Stability of positronium negative ion in non-ideal classical plasmas

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Abstract

The stability of positronium negative ion embedded in non-ideal classical plasmas has been investigated theoretically within the framework of Rayleigh-Ritz variational method by computing its ground state energy quite accurately. A pseudopotential, derived from a solution of Bogolyubovs hierarchy equations, has been used to describe interactions among the charged particles in plasma. A large basis set is utilized to compute accurately various quantities, such as binding energy, cusp values, annihilation rate, associated with the ground state of the ion. A detailed study has been made on the effects of non-ideality of plasma on those quantities. In particular, special emphasis is given to determine the ranges of plasma screening parameters within which the ion remains stable.
Keywords: positronium negative ion, stability, annihilation rate, non-ideal plasma, pseudopotential.

1 Introduction

An electron (e^-) and its anti-particle (positron, e^+), interacting with Coulomb potential (CP) can form bound states. The resulting bound states is called positronium atom (Ps). This atom resembles hydrogen atom, except the reduced mass is half of the hydrogen atom, which leads to the doubling of first Bohr radius and halving the energy levels of Ps than of the hydrogen atom. Another e^- can be attached weakly to the ground state of Ps to form what is known as positronium negative ion (Ps^-). This ion exists only in the ground state (^1S) which is stable against disassociation into an e^- and Ps, but unstable against e^++e^- annihilation. As the ion consists of three leptons (e^-, e^-, e^+) of equal mass, it provides an ideal platform for testing quantum three-body problems. The stability of Ps^- was first discussed by Wheeler [1] by proposing its formation through the interaction of a photon with atomic electrons. Subsequently Mills [2] observed this ion by using a beam foil method of

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production. Since its observation, the ion has been drawing the attention of both theorists and experimentalists due to its applicability in various branches of physics, such as solid state physics, astrophysics, plasma physics including modern communication devices [3]. A number of theoretical investigations has been performed so far to study various aspects of Ps^-. Particular mention may be made of the works attempting to calculate accurately the ground state energy [[4]-[12]], annihilation rate [[11]-[14]], resonance states [[15]-[19]], photodetachment cross section [[20]-[23]], polarizability [[24]-[26]] etc. At the same time, several experiments were also performed to study the properties of Ps^- [[27]-[34]]. These have been well documented in the review article of Nagashima [35].

It is well known that an atom or an ion behaves differently, when it is embedded in finite density plasma at a given temperature. The plasma screening alters the interaction potentials of the constituent charged particles of the embedded atom. As a result, stability and other structural properties, such as energy levels, line shapes, ionization potential, transitions, line merging etc of the embedded atom may suffer considerable changes in contrast to a free atom [[36]-[38]]. Studies of those changes are important for interpreting data for astrophysical, laser induced and ultra-cold plasmas [[39]-[43]].

Screened potential in plasma depends on the state of plasma, and in few cases it can be approximated in closed forms. The ratio of mean inter-particle interaction and the mean kinetic energy of the thermal motion, called the coupling parameter γ, serves to characterise the state of plasma. If $\gamma = 0$, plasma is called ideal plasma (in strict sense). For such plasmas, the average distance between particles is large enough so that their mean interaction can be neglected. With the increase in plasma density, average distance between particles starts diminishing, leading γ to increase gradually. When $\gamma \neq 0$, plasma is called non-ideal plasma [44]. Thus γ serves as a measure of the non-ideality of plasma. So, γ is also called non-ideal plasma parameter. When $\gamma \ll 1$ (weak limit of non-ideality), the plasma is called weakly coupled plasma. For a weakly coupled plasma, the screened interaction can be represented by the Debye-Huckel potential (DHP) of the form [45]:

$$V_{\rm DH}(r) = \frac{e^{-r/r_D}}{r}, \tag{1}$$

where r_D is called the Debye length. In terms of density n_e and temperature T_e, it is given by $r_D = (K_B T_e/4\pi e^2 n_e)^{1/2}$, where K_B and e respectively denote the Boltzmann constant, electronic charge. r_μ $(= 1/r_D)$ is often called plasma screening parameter. On the other side, for non-ideal classical plasmas (NICP) with no degeneration quantum effects, the screened interaction can be described by a pseudopotential which is obtained from a sequential solution of the chain of Bogolyubov equations [46] by taking into account the collective events and the screening effects of plasma. That pseudopotential or effective potential is of the form (in a.u.) [46]:

$$V(r) = \frac{[10 + \gamma(e^{-\sqrt{\gamma}r/r_D} - 1)(1 - e^{-2r/r_D})]}{10[1 + c(\gamma)]} \frac{e^{-r/r_D}}{r}, \tag{2}$$

where $c(\gamma)$ is known as the correction function. In terms of r_D, γ is given by $\gamma = e^2/(r_D K_b T_e)$. The correction function $c(\gamma)$ is given for a number of values of γ in the range $0 \leq \gamma \leq 4.5$ [46]. It should be mentioned that the above pseudopotential appropriately represents the particle interaction of a NICP for $0 \leq \gamma \leq 4.0$. It reduces to the DHP in the form of equation (1) in

the weak limit of non-ideality ($\gamma \ll 1$). In a number of investigations such a pseudopotential has been used to describe the screening of NICPs [[46]-[56]]. In this context it is worthy to mention that it is possible to obtain pseudopotentials which include the quantum mechanical effects in non-ideal plasmas [[57]-[61]].

In this paper our objective is to study various properties of Ps^- embedded in NICPs, such as bound states, annihilation rate, cusps etc. In particular, emphasis will given on the stability of the ion. We work within the framework of Rayleigh-Ritz variational method which is the most powerful and effective method for studying three-body problems of bound states. We consider the density and temperature of the plasma in the ranges $2.7 \times [10^{23} - 10^{26}]$ m^{-3} and $[1 - 10] \times 10^4$ K respectively so that $\gamma \in [0, 4.0]$ approximately. Our endeavour will be to make a detailed study on the changes of various properties of Ps^- due to change in the non-ideality of the underlying plasma. Such a study is of fundamental interest in the context of astrophysics, since existence of positrons in several astrophysical environments has been conclusively proved [[62]-[64]]. It should be mentioned here that some studies on the properties of Ps^- in weakly coupled plasma environments have been reported in the literature [[65]-[69]], but, to the best of our knowledge, effects of non-ideal plasmas on Ps^- have not been reported so far.

2 Theory and calculations

Let $\vec{r}_1$, and $\vec{r}_2$ be the coordinates of two electrons relative to the positron and and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$. With such choice of coordinates, the intrinsic part of the non-relativistic Hamiltonian of Ps^- implanted in NICP which is described by the pseudopotential in the form of equation (2) is given by (in reduced atomic units)

$$H^{Ps^-} = -\frac{1}{2}\nabla_1^2 - \frac{1}{2}\nabla_2^2 - \frac{1}{2}\nabla_1 \cdot \nabla_2 - C_V(r_1;\gamma)\frac{e^{-r_1/r_D}}{r_1} - C_V(r_2;\gamma)\frac{e^{-r_2/r_D}}{r_2} + C_V(r_{12};\gamma)\frac{e^{-r_{12}/r_D}}{r_{12}}, \tag{3}$$

where $C_V(r;\gamma) = [10 + \gamma(e^{-\sqrt{\gamma}r/r_D} - 1)(1 - e^{-2r/r_D})]/[10\{1 + c(\gamma)\}]$. For a given value of $\gamma \in [0, 4.0]$, $c(\gamma)$ is computed by fitting a cubic polynomial with four values of γ in succession from the Table I of Ref. [46]. Note that here we use same screening parameters to represent the screened positron-electron and electron-electron interactions. In this regard it is important to state that screening in any form is a feature in plasma. Thus, the results in this paper, of course based on the validity of the pseudopotential model, show general qualitative features. These have to be refined when there exists testimonies that the model is not a fair approximation to the interaction potential.

We solve the Schrodinger equation $H^{Ps^-}\Psi(r_1, r_2, r_{12}) = E_{Ps^-}\Psi(r_1, r_2, r_{12})$, $(E_{Ps^-} < 0)$ with in the framework of Rayleigh-Ritz variational method to determine the ground state energy E_{Ps^-} and corresponding wave functions $\Psi(r_1, r_2, r_{12})$ of Ps^-. That requires the minimization of the Rayleigh quotient:

$$E_{Ps^-}[\Psi] = \frac{<\Psi|H^{Ps^-}|\Psi>}{<\Psi|\Psi>} \tag{4}$$

by means of trial wave function Ψ. In this work, we choose Ψ as:

$$\begin{aligned}\Psi(r_1, r_2, r_{12}) &= \sum_{j=1}^{N} C_j |\psi_j(r_1, r_2, r_{12}; A, l_j, m_j, n_j) > \\ &= \sum_{j=1}^{N} C_{l_j m_j n_j} (1 + P_{12}) e^{-A(r_1+r_2)} r_1^{l_j} r_2^{m_j} r_{12}^{n_j} \ , \end{aligned} \tag{5}$$

where C_j's (or $C'_{l_j m_j n_j}$s, l_j, m_j, n_j are non-negative integers) are linear expansion coefficients, A is a non-linear variational parameter and P_{12} is an electron exchange operator such that $P_{12} f(r_1, r_2) = f(r_2, r_1)$ for an arbitrary function f. The summation in the wave function (5) is disintegrated to expand Ψ by considering the sum $\omega_j = l_j + m_j + n_j$. We set $\omega_j = 0, 1, 2, \cdots$ so that ω_0 corresponds to $N = 1$, ω_1 corresponds to $N = 3$, ω_2 corresponds to $N = 7$, and so on. Putting equation (5) in equation (4) we obtain

$$E_{Ps^-}[\Psi] = \frac{\sum_{i=0}^{N} \sum_{j=0}^{N} C_i^* C_j H_{ij}^{Ps^-}}{\sum_{i=0}^{N} \sum_{j=0}^{N} C_i^* C_j S_{ij}^{Ps^-}}, \tag{6}$$

where

$$\begin{aligned} H_{ij}^{Ps^-} &= < \psi_i(r_1, r_2, r_{12}; A, l_i, m_i, n_i) | H^{Ps^-} | \psi_i(r_1, r_2, r_{12}; A, l_j, m_j, n_j) > \quad \text{and} \\ S_{ij}^{Ps^-} &= < \psi_i(r_1, r_2, r_{12}; A, l_i, m_i, n_i) | \psi_i(r_1, r_2, r_{12}; A, l_j, m_j, n_j) > \end{aligned} \tag{7}$$

are respectively the Hamiltonian matrix elements and overlap matrix elements. Carrying out a little algebra, it can shown that solving equation (6) for E_{Ps^-} leads to calculating the least eigen value of the matrix $\tilde{S}^{-1}\tilde{H}$, where $\tilde{H} = [H_{ij}^{Ps^-}]$ and $\tilde{S} = [S_{ij}^{Ps^-}]$. The least eigen value and corresponding eigen function have been computed by employing Q-R algorithm on the transformed matrix in the Hessenberg form [70]. In order to determine the bound state energies and corresponding wave functions, we follow the method as described in our previous paper [56].

The optimized ground state wave function Ψ is used to compute various quantities associated with the ground state of Ps^-, such as various geometrical expectations, cusp values, annihilation rate etc. The cusp quantities can be obtained from the following equations [11]:

$$v_i = \frac{\left\langle \Psi \left| \delta(\vec{r}_i) \frac{\partial}{\partial r_i} \right| \Psi \right\rangle}{< \Psi | \delta(\vec{r}_i) | \Psi >}, \quad i = 1, 2, \quad (\text{electron} - \text{positron cusp}) \tag{8}$$

$$v_{12} = \frac{\left\langle \Psi \left| \delta(\vec{r}_{12}) \frac{\partial}{\partial r_{12}} \right| \Psi \right\rangle}{< \Psi | \delta(\vec{r}_{12}) | \Psi >}, \qquad (\text{electron} - \text{electron cusp}), \tag{9}$$

where δ denote the Dirac-delta function. The positron in Ps^- can annihilate one of the electrons as a result of which energy (photon) is released. The number of photons could be one, two, three or more. Various photon annihilation rates can be calculated from the knowledge of the delta functions concerned [11]. However, total annihilation rate Γ is given

by the sum of the two-photon and three-photon annihilation rates. It is calculated by the following formula [11]:

$$\begin{aligned}\Gamma &= 2\pi\alpha^4 a_0^{-1}\left[1-\frac{\alpha(204-19\pi^2)}{12\pi}\right] < \Psi|\delta(\vec{r}_1)|\Psi > \\ &= 100.61745997357 < \Psi|\delta(\vec{r}_1)|\Psi > \qquad (\text{in ns}^{-1}),\end{aligned} \tag{10}$$

where c is the speed of light in vacuum, α is the fine structure constant and a_0 is the first Bohr's radius. The reciprocal of Γ gives us the lifetime τ of Ps^-, that is $\tau = 1/\Gamma$.

3 Results and discussion

First we examine the convergence of the scheme that we have employed here with respect to the increase in the number of terms in the wave function (5). The ground state energies for various values of r_D and γ obtained by increasing number of terms in (5) are held up in Table 1. From this table we observe that convergent results, correct up to eight places of decimal, can be obtained by using 1078 (ω_{21}) number of terms in the wave function (5). Some results related to the ground state of energy of Ps^- in vacuum as well as in weakly coupled plasma environments are available in the literature. In Table 2 we put up a comparison of our present results with some of the reliable corresponding results available in the literature. The comparison shows an excellent agreement for free-atomic case as well as for the case of weakly coupled plasmas.

Having done these, we now move to study the effects of non-ideality of plasma on the properties of Ps^-. We study the effects of non-ideality of plasma on a number of quantities pertaining to the ground state of Ps^-. These include the ground state energies of Ps^- and Ps, various geometrical expectation values, cusps and total annihilation rate. Values of those quantities for a number of densities and temperatures lying in the ranges $2.7 \times [10^{23} - 10^{26}]$ m^{-3} and $[1 - 10] \times 10^4$ K respectively are presented in Tables 3 - 6. From these tables we note that for a given temperature, E_{Ps^-} increases with increasing density. This indicates that interaction potentials get weaker with increasing density at a given temperature. As a result, average distances among positron and electrons increase (as evident from Tables 3 - 6), leading to decrease in the total annihilation rate or increased lifetime of Ps^-. Exactly opposite thing takes place, when temperature increases but density remains the same. However, we observe that Ps^- remains stable in the above-stated ranges of temperature and density with lifetime ranging from 0.48034336 ns to 2.97346772 ns.

The goodness of the optimized wave function Ψ can be estimated by studying its properties. For example, by definition, expectation values of the delta functions are measures of local behaviour of the wave function. For the unscreened case, our computed values are in excellent agreement with results of Refs. [5, 11]. Here, we find that they are being gradually reduced with increasing non-ideality. The cusp quantities serve as a measure of accuracy of wave function near the points of coalescence. For a system having two charged particles with charges q_1 and q_2, it is given by $v_{12} = \mu q_1 q_2$, where μ is the reduced mass of the system [14]. So, for the unscreened case, positron-electron cusp should be -0.5 a.u. and electron-electron

cusp should be 0.5 a.u. What we have computed are exactly same for the unscreened case. However, with the increase in non-ideality cusp values decrease, but they stay very near to their respective values for the unscreened case. Another important quantity is the ratio of the average potential energy to the average kinetic energy, which according to the virial theorem, must be -2 for the unscreened case. What we find here is exactly same for the unscreened case. However, it decreases slowly with increasing non-ideality.

We now focus our attention to study the effect of non-ideality on the stability of Ps^-. In Figure 1, ground state energy of Ps^- as a function of non-ideal parameter is plotted for various Debye lengths. We note that for a given Debye length, E_{Ps^-} increases with increasing γ. This means that increase in the non-ideality gradually leads the ion towards instability. We have tried to compute quite exactly the critical values of r_D and γ (the respective values of r_D and γ beyond which Ps^- does not exist) giving the range of stability of Ps^-. These are shown in Table 7. We find that for weakly coupled plasmas ($\gamma = 0$), critical value of r_D is 1.700420 a_o which is better than previously reported values [65, 66]. Moreover, if the Debye length is relaxed to increase, critical value of non-ideal parameter increases such that when $r_D = 0.33a_0$, $\gamma = 2.996616$. In Figure 2 we put up a graphical presentation of electron affinity Ps_{ea} of Ps. As expected, we find that, for a given Debye length, it decreases with increasing γ.

4 Conclusions

In conclusion, we have made an inclusive study on the properties of Ps^- embedded in NICP. It has been found that increase in non-ideality as well as the plasma screening effect leads the ion towards instability. However, the ion remains stable for the density and temperature lying the ranges $2.7 \times [10^{23} - 10^{26}]$ m^{-3} and $[1 - 10] \times 10^4$ K respectively with lifetime varying from 0.48034336 ns to 2.97346772 ns. We show up the values of non-ideal plasma parameter and Debye length beyond which ion does not exist. We hope that our results will be useful in interpreting various astrophysical data relating to positron and in developing our understanding regarding kinetic properties of non-ideal plasmas.

Acknowledgements: The work has been supported by the Science and Engineering Research Board, India through the Research project (FILE NO. EMR/2017/004985).

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Table 1: Convergence of the ground state energy of Ps^-.

		$-E_{Ps^-}$ (a.u.)			
r_μ (a_0^{-1})	γ	$N = 715(\omega_{18})$	$N = 825(\omega_{19})$	$N = 946(\omega_{20})$	$N = 1078(\omega_{21})$
0.00	0.0	0.2620050700	0.2620050701	0.2620050702	0.2620050702
0.10	0.1	0.1575887532	0.1575887533	0.1575887534	0.1575887534
0.15	0.5	0.0829082162	0.0829082164	0.0829082164	0.0829082164
0.25	0.7	0.0292130242	0.0292130246	0.0292130249	0.0292130250
0.25	1.1	0.0188974629	0.0188974634	0.0188974637	0.0188974639

Table 2: Comparison of the ground state energy and annihilation rate of Ps^- and ground state energy of Ps.

r_μ (a_0^{-1})	γ	$-E_{Ps^-}$ (a.u.)	$-E_{Ps}$(a.u.)	$\Gamma(\mathrm{ns}^{-1})$
0.00	0.0	0.2620050702	0.2500000000	2.0928027667
		0.2620050702[a]		2.086121720[a]
		0.262005068[b]	0.250000000[b]	
		0.2620050702[c]	0.250000000[c]	2.086121817[c]
0.05	0.0	0.2149738357	0.2035290153	2.0718671824
		0.214973833[b]	0.203529015[b]	
		0.2149738358[c]	0.203529015[c]	
0.10	0.0	0.1736181599	0.1634042557	2.0104151518
		0.173618156[b]	0.163340426[b]	
		0.1736181600[c]	0.163340426[c]	2.003997348[c]
0.25	0.0	0.0798460755	0.0740585109	1.6204762788
		0.079845972[b]	0.074058510[b]	
		0.0798460757[c]	0.074058510[c]	

[a] Results of Frolov [11], [b] Results of Saha *et al* [65], [c] Results of Kar and Ho [66, 67]

Table 3: Ground state energy, cusps, various expectation values, annihilation rate etc of Ps^- embedded in NICP for various plasma temperatures and densities.

	$n_e = 2.7 \times 10^{23}\ m^{-3}$				
T(in K)	10^4	2×10^4	5×10^4	8×10^4	10^5
γ	0.12582166	0.04448467	0.01125383	0.00556058	0.00397883
r_D (in a_0)	250.97041160	354.92575984	561.18690068	709.85151968	793.63812597
A_1	0.23417808	0.24970264	0.25768250	0.25932792	0.25983485
A_2	0.22327927	0.23813720	0.24579531	0.24738180	0.2478721211
A_3	0.32383632	0.33356595	0.33815852	0.33899199	0.33922621
A_4	0.14830968	0.15276645	0.15487021	0.15525204	0.15535935
A_5	5.76082218	5.59267717	5.51666138	5.50308305	5.49927845
A_6	8.97079960	8.70900424	8.59065438	8.56951558	8.56359287
A_7	106.64581179	100.50921972	97.79447122	97.31339562	97.17879993
A_8	0.01794389	0.01960974	0.02043056	0.02058191	0.02062458
A_9	0.00014801	0.00016175	0.00016853	0.00016978	0.00017013
A_{10}	0.47648968	0.49079485	0.49754622	0.49877081	0.49911468
A_{11}	0.47593807	0.49021867	0.49697898	0.49819839	0.49854099
A_{12}	1.98419977	1.98912058	1.99319505	1.99462851	1.99519718
A_{13}	1.81125662	1.97940700	2.06226070	2.07753823	2.08184466

$A_1 \equiv -E_{Ps^-}$ (in a.u.), $A_2 \equiv -E_{Ps}$ (in a.u.), $A_3 \equiv < 1/r_1 > (a_0{}^{-1})$, $A_4 \equiv < 1/r_{12} > (a_0{}^{-1})$, $A_5 \equiv < r_1 > (a_0)$, $A_6 \equiv < r_{12} > (a_0)$, $A_7 \equiv < r_1{}^2 + r_2{}^2 > (a_0{}^2)$, $A_8 \equiv < \delta(\vec{r}_i > $ (a.u.), $A_9 \equiv < \delta(\vec{r}_{12}) > $ (a.u.), $A_{10} \equiv -v_i(a_0{}^{-1})$, $A_{11} \equiv v_{12}(a_0{}^{-1})$, $A_{12} \equiv - < P.E. > / < K.E. >$, $A_{13} \equiv \Gamma$ (in ns^{-1})

Table 4: Same as Table 3.

	$n_e = 2.7 \times 10^{24}\ m^{-3}$				
T(in K)	10^4	2×10^4	5×10^4	8×10^4	10^5
γ	0.39788302	0.14067289	0.03558774	0.01758411	0.01258217
r_D (in a_0)	79.36381260	112.23738014	177.46287992	224.47476027	250.97041160
A_1	0.18764304	0.22724271	0.24874440	0.25366209	0.25521355
A_2	0.17859010	0.21646887	0.23710311	0.24184334	0.24334340
A_3	0.29559703	0.32217178	0.33473158	0.33723554	0.33795559
A_4	0.13536248	0.14754116	0.15329803	0.15444594	0.15477608
A_5	6.31355446	5.79145589	5.57349962	5.53196574	5.52012995
A_6	9.83065698	9.01818888	8.67903051	8.61440949	8.59599705
A_7	128.13985372	107.79894063	99.82635950	98.34144322	97.92022064
A_8	0.01365515	0.01767278	0.01981769	0.02026489	0.02039470
A_9	0.00011255	0.00014573	0.00016345	0.00016715	0.00016822
A_{10}	0.43512480	0.47412223	0.49253941	0.49620894	0.49726351
A_{11}	0.43460047	0.47358090	0.49195573	0.49564771	0.49669753
A_{12}	1.94644020	1.96492208	1.97846587	1.98305219	1.98485610
A_{13}	1.37835119	1.78389089	2.00039718	2.04553887	2.05864069

Table 5: Same as Table 3.

	$n_e = 2.7 \times 10^{25}\ m^{-3}$				
T(in K)	10^4	2×10^4	5×10^4	8×10^4	10^5
γ	1.25821658	0.44484674	0.11253831	0.05560584	0.03978830
r_D (in a_0)	25.09704116	35.49257598	56.11869007	70.98515197	79.36381260
A_1	0.10312101	0.16880681	0.22344603	0.23674935	0.24120125
A_2	0.09748625	0.16019035	0.21252518	0.22533402	0.22963275
A_3	0.23806847	0.29025619	0.32505057	0.33196039	0.33405987
A_4	0.10884309	0.13284896	0.14883596	0.15201267	0.15297815
A_5	7.88171635	6.44051378	5.74324058	5.62210926	5.58626948
A_6	12.25954919	10.02500853	8.94209626	8.75400929	8.69837250
A_7	200.76256282	133.56140974	106.06667939	101.61198894	100.31177090
A_8	0.00719036	0.01296298	0.01816635	0.01934092	0.01970736
A_9	0.00005870	0.00010649	0.00014964	0.00015941	0.00016246
A_{10}	0.35231587	0.42809855	0.47865905	0.48867205	0.49171034
A_{11}	0.35188932	0.42760737	0.47809846	0.48810588	0.49113057
A_{12}	1.80105938	1.88278127	1.93198053	1.94683498	1.95256780
A_{13}	0.72579502	1.30848319	1.83371144	1.95227271	1.98926064

Table 6: Same as Table 3.

	$n_e = 2.7 \times 10^{26}\ m^{-3}$				
T(in K)	10^4	2×10^4	5×10^4	8×10^4	10^5
γ	3.97883017	1.40672890	0.35587739	0.17584111	0.12582166
r_D (in a_0)	7.93638126	11.22373801	17.74628799	22.44747603	25.09704116
A_1	0.02380007	0.06939273	0.15838598	0.1907901601	0.20184767
A_2	0.02202632	0.06504784	0.14967022	0.1806869658	0.19131224
A_3	0.17058560	0.22515446	0.29752923	0.31684978	0.32249013
A_4	0.07389565	0.10182122	0.13591050	0.14490393	0.14753115
A_5	12.22360486	8.54967842	6.31776907	5.91350211	5.80463682
A_6	19.01632039	13.27796847	9.82650798	9.20161262	9.03346686
A_7	519.20362516	241.34680543	129.17095480	112.83638261	108.62567052
A_8	0.00333176	0.00637107	0.01409410	0.01692908	0.01782105
A_9	0.00001958	0.00004883	0.00011438	0.00013836	0.00014595
A_{10}	0.29779221	0.34449591	0.44191307	0.46870899	0.47648953
A_{11}	0.29682532	0.34399033	0.44136527	0.46815740	0.47594160
A_{12}	1.34483369	1.59814699	1.78730691	1.83686978	1.85484686
A_{13}	0.33630767	0.64309607	1.42265812	1.70882121	1.79885730

Table 7: Critical value of r_μ (in a_0^{-1}) and γ and the corresponding lifetime τ (in ns) of Ps^-.

r_{μ_c}	0.30	0.35	0.40	0.45	0.50	0.588090
γ_c	2.996616	1.856900	1.181457	0.712813	0.428837	0.0
τ	85.41102671	71.74867101	58.74212989	48.25867106	39.78714990	12.63984545

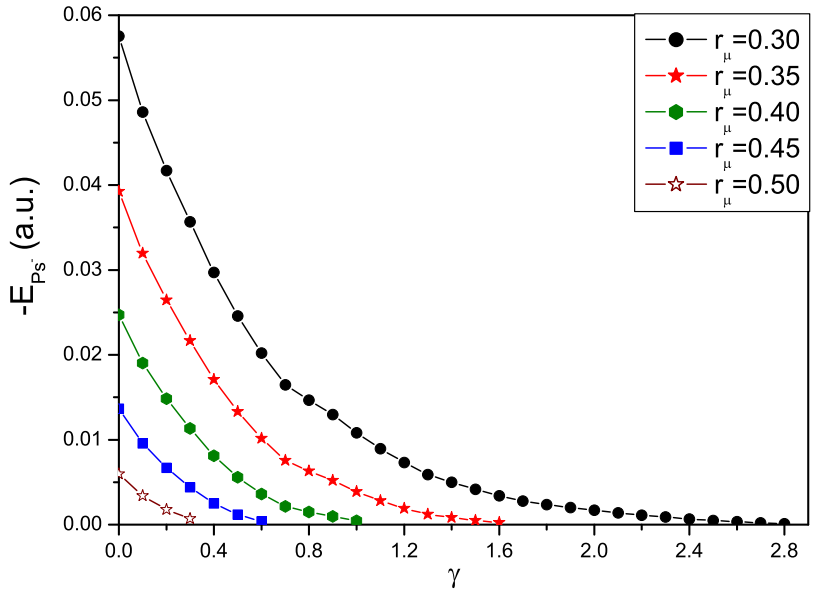

Figure 1: Ground state energy of Ps^- as a function of γ for different values of r_μ (in a_0^{-1}).

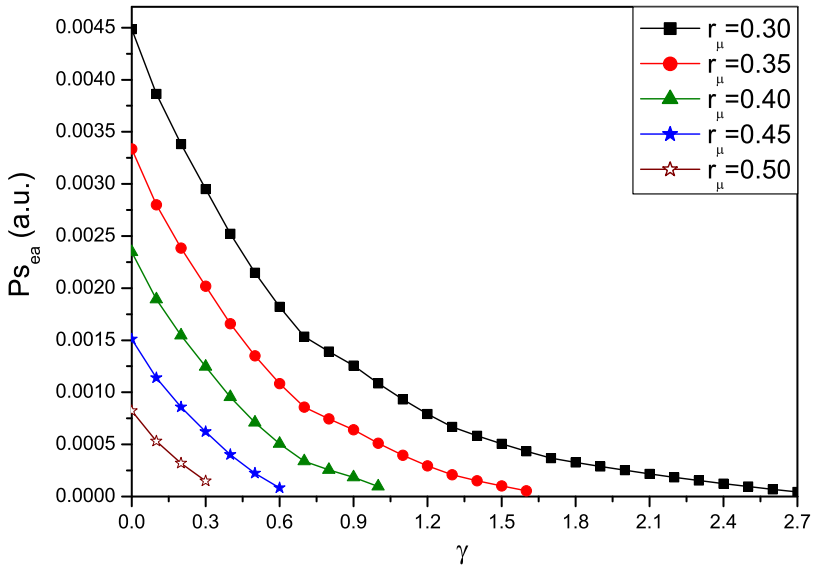

Figure 2: Electron affinity of Ps as a function of γ for different values of r_μ (in a_0^{-1}).