* 29, pp. 174-210, 1933.

[8]Hwang, K.S., Jang, S.P., Choi, S.U., "Flow and convective heat transfer characteristics of water-based Al2O3 nanofluids in fully developed laminar flow regime," *International journal of heat and mass transfer* vol. 52, pp. 193-199, 2009.

[9]Levy, S., Fuller, R.A., Niemi, R.O., "Heat transfer to water in thin rectangular channels," *Journal of Heat Transfer*, vol. 81, pp. 129-140, 1959.

[10]Battista, E., Perkins, H.C., "Turbulent Heat and Momentum Transfer in a Square Duct with Moderate Property," *Int.J. Heat Mass Transfer*, vol. 13, 1063-1065, 1970.

[11]Campbell, D.A., Perkins, H.C., "Variable property turbulent heat ans momentum transfer for air in a vertical rounded corner triangular duct," *Int. J. Heat Mass Transfer*, vol. 11, 1968.