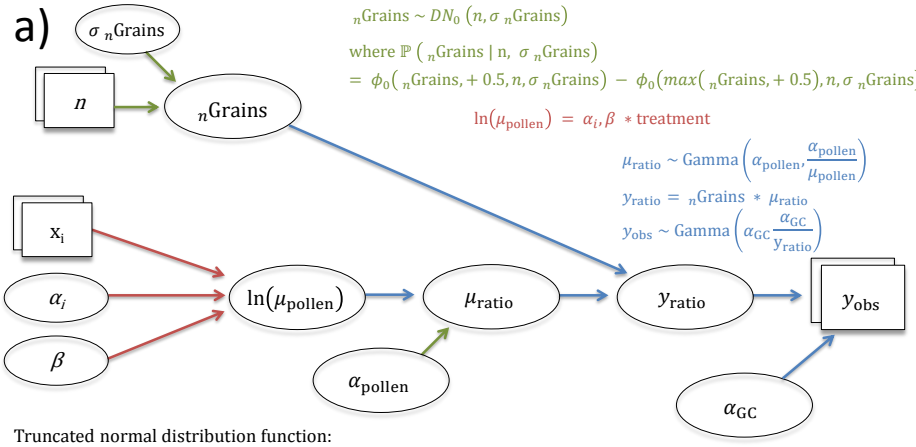
a)



$${}_n\text{Grains} \sim DN_0\left(n, \sigma\ {}_n\text{Grains}\right)$$

where $\mathbb{P}\left({}_n\text{Grains} \mid \text{n},\ \sigma\ {}_n\text{Grains}\right)$

$$= \phi_0\left({}_n\text{Grains}, +0.5, n, \sigma\ {}_n\text{Grains}\right) - \phi_0\left(max\left({}_n\text{Grains}, +0.5\right), n, \sigma\ {}_n\text{Grains}\right)$$

$$\ln(\mu_{\text{pollen}}) = \alpha_i, \beta * \text{treatment}$$

$$\mu_{\text{ratio}} \sim \text{Gamma}\left(\alpha_{\text{pollen}}, \frac{\alpha_{\text{pollen}}}{\mu_{\text{pollen}}}\right)$$

$$y_{\text{ratio}} = {}_n\text{Grains} * \mu_{\text{ratio}}$$

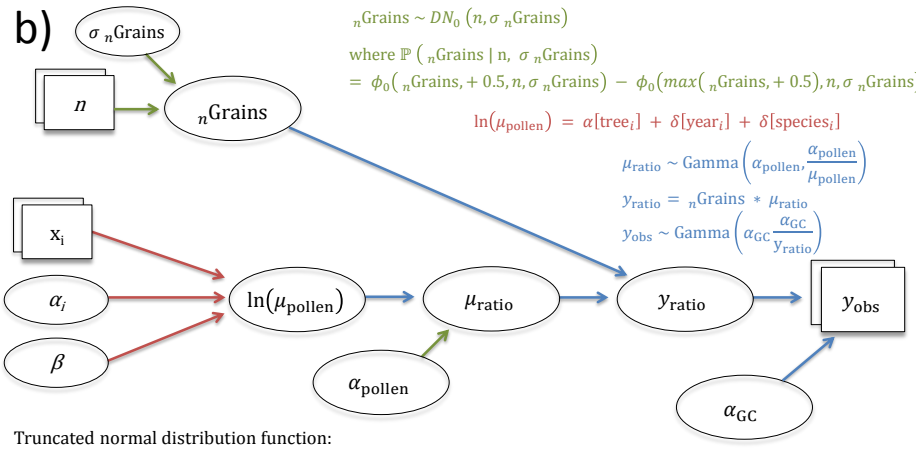
$$y_{\text{obs}} \sim \text{Gamma}\left(\alpha_{\text{GC}}, \frac{\alpha_{\text{GC}}}{y_{\text{ratio}}}\right)$$

Truncated normal distribution function:

$$\phi_0(x, \mu, \sigma) = \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\left[1 - \phi\left(\frac{0-\mu}{\sigma}\right)\right]}$$

$$= \frac{\int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{t^2}{2}}\, dt}{\left[\sqrt{2\pi} - \int_{-\infty}^{\frac{\mu}{\sigma}} e^{-\frac{t^2}{2}}\, dt\right]}$$

b)



$${}_n\text{Grains} \sim DN_0\left(n, \sigma\ {}_n\text{Grains}\right)$$

where $\mathbb{P}\left({}_n\text{Grains} \mid \text{n},\ \sigma\ {}_n\text{Grains}\right)$

$$= \phi_0\left({}_n\text{Grains}, +0.5, n, \sigma\ {}_n\text{Grains}\right) - \phi_0\left(max\left({}_n\text{Grains}, +0.5\right), n, \sigma\ {}_n\text{Grains}\right)$$

$$\ln(\mu_{\text{pollen}}) = \alpha[\text{tree}_i] + \delta[\text{year}_i] + \delta[\text{species}_i]$$

$$\mu_{\text{ratio}} \sim \text{Gamma}\left(\alpha_{\text{pollen}}, \frac{\alpha_{\text{pollen}}}{\mu_{\text{pollen}}}\right)$$

$$y_{\text{ratio}} = {}_n\text{Grains} * \mu_{\text{ratio}}$$

$$y_{\text{obs}} \sim \text{Gamma}\left(\alpha_{\text{GC}}, \frac{\alpha_{\text{GC}}}{y_{\text{ratio}}}\right)$$

Truncated normal distribution function:

$$\phi_0(x, \mu, \sigma) = \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\left[1 - \phi\left(\frac{0-\mu}{\sigma}\right)\right]}$$

$$= \frac{\int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{t^2}{2}}\, dt}{\left[\sqrt{2\pi} - \int_{-\infty}^{\frac{\mu}{\sigma}} e^{-\frac{t^2}{2}}\, dt\right]}$$