The Optimum Conditions of the Barlat-Lian Yield Criteria to Investigate the Formability of FCC and BCC Materials

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This paper presents the optimum exponent of Barlat-Lian (1989) yield function which defines the yield surface shape for different materials under plane stress condition. The predictions of the optimum exponents of Barlat-Lian (1989) yield function are derived and then compared with the experimental data for these materials to allow a better representation of the plastic behavior of the orthotropic sheet metals. The results show that contrary to what it was supposed to be up to now the optimum exponent of Barlat-Lian could be have different values for verity of face and body centered cubic types of materials case by case.

**Nomenclature:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Principal stress components |  | Thickness of sheet |
|  | Yield function |  | Initial surface roughness |
|  | Barlat-Lian yield exponent |  | Grain size coefficient |
| , | Anisotropy coefﬁcients at and from rolling direction |  | Effective strain |
|  | Normal anisotropy coefﬁcient |  | Grain size |
|  | Principal strain component |  | Initial coefﬁcient of heterogeneity |
| dγ | Proportionality factor |  | Coefﬁcient of heterogeneity |

*Conflict of Interest: The authors declare that they have no conflict of interest*

**Introduction**

In order to achieve the safe formability against occurred defects throughout forming process, forming limit diagram (FLD) commonly is used. The FLD is a plot of the major strains versus the minor strain which is obtained at the onset of localized necking condition (Nguyen et al., 2009). This curve defines the maximum permissible major strain with respect to the minor strain above which failure may be occurred in the deformed sheet (Darabi et al., 2017; Alipour et al., 2019; Habibi et al., 2018a; Al-Ghamdi and Hussain, 2016; Ahmed et al., 2010; Cui et al., 2019; Habibi et al., 2018b). The plasticity plays a fundamental role in designing metal products and controlling the process. In plasticity, yield functions are critical because they provide the yielding point of the material and also when used within an associated flow rule scheme, they describe the plastic flow of the metal accurately (Otorabad et al., 2018; Cardoso and Adetoro, 2017; Bae et al., 2017; Khalatbari et al., 2015; Hosseini et al., 2018; Babaei and Darvizeh, 2011). The FLD concept was introduced by Keeler (Ghazanfari et al., 2016; Keeler, 1968) and Goodwin (Habibi et al., 2016; Goodwin, 1968) for the first time in 1968. Based on geometric imperfection, another well-known methodology was proposed by Marciniak and Kuczynski (M-K) (Marciniak and Kuczyński, 1967; Habibi et al., 2017), which was studied and used extensively in prediction of sheet metal forming. They developed another approach for localized necking prediction. The latter is based on the introduction of an initial imperfection, which ultimately triggers the occurrence of localized necking. Hill (Hill, 1952) was one of the pioneers in developing the yield functions for plastic anisotropy. Barlat and Lian (Barlat and Lian, 1989) extended the proposed isotropic approach by Hosford (Hosford, 1972) to planar anisotropy by using stress tensor invariants (including shear stress terms). They attributed the amount of 6 to body center cubic (BCC) and 8 to face centered cubic (FCC) as exponent of yield criteria (Banabic, 2010). Besides Hill’s quadratic yield model, many other anisotropic yield models have been developed. For example, the famous plane stress yield function, Yld2000-2d, was proposed by Barlat et al. (Barlat et al. , 2003) to describe the anisotropic plastic deformation of sheet metals. The Yld2000-2d yield function involves eight parameters which can be determined by the yield stresses and r-values along 0°, 45°, and 90°, and the equibiaxial tension direction (Wang et al., 2019; Dick et al. ,2016). To provide more accurate predictions, yield locus expressions with more coefficients have been introduced, which necessitate more material tests for calibration. For example, the Yld2004-18p (Yoon et al., 2006) yield function includes 18 parameters; the criterion BBC2008 (Comsa et al., 2008) needs 16 or 24 parameters; and the CPB06 (Plunkett et al. , 2008) yield locus may contain 28 anisotropy coefficients.

In this study, a precise procedure in getting the best exponent in Barlat-Lian yield criterion is exhibited by approaching from the theoretical viewpoints. The ability of this yield criterion to represent the plastic behavior for different sheet metals is investigated.

**Mathematical Formulation**

**Barlat-Lian Theory**

In 1989, Barlat-Lian published a new generalization of Hosford’s criterion [10] for materials exhibiting planar anisotropy by introducing the following yield function. Eq. 1 shows the Barlat-Lian yield function. Here f is yield function while and are invariants of the stress tensor while is an integer exponent of Barlat-Lian yield criterion (Eq.1); this exponent is 8 for FCC and 6 for BCC materials (Hosford, 1979). Where and coefficients are obtained from Eq.2 & 3 while and are principle stresses and is the effective stress.

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |

The coefficients and can be determined as Eq. 4, 5. The and are anisotropy coefficients.

In order to evaluate these parameters, samples at 0°, 45°, and 90° with respect to the rolling direction have to be machined from the metal blank and tested under uniaxial tensile conditions and can be derived while the strains in the width and thickness are  and respectively.

|  |  |
| --- | --- |
|  | (4) |
|  | (5) |

For planar isotropy (), the coefficient () and parameters c is as follows:

|  |  |
| --- | --- |
|  | (6) |

Where dγ is the plastic multiplier and is the plastic potential defined by the yield criterion. The flow rule can be written as Eq.7:

|  |  |
| --- | --- |
|  | (7) |

Using Barlat-Lian yield criterion, the effective strain increment-effective stress ration is expressed as:

|  |  |
| --- | --- |
|  | (8) |

**Marciniak and Kuczynski (M-K) Model**

Plastic instability (Marzani et al., 2012; Banichuk et al., 2013) in sheets is investigated by Marciniak and Kuczynski. The existence of an initial imperfection has been assumed by M-K model, a groove through the entire material due to surface roughness, which leads a locally weak area. In this study, the plane stress condition is supposed, and the roll direction is also taken as a principal direction of strain. In the M-K model, the homogeneous and inhomogeneous zones have referred as ”A,” and ”B” respectively. This model for a metal sheet under biaxial tension is shown in Fig 1.

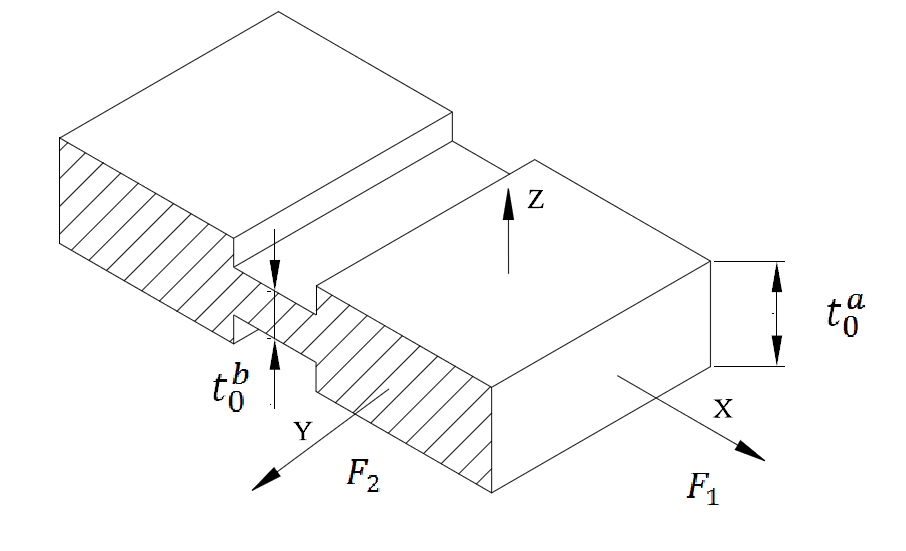


Fig. 1. Barlat-Lian model

The coefficient of initial heterogeneity in the general framework of M-K model is expressed as Eq. 9:

|  |  |
| --- | --- |
|  | (9) |

Where and are the initial thickness of homogenous and inhomogeneous regions, respectively. The thickness of both regions changes during metal forming process which is defined as Eq. 10:

|  |  |
| --- | --- |
|  | (10) |

Where and are the thickness strains. Also, the initial thickness of heterogeneous area ( can be explained based on the homogenous region ( as follow:

|  |  |
| --- | --- |
|  | (11) |

Here is the initial surface roughness, is the coefficient of grain size, is the grain size and is the effective strain in the inhomogeneous region(Barlat and Lian, 1989; Gronostajski and Zimniak, 1992; Assempour and Nurcheshmeh, 2003). Hence, the coefficient of heterogeneity can be defined as:

|  |  |
| --- | --- |
|  | (12) |

The compatibility requirement between the regions “A” and “B” yields to equality of strains at direction “2” as Eq.13:

|  |  |
| --- | --- |
|  | (13) |

Both safe (A) and defected (B) zones are loaded by a similar force (F1). This equilibrium condition is expressed as below:

|  |  |
| --- | --- |
|  | (14) |

For the Incompressibility material, the volume is constant and the deformation defines the relations between strains. Eq. 15 shows the volume strains are constant in this relation.

|  |  |
| --- | --- |
|  | (15) |

It has been assumed that the localization occurs in the groove when a critical strain reaches in the homogeneous region based on M-K model. Then, the values of strain increments in two regions of homogeneous and heterogeneous are compared with specific amount (e.g.,) and the material major and minor strain limits are obtained for the forming limit diagrams (Hashemi and Karajibani, 2018).

**Identification Procedure**

In experimental procedure, two type of stainless steel including AISI 321, 430 and copper, aluminum alloys including 7075-T6 and 3003-H24 (Dariani and Azodi, 2003) and carbon steel St-12 (Sadough et al., 1999) were used. Stretching the metallic sheet samples was done by a hemispherical punch to obtain the FLD. The punch-stretch apparatus sheets were prepared according to (Shakeri et al., 2000). The tensile testing is performed at a crosshead speed 5 mm/min. The initial circular grids were printed on surface of sheets with 3 mm diameters in order to measure the minor and major diameters of ovals after notching and deforming which illustrate the FLD. The strains were measured directly from specimens by using a profile projector. The scheme of hemispherical punch and its setup are shown in Fig.2 before and after sheet forming process.

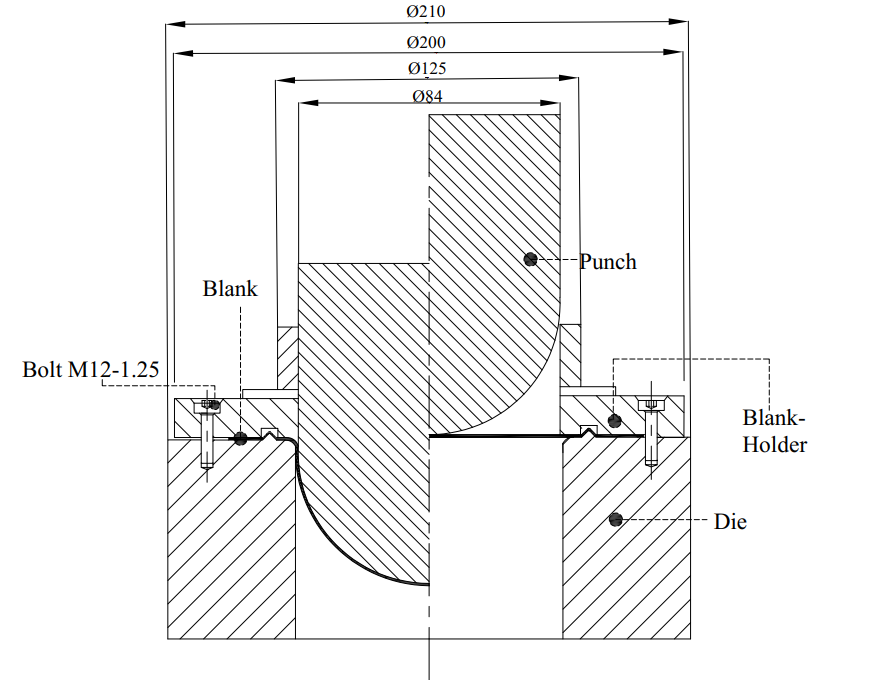


Fig. 2 scheme of hemispherical punch and its dimensions

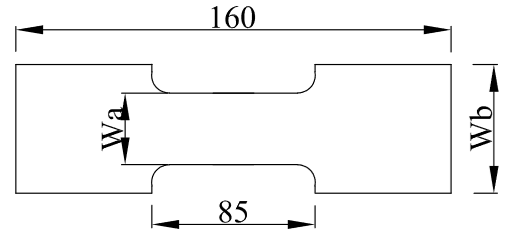
Fig.3 shows the circular and rectangular specimens with various widths. Table 1 shows the properties of used materials.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Samples (a) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **(a)** | Samples (a) 1 2 3 4 5 6 7 8 9 L (mm) 25 25 30 30 40 40 45 45 45 R (mm) 2.5 5 2.5 5 5 10 2.5 5 10 |  |  |  |  |  |  |  |  |

&

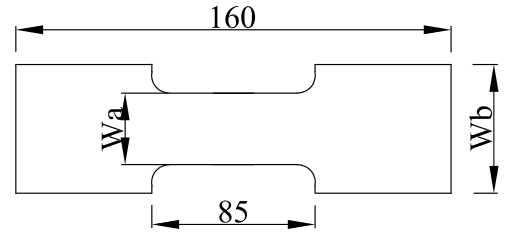
|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Samples (a) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| L (mm) | 25 | 25 | 30 | 30 | 40 | 40 | 45 | 45 | 45 |
| R (mm) | 2.5 | 5 | 2.5 | 5 | 5 | 10 | 2.5 | 5 | 10 |

**(b)**

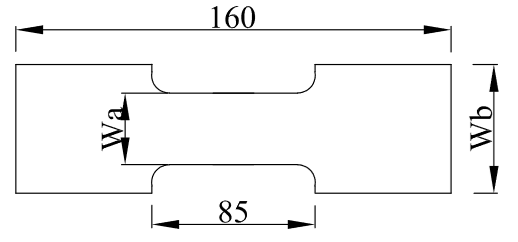


&

**(b)**



&



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Samples (b) | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| L (mm) | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| R (mm) | 25 | 35 | 45 | \_ | \_ | \_ | \_ | \_ |

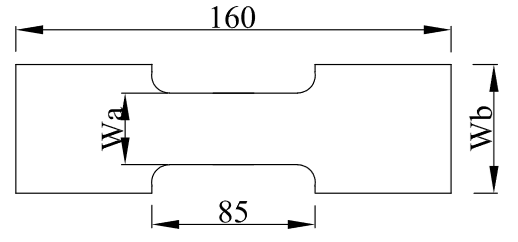
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|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Samples (b) | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| L (mm) | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| R (mm) | 25 | 35 | 45 | \_ | \_ | \_ | \_ | \_ |

&

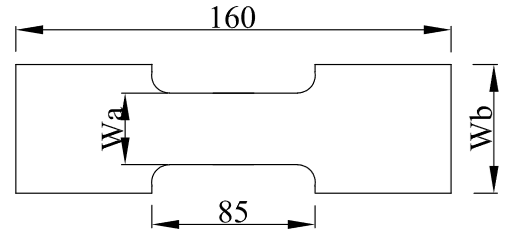
|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Samples (b) | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| L (mm) | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| R (mm) | 25 | 35 | 45 | \_ | \_ | \_ | \_ | \_ |

**(c)**



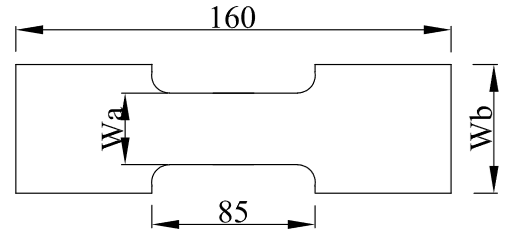
&

**(c)**



&

**(c)**



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Material |  |  |  |  |  |
| AISI 321 (Dariani and Azodi, 2003) | 0.012 | 0.29 | 0.84 | 0.025 | 0.005 |
| AISI 430 (Dariani and Azodi, 2003) | 0.009 | 0.23 | 1.04 | 0.028 | 0.005 |
| AA 7075-T6 (Dariani and Azodi, 2003) | 0 | 0.09 | 0.75 | 0.05 | 0.0055 |
| AA 3003-H24 (Dariani and Azodi, 2003) | 0.003 | 0.08 | 0.715 | 0.044 | 0.0055 |
| Copper (Dariani and Azodi, 2003) | 0.005 | 0.36 | 0.73 | 0.035 | 0.0055 |
| St 12 (Sadough et al., 1999) | 0.01 | 0.3 | 1.21 | 0.025 | 0.006 |

**Comparison with Experiments**

The curved fitting of experimental values are obtained with Matlab software and evaluating of different powers of Barlat-Lian yield criteria can be estimated with comparison between experimental and achieved numerical strains of different powers of Barlat-Lian yield criteria. The R-square is expressed in Eqs.16,17 and 18.Where SSE and SST are the sum of squared errors of our regression model and the sum of squared errors of our baseline model respectively. The factor gives the overall predictive capabilities of the obtained model and Minimum differences are the best fitted values. Root Mean Squared Error (RMSE) is also known as the fit standard error and the standard error of the regression and closer to 0 indicates a better fit (Eq.19).

|  |  |
| --- | --- |
|  | (16) |
|  | (17) |
|  | (18) |
|  | (19) |

**Results and Discussion**

Fig 4 shows the forming limit diagrams for different expotent of Barlat-Lian which is applied for AA3003-H24 and the table 2 demonstrates that a=5.9 may have the best fitting with experimental results. Aluminum is F.C.C material and the predefined exponent is 8 but the results show that the best power amount is 5.9.

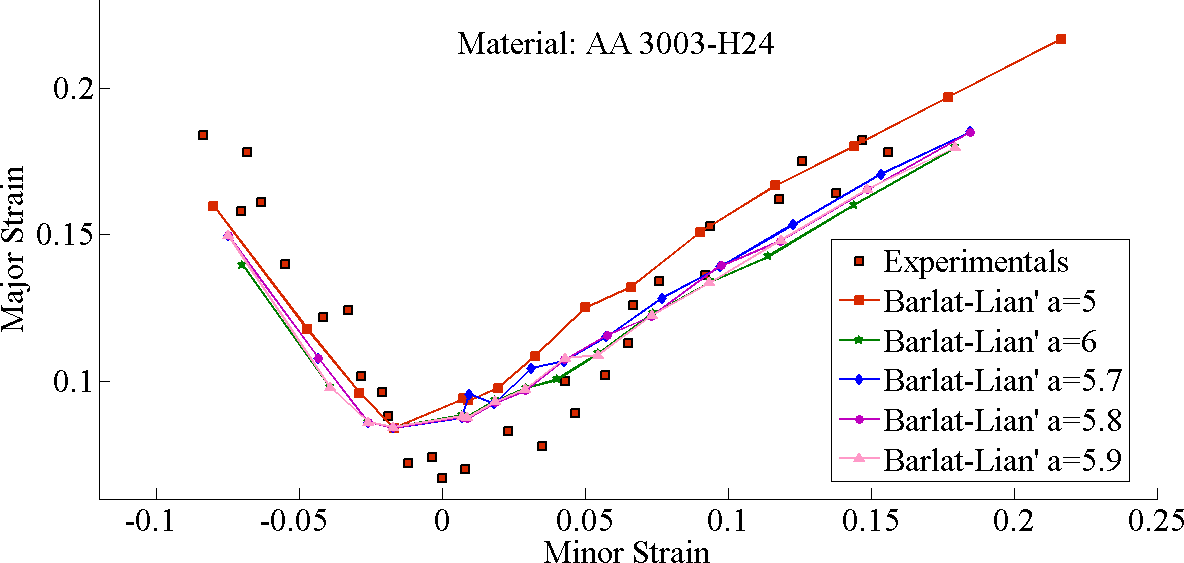


Fig. 4 Forming limit diagrams of AA3003-H24 sheet (Experimental data from Dariani and Azodi, 2003)

Table 2. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

The numerical results are achieved to investigate the best fitting curve for AA7075-T6 in comparison with experimental data based on Fig.5 and the table 3 shows that a=6.6 can present the least RMSE value. Although AA7075-T6 is face centered cubic (F.C.C), amount of archived Barlat-lian power is 6.6.

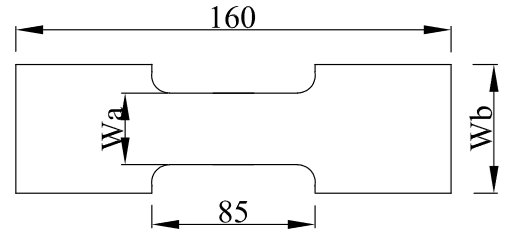


Fig. 5 Forming limit diagrams of AA7075-T6 sheet (Experimental data from Dariani and Azodi, 2003)

Table 3. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

Another tests and numerical approaches have been done to determine the best power of Barlat Lian yield criteria for AISI 321 in Fig.6 . It can be seen the best fitting results are derived from a=4.6 as it is shown in table 4. AISI 321 is titanium (Ti)-stabilized austenitic stainless steel and has B.C.C structure. It has been assumed in all stainless steel, a=6 otherwise in this study obtained a is 4.6.

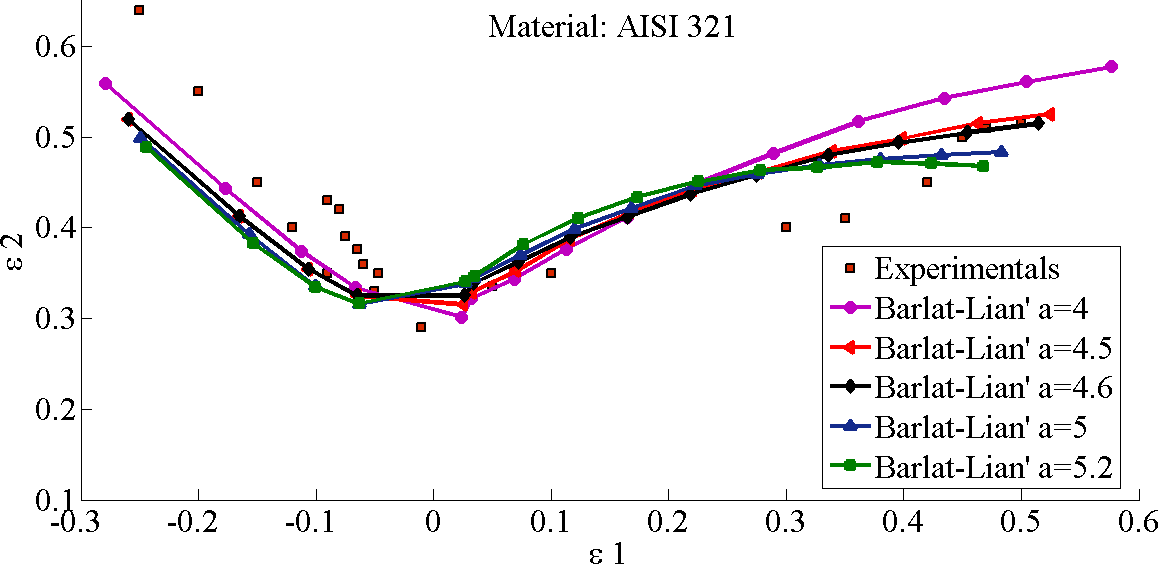


Fig. 6 Forming limit diagrams of stainless steel AISI 321 sheet (Experimental data from Dariani and Azodi, 2003)

Table 4. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

Figure 7 and table 5 show the results of numerical and experimental method to obtain the best exponent of Barlat-Lian for AISI 430. AISI 430 has the B.C.C structure and Contrary to previous assumptions has the least square in regression.

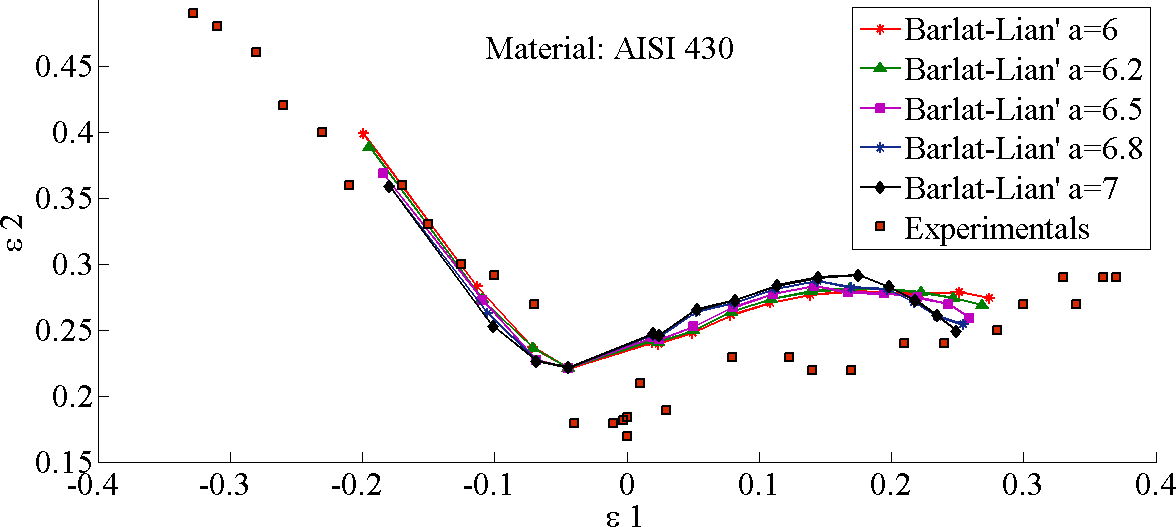


Fig. 7 Forming limit diagrams of stainless steel AISI 430 sheet (Experimental data from Dariani and Azodi, 2003)

Table 5. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

Copper was studied to derive the best exponent of Barlat-Lian. It can be seen that the best power is a=6.5 in table 6, while Fig 7 shows that most of left sided is not covered by numerical results. So the r squares have the large amount in comparison with other materials in this study. Cu is F.C.C material and can cover experimental results sufficiently although it had been assumed that this amount is 8 for Copper.

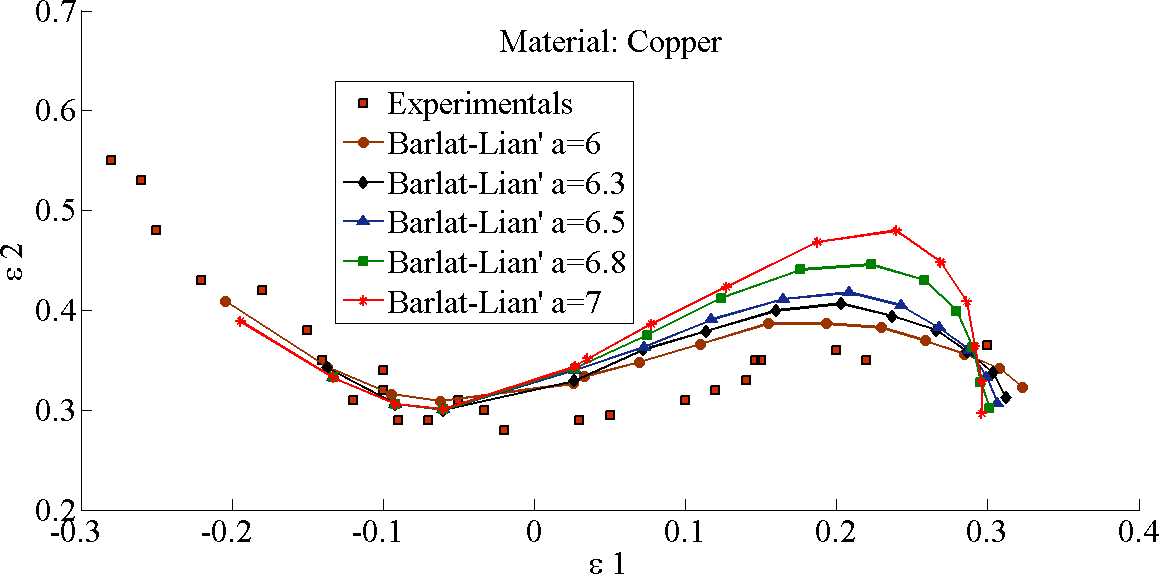


Fig. 7 Forming limit diagrams of Copper sheet (Experimental data from Dariani and Azodi, 2003)

Table 6. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

Figure 8 and Table 7 represent the best achieved amount of Barlat-Lian exponent for St 12. The investigation shows that a=4.8 is the best curve fitted value. St 12 is low carbon steel and has the FCC structure and archived Barlat-Lian exponent is different by assumption.

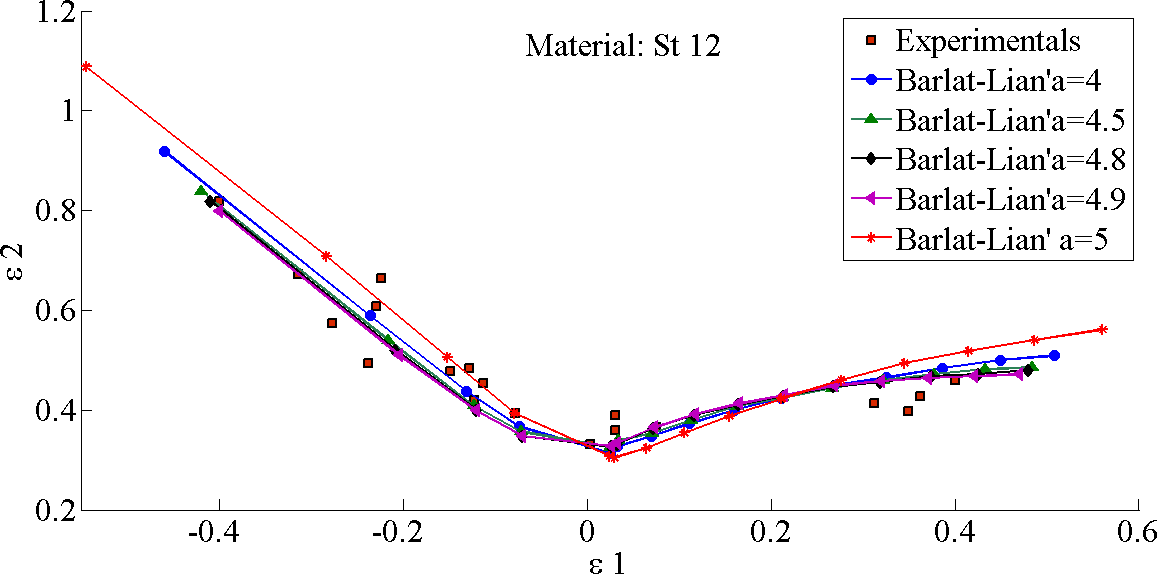


Fig. 8 St 12 Forming limit diagrams of St-12 sheet (Experimental data from Sadough et al., 1999)

Table 7. RMSE for different powers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **St 12** | a=4 | a=4.5 | a=4.8 | a=4.9 | a=5 | a=6 | a=8 |
| RMSE | 0.703829 | 0.581941 | 0.565814 | 0.618761 | 0.634552 | 1.089167 | 1.273228 |

Table 8 shows the best power of Barlat-Lian for different materials.

Table 8 the best power of Barlat-Lian for different materials.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Material | AA3003-H24 | AA7075-T6 | AISI 321 | AISI 430 | Copper | St 12 |
| Best power | 5.9 | 6.7 | 4.6 | 6.5 | 6.5 | 4.8 |

**Conclusions:**

* Evaluation of r-squares for considered materials shows that the FLD of Aluminum alloys can be predicted well by Barlat-Lian yield criteria though FLD of stainless steels whether low or high carbon, associated with this yield criteria have less accuracy. The derived FLD of Copper has the highest amount of RMSE that comes from less compatibility of this yield theory for this material.
* For the aluminum alloys including AA3003-H24 and AA7075-T6, the exponent of Barlat-Lian are 5.9 and 6.6 respectively. The assumption presented this amount for FCC material like aluminum equals 8.
* The investigated Stainless steel including AISI 321 and AISI 430 demonstrate better power of Barlat-Lian yield criteria are 4.6 and 6.5 respectively however the theory assumed this amount for B.CC material equals 6.
* Although Copper is F.C.C material and anticipates this power 8, the achieved exponent is 6.5.
* For Low carbon stainless steel like St 12 with F.C.C structure, the exponent of Barlat-Lian hypothesis is 4.8.

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