Dynamics of an IPM pest-predator model with impulses and stage structure on predator population *

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Abstract. Integrated pest management(IPM) has been promoted as an environmentally friendly pest control approach. It utilizes a combination of control methods to control pest populations in agricultural and forestry systems. In this paper, we propose an IPM pest-predator model with impulses and stage structure on predator population, where the predator population is divided into two stages, a juvenile stage and a mature stage. The mature predator's predation conversion for production of new predators. This kind of stage-structured pest-predator model has been omitted in the mathematical models for integrated pest management. The dynamical properties for the pest-extinction solution and permanence of system (2.1) are established. The simulations are employed to support the proofs. Our results provide a good balance between the biological control and chemical control for integrated pest management.

Keywords: Stage-structured pest-predator model; Releasing juvenile predator; Spaying pesticides; Pest-extinction

1 Introduction

Pests threaten to the environments, from health issues to property damage. Spraying chemical pesticides for controlling pest is less expensive and destroys pest rapidly but causes high environmental loss. While biological control are lengthy and expensive process to use, but with very little environmental loss. Considering with aspect of environmental loss and economic costs of controls, combination of chemical and biological agents would give better outcomes $[1, 2]$. Integrated pest management $[3-6]$ has been promoted as an environmentally friendly pest control approach, which utilizes a combination of control methods to control pest populations in agricultural and forestry systems. Recently, integrated pest management has attracted the attention of many

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mathematical biologist [7 − 10]. Sun et al. [11] presented an integrated pest management predator-prey model, where the yield of releases of predator and the strength of pesticide spraying are linearly dependent on the selected control level, they aimed at providing a good balance between the biological control and chemical control. These works omitted stage-structured pest-predator models in the mathematical models for integrated pest management. Akman et al. [12] constructed a stage structured impulsive integrated pest management with added prey refuge, they discussed conditions for globally asymptotically stability of the pest eradication solution and the permanence of the model. Obviously, it is unreasonable for considering the releasing predator and spraying pesticides at the same moments in the reality of integrated pest management in [12].

2 The model

Inspired by the above discussion, we propose an IPM pest-predator model with impulses and stage structure on predator population

$$\begin{cases} \left.\begin{aligned} &\frac{dx(t)}{dt} = x(t)(a - bx(t)) - \frac{\beta x(t)y_2(t)}{1+\theta x(t)}, \\ &\frac{dy_1(t)}{dt} = \frac{k\beta x(t)y_2(t)}{1+\theta x(t)} - (c+d_1)y_1(t), \\ &\frac{dy_2(t)}{dt} = cy_1(t) - d_2y_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} &\triangle x(t) = 0, \\ &\triangle y_1(t) = p, \\ &\triangle y_2(t) = 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} &\triangle x(t) = -\mu x(t), \\ &\triangle y_1(t) = -\mu_1 y_1(t), \\ &\triangle y_2(t) = -\mu_2 y_2(t), \end{aligned}\right\} t = (n+1)\tau, \end{cases} \tag{2.1}$$

where $x(t)$ is the density of the pest population at time t. $y_1(t)$ is the density of the immature predator (natural enemy) population at time t. $y_2(t)$ is the density of mature predator (natural enemy) population at time t. Intrinsic rate of natural increase and density dependence rate of pest population are denoted by $a > 0, b > 0$ respectively. $\frac{a}{b}$ denotes the carrying capacity of the prey population. The term $\frac{\beta x(t)}{1+\theta x(t)}$ represents Holling type II function response of the mature predator with $\theta > 0$. The mature predator population consumes pest population with predation coefficients $\beta > 0$. Conversion rate $k > 0$ represents the conversion of feeding to production of new predators. $c > 0$ is called the rate of immature predator population turning into mature predator population. $d_1 > 0$ is the natural death rate of the immature predator population. d_2 is the natural death rate of the mature predator population. $p > 0$ is the releasing amount of the immature predator population at $t = (n + l)\tau$. $0 < \mu < 1$ is the effect of spraying pesticides on pest population at $t = (n + 1)\tau$. $0 < \mu_1 < 1$ is the effect of spraying pesticides on immature predator population at $t = (n+1)\tau$. $0 < \mu_2 < 1$ is the effect of spraying pesticides on mature predator population at $t = (n+1)\tau$. $0 < l < 1$ is the interval between spraying pesticides moments and releasing predator population moments. $\tau > 0$ represents the impulsive releasing predator population period or impulsive spraying pesticides period.

3 Some lemmas

Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand of system (2.1). The solution of system (2.1), denoted by $Z(t) = (x(t), y_1(t), y_2(t))^T$, is a piecewise continuous function Z: $R_+ \to R^3_+$, where $R_+ = [0, \infty)$, $R^3_+ = \{Z \in R^3 : Z > 0\}$. $Z(t)$ is continuous on $(n\tau, (n+l)\tau] \times R^3_+$ and $((n+l)\tau, (n+1)\tau] \times R^3_+ (n \in Z_+, 0 \leq l \leq 1)$. According to Reference[13, 14], the global existence and uniqueness of solutions of system (2.1) is guaranteed by the smoothness properties of f, which denotes the mapping defined by right-side of system (2.1).

Considering the auxiliary system

$$\begin{cases} \left.\begin{aligned} \frac{du_1(t)}{dt} &= \lambda - (c+d_1)u_1(t), \\ \frac{du_2(t)}{dt} &= cy_1(t) - d_2u_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle u_1(t) &= p, \\ \triangle u_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle u_1(t) &= -\mu_1 u_1(t), \\ \triangle u_2(t) &= -\mu_2 u_2(t), \end{aligned}\right\} t = (n+1)\tau. \end{cases} \tag{3.1}$$

We can easily obtain the analytic solution of system (3.1) between impulses

$$\begin{cases} u_1(t) = \begin{cases} \dfrac{\lambda(1 - e^{-(c+d_1)(t-n\tau)})}{c+d_1} \\ \qquad +u_1(n\tau^+)e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ \dfrac{\lambda(1 - e^{-(c+d_1)(t-(n+l)\tau)})}{c+d_1} \\ \quad +u_1((n+l)\tau^+)e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ u_2(t) = e^{-d_2(t-n\tau)}[\dfrac{c(1 - e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times u_1(n\tau^+) \\ \qquad\qquad +u_2(n\tau^+)], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{3.2}$$

We have the following stroboscopic map of system (3.1) with considering the third, fourth,fifth and sixth equations of system (3.1)

$$\begin{cases} u_1((n+1)\tau^+) = (1-\mu_1)e^{-(c+d_1)\tau}u_1(n\tau^+) \\ \qquad +\dfrac{(1-\mu_1)\lambda(1-e^{-(c+d_1)\tau})}{c+d_1} + (1-\mu_1)pe^{-(c+d_1)(1-l)\tau}, \\ u_2((n+1)\tau^+) = \dfrac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} \times u_1(n\tau^+) \\ \qquad +(1-\mu_2)e^{-d_2\tau}u_2(n\tau^+). \end{cases} \tag{3.3}$$

Making notations as

$$A = (1-\mu_1)e^{-(c+d_1)\tau} < 1,$$

$$B = \frac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} > 0,$$

$$C = (1-\mu_2)e^{-d_2\tau} < 1,$$

and

$$D = \frac{(1-\mu_1)\lambda(1-e^{-(c+d_1)\tau})}{c+d_1} + (1-\mu_1)pe^{-(c+d_1)(1-l)\tau} > 0,$$

we can rewrite system (3.3) as

$$\begin{cases} u_1((n+1)\tau^+) = Au_1(n\tau^+) + D, \\ u_2((n+1)\tau^+) = Bu_1(n\tau^+) + Cu_2(n\tau^+). \end{cases} \tag{3.4}$$

The unique positive fixed points of (3.4) (or (3.3)) is obtained as $G(u_1^*, u_2^*)$, where

$$\begin{cases} u_1^* = \dfrac{D}{1-A} > 0, \\ u_2^* = \dfrac{B}{1-C} \times u_1^* > 0. \end{cases} \tag{3.5}$$

Then, we can obtain auxiliary lemmas.

Lemma 3.1. The fixed point $G(u_1^*, u_2^*)$ of (3.4) (or (3.3)) is globally asymptotically stable.

Proof. For convenience, we make a notation as $(u_1^n, u_2^n) = (u_1(n\tau^+), u_2(n\tau^+))$. The linear form of (3.4) (or (3.3)) can be written as

$$\begin{pmatrix} u_1^{n+1} \\ u_2^{n+1} \end{pmatrix} = M \begin{pmatrix} u_1^n \\ u_2^n \end{pmatrix}. \tag{3.6}$$

Obviously, the near dynamics of $G(u_1^*, u_2^*)$ of (3.4) (or (3.3)) is determined by linear system (3.6). The stabilities of $G(u_1^*, u_2^*)$ is determined by the eigenvalue of M less than 1. We can know the eigenvalue of M less than 1, if M satisfies the *Jury* criteria [9]

$$1 - trM + \det M > 0. \tag{3.7}$$

with

$$M = \begin{pmatrix} A & 0 \\ B & C \end{pmatrix}, \tag{3.8}$$

Calculating

$$\begin{aligned} 1 - trM + \det M &= 1 - [(A+C) + AC] \\ &= (1-A)(1-C) > 0. \end{aligned}$$

From *Jury* criteria, $G(u_1^*, u_2^*)$ is locally stable. Furthermore, it is globally asymptotically stable. This completes the proof.

Similarly with Reference [15], the following lemma can easily be proved.

Lemma 3.2. The periodic solution $(\widetilde{u_1(t)}, \widetilde{u_2(t)})$ of system (3.1) is globally asymptotically stable, where

$$\begin{cases} \widetilde{u_1(t)} = \begin{cases} \dfrac{\lambda(1-e^{-(c+d_1)(t-n\tau)})}{c+d_1} \\ \qquad +u_1^* e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ \dfrac{\lambda(1-e^{-(c+d_1)(t-(n+l)\tau)})}{c+d_1} \\ \quad +u_1^{**} e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ \widetilde{u_2(t)} = e^{-d_2(t-n\tau)}[\dfrac{c(1-e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times u_1^* \\ \qquad +u_2^*], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{3.9}$$

where u_1^*, u_2^* are determined as (3.5), and $u_1^{**} = \frac{\lambda(1-e^{-(c+d_1)l\tau})}{c+d_1} + u_1^* e^{-(c+d_1)l\tau} + p$.

If $x(t) = 0$, one can have the subsystem of system (2.1) as

$$\begin{cases} \left.\begin{aligned} \frac{dy_1(t)}{dt} &= -(c+d_1)y_1(t), \\ \frac{dy_2(t)}{dt} &= cy_1(t) - d_1 y_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau \\ \left.\begin{aligned} \triangle y_1(t) &= p, \\ \triangle y_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle y_1(t) &= -\mu_1 y_1(t), \\ \triangle y_2(t) &= -\mu_2 y_2(t), \end{aligned}\right\} t = (n+1)\tau. \end{cases} \tag{3.10}$$

We can easily obtain the analytic solution of system (3.10) between impulses

$$\begin{cases} y_1(t) = \begin{cases} y_1(n\tau^+)e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ y_1((n+l)\tau^+)e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ y_2(t) = e^{-d_2(t-n\tau)}[\dfrac{c(1-e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times y_1(n\tau^+) \\ \qquad +y_2(n\tau^+)], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{3.11}$$

We have the following stroboscopic map of system (3.10) with considering the third, fourth,fifth and sixth equations of system (3.10)

$$\begin{cases} y_1((n+1)\tau^+) = (1-\mu_1)e^{-(c+d_1)\tau}y_1(n\tau^+) + (1-\mu_1)pe^{-(c+d_1)(1-l)\tau}, \\ y_2((n+1)\tau^+) = \dfrac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} \times y_1(n\tau^+) \\ \qquad\qquad\qquad\qquad +(1-\mu_2)e^{-d_2\tau}y_2(n\tau^+). \end{cases} \tag{3.12}$$

Obviously, system (3.10) is equivalent to system (3.1) with $\lambda = 0$. Furthermore, one can similarly have

Lemma 3.3. The fixed point $G(y_1^*, y_2^*)$ of (3.12) is globally asymptotically stable.Here y_1^* and y_2^* are defined as

$$\begin{cases} y_1^* = \dfrac{D_1}{1-A_1} > 0, \\ y_2^* = \dfrac{B_1}{1-C_1} \times y_1^* > 0. \end{cases} \tag{3.13}$$

with

$$A_1 = (1-\mu_1)e^{-(c+d_1)\tau} < 1,$$
$$B_1 = \frac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} > 0,$$
$$C_1 = (1-\mu_2)e^{-d_2\tau} < 1,$$

and

$$D_1 = (1-\mu_1)pe^{-(c+d_1)(1-l)\tau} > 0,$$

Lemma 3.4. The periodic solution $(\widetilde{y_1(t)}, \widetilde{y_2(t)})$ of system (3.10) is globally asymptotically stable, where

$$\begin{cases} \widetilde{y_1(t)} = \begin{cases} y_1^* e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ y_1^{**} e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ \widetilde{y_2(t)} = e^{-d_2(t-n\tau)}[\dfrac{c(1-e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times y_1^* \\ \qquad\qquad\qquad +y_2^*], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{3.14}$$

where y_1^*, y_2^* are determined as (3.13), and $y_1^{**} = y_1^* e^{-(c+d_1)l\tau} + p$.

Remark 3.5. From lemma 3.4., there exists a ε_0 small enough, we can obtain that

$$y_1(t) \geq (y_1^* + y_1^{**}) - \varepsilon_0 \triangleq \rho_1,$$

and

$$y_2(t) \geq (\frac{c(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} y_1^* + y_2^*) - \varepsilon_0 \triangleq \rho_2.$$

Lemma 3.6. For each solution $(x(t), y_1(t), y_2(t))$ of system (2.1), there exists a constant $M_1 > 0$ such that $x(t) \le M_1, y_1(t) \le M_1, y_2(t) \le M_1$ with all t large enough.

Proof. Defining $V(t) = kx(t) + y_1(t) + y_2(t)$, and choosing $\xi = \min\{d_1, d_2\}$, for $t \neq (n+l)\tau$ and $t \neq (n+1)\tau$, we have

$$\begin{aligned} &D^+V(t) + \xi V(t) \\ &= kx(t)[(a-\xi) - bx(t)] - (d_1 - \xi)y_1(t) - (d_2 - \xi)y_2(t) \\ &\le -kb(x(t) - \frac{a-\xi}{2b})^2 + \frac{k(a-\xi)^2}{4b} \le M_0, \end{aligned}$$

where $M_0 = \frac{k(a-\xi)^2}{4b}$. When $t = (n+l)\tau, V((n+l)\tau^+) = V((n+l)\tau) + p$. When $t = (n+1)\tau, V((n+1)\tau^+) \le V((n+1)\tau)$. By lemma 2. [5], for $t \in (n\tau, (n+1)\tau]$ we have

$$\begin{aligned} V(t) &\le V(0)\exp(-\xi t) + \int_0^t M_0 \exp(-\xi(t-s))ds + \sum_{0<n\tau<t} p\exp(-\xi(t-n\tau)) \\ &= V(0)\exp(-\xi t) + \frac{M_0}{\xi}(1-\exp(-\xi t)) + p\frac{\exp(-\xi(t-\tau)) - \exp(-\xi(t-(n+1)\tau))}{1-\exp(\xi\tau)} \\ &< V(0)\exp(-\xi t) + \frac{M_0}{\xi}(1-\exp(-\xi t)) + \frac{p\exp(-\xi(t-\tau))}{1-\exp(\xi\tau)} + \frac{p\exp(\xi\tau)}{\exp(\xi\tau)-1} \\ &\to \frac{M_0}{\xi} + \frac{p\exp(\xi\tau)}{\exp(\xi\tau)-1}, \quad as \quad t\to\infty. \end{aligned}$$

So $V(t)$ is uniformly ultimately bounded. Furthermore, for any $\varepsilon > 0$ small enough, there exists $k_0 > 0$ large enough, one can have

$$V(t) < \frac{M_0}{\xi} + \frac{p\exp(\xi\tau)}{\exp(\xi\tau)-1} + \varepsilon,$$

for $t > k_0\tau + \tau_1$. Making notations as $M_1 \triangleq \frac{M_0}{\xi} + \frac{\mu\exp(\xi\tau)}{\exp(\xi\tau)-1} + \varepsilon$, we have $V(t) \le M_1$. By the definition of $V(t)$, we also have $x(t) \le M_1, y_1(t) \le M_1, y_2(t) \le M_1$ for t large enough. The proof is complete.

4 The dynamics

Firstly, we prove the prey-extinction periodic solution $(0, \widetilde{y_1(t)}, \widetilde{y_2(t)})$ of system (2.1) is globally asymptotically stable. Then, we prove System (2.1) is permanent.

Theorem 4.1. If

$$\begin{aligned} \ln\frac{1}{1-\mu_1} &> a\tau \\ &-\frac{\beta c y_1^*}{c+d_1-d_2} \times (\frac{1-e^{-d_2\tau}}{d_2} - \frac{1-e^{-(c+d_1)\tau}}{c+d_1}) - \frac{y_2^*}{d_2} \times (1-e^{-d_2\tau}), \end{aligned} \tag{4.1}$$

holds, the prey-extinction periodic solution $(0, \widetilde{y_1(t)}, \widetilde{y_2(t)})$ of System (2.1) is globally asymptotically stable. Where y_1^* and y_2^* are defined as (3.13).

Proof. Firstly, we prove the local stability. Define $x(t) = x(t), z_1(t) = y_1(t) - \widetilde{y_1(t)}, z_2(t) = y_2(t) - \widetilde{y_2(t)}$, we have the following linearly similar system of system (2.1)

$$\begin{pmatrix} \frac{dx(t)}{dt} \\ \frac{dz_1(t)}{dt} \\ \frac{dz_2(t)}{dt} \end{pmatrix} = \begin{pmatrix} a - \beta\widetilde{y_2(t)} & 0 & 0 \\ k\beta\widetilde{y_2(t)} & -(c+d_1) & 0 \\ 0 & c & -d_2 \end{pmatrix} \begin{pmatrix} x(t) \\ z_1(t) \\ z_2(t) \end{pmatrix}.$$

The fundamental solution matrix can be obtained

$$\Phi(t) = \begin{pmatrix} \exp(\int_0^t (a - \beta\widetilde{y_2(s)})ds) & 0 & 0 \\ * & \exp[-(c+d_1)t] & 0 \\ \dagger & \ddagger & \exp(-d_2 t) \end{pmatrix}.$$

There is no need to calculate the exact form of $*$, $\dagger$ and $\ddagger$ as they are not required in the analysis that follows. The linearization of the fourth, fifth and sixth equations of system (2.1) is

$$\begin{pmatrix} x((n+l)\tau^+) \\ z_1((n+l)\tau^+) \\ z_2((n+l)\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x((n+l)\tau) \\ z_1((n+l)\tau) \\ z_2((n+l)\tau) \end{pmatrix}.$$

The linearization of the seventh, eighth and ninth equations of system (2.1) is

$$\begin{pmatrix} x((n+1)\tau^+) \\ z_1((n+1)\tau^+) \\ z_2((n+1)\tau^+) \end{pmatrix} = \begin{pmatrix} 1-\mu & 0 & 0 \\ 0 & 1-\mu_1 & 0 \\ 0 & 0 & 1-\mu_2 \end{pmatrix} \begin{pmatrix} x((n+1)\tau) \\ z_1((n+1)\tau) \\ z_2((n+1)\tau) \end{pmatrix}.$$

The stability of the prey-extinction periodic solution $(0, \widetilde{y_1(t)}, \widetilde{y_2(t)})$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1-\mu & 0 & 0 \\ 0 & 1-\mu_1 & 0 \\ 0 & 0 & 1-\mu_2 \end{pmatrix} \Phi(\tau),$$

which are

$$\lambda_1 = (1-\mu)\exp[\int_0^\tau (a - \beta\widetilde{y_2(s)})ds],$$

$$\lambda_2 = (1-\mu_1)\exp[-(c+d_1)\tau] < 1,$$

and

$$\lambda_3 = (1-\mu_2)e^{-d_2\tau} < 1.$$

According the conditions of this theorem, we easily know that $(1-\mu)\exp(\int_0^\tau(a-\beta\widetilde{y_2(s)})ds)<1$, then $\lambda_1<1$. From the Floquet theory [5], the prey-extinction $(0,\widetilde{y_1(t)},\widetilde{y_2(t)})$ is locally stable.

The following work is to prove the global attraction, choose a small enough $\varepsilon>0$ such that

$$\rho=(1-\mu)e^{\int_0^\tau[a-\beta(\widetilde{y_2(s)}-\varepsilon)]ds}<1.$$

From the second equation of system (2.1), we notice that $\frac{dy_1(t)}{dt}\geq-(c+d_1)y_1(t)$, so we consider following comparative impulsive differential equation

$$\begin{cases}\left.\begin{aligned}\frac{dn_1(t)}{dt}&=-(c+d_1)n_1(t),\\ \frac{dn_2(t)}{dt}&=cn_1(t)-d_2n_2(t),\end{aligned}\right\}t\neq(n+l)\tau,t\neq(n+1)\tau,\\ \left.\begin{aligned}&\triangle n_1(t)=p,\\ &\triangle n_2(t)=0,\end{aligned}\right\}t=(n+l)\tau,\\ \left.\begin{aligned}&\triangle n_1(t)=-\mu_1n_1(t),\\ &\triangle n_2(t)=-\mu_2n_2(t),\end{aligned}\right\}t=(n+1)\tau.\end{cases}\tag{4.2}$$

From lemma 3.4. and comparison theorem of impulsive equation (see [5]), we have $y_1(t)\geq n_1(t),y_2(t)\geq n_2(t)$ and $n_1(t)\to\widetilde{y_1(t)},n_2(t)\to\widetilde{y_2(t)}$ as $t\to\infty$, that is

$$\begin{cases}y_1(t)\geq n_1(t)\geq\widetilde{y_1(t)}-\varepsilon,\\ y_2(t)\geq n_2(t)\geq\widetilde{y_2(t)}-\varepsilon,\end{cases}\tag{4.3}$$

for all t large enough. For convenience, we may assume(4.2) hold for all $t\geq0$. From (2.1) and (4.3), we get

$$\begin{cases}\frac{dx(t)}{dt}\leq x(t)[a-\beta(\widetilde{y_2(t)}-\varepsilon)],t\neq(n+1)\tau,\\ \triangle x(t)=-\mu x(t),t\neq(n+1)\tau.\end{cases}\tag{4.4}$$

So $x((n+1)\tau)\leq(1-\mu)x(n\tau^+)\exp(\int_{n\tau}^{(n+1)\tau}(a-\beta(\widetilde{y_2(s)}-\varepsilon))ds)$, hence $x((n+1)\tau)\leq x(0^+)\rho^n$ and $x((n+1)\tau)\to0$ as $n\to\infty$, Since $0<x(t)\leq x((n+1)\tau)e^{a\tau}$ for $n\tau<t\leq(n+1)\tau$, therefore $x(t)\to0$ as $t\to\infty$.

Next we prove that $y_1(t)\to\widetilde{y_1(t)},y_1(t)\to\widetilde{y_2(t)}$ as $t\to\infty$. For $\varepsilon\geq0$ small enough, there must exist a $t_0>0$ such that $0<x(t)<\varepsilon$ for all $t\geq t_0$. Without loss of generality, we assume that $0<x(t)<\varepsilon$ for all $t\geq0$. Then, for the second equation of system (2.1), we have

$$-(c+d_1)y_1(t)\leq\frac{dy_1(t)}{dt}\leq\frac{k\beta\varepsilon}{1+\theta\varepsilon}-(c+d_1)y_1(t),\tag{4.5}$$

then, we have $w_1(t) \le y_1(t) \le m_1(t), w_2(t) \le y_2(t) \le m_2(t)$, and $w_1(t) \to \widetilde{y_1(t)}, w_2(t) \to \widetilde{y_2(t)}, m_1(t) \to \widetilde{m_1(t)}, m_2(t) \to \widetilde{m_2(t)}$ as $t \to \infty$. While $(w_1(t), w_2(t))$ and $(m_1(t), m_2(t))$ are the solutions of

$$\begin{cases} \left.\begin{aligned} \frac{dw_1(t)}{dt} &= -(c+d_1)w_1(t), \\ \frac{dw_2(t)}{dt} &= cw_1(t) - d_2w_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle w_1(t) &= p, \\ \triangle w_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle w_1(t) &= -\mu_1 w_1(t), \\ \triangle w_2(t) &= -\mu_2 w_2(t), \end{aligned}\right\} t = (n+1)\tau, \end{cases} \tag{4.6}$$

and

$$\begin{cases} \left.\begin{aligned} \frac{dm_1(t)}{dt} &= \frac{k\beta M_1\varepsilon}{1+\theta\varepsilon} - (c+d_1)m_1(t), \\ \frac{dm_2(t)}{dt} &= cm_1(t) - d_2m_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle m_1(t) &= p, \\ \triangle m_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle m_1(t) &= -\mu_1 m_2(t)), \\ \triangle m_2(t) &= -\mu_1 m_2(t)), \end{aligned}\right\} t = (n+1)\tau, \end{cases} \tag{4.7}$$

Here $(\widetilde{m_1(t)}, \widetilde{m_2(t)})$ can be expressed as

$$\begin{cases} \widetilde{m_1(t)} = \begin{cases} \dfrac{\frac{k\beta M_1\varepsilon}{1+\theta\varepsilon}(1-e^{-(c+d_1)(t-n\tau)})}{c+d_1} \\ \qquad +m_1^* e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ \dfrac{\frac{k\beta M_1\varepsilon}{1+\theta\varepsilon}(1-e^{-(c+d_1)(t-(n+l)\tau)})}{c+d_1} \\ \qquad +m_1^{**} e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ \widetilde{m_2(t)} = e^{-d_2(t-n\tau)}[\dfrac{c(1-e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times m_1^* \\ \qquad +m_2^*], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{4.8}$$

where m_1^*, m_2^* are determined as

$$\begin{cases} m_1^* = \dfrac{D_2}{1-A_2} > 0, \\ m_2^* = \dfrac{B_2}{1-C_2} \times u_1^* > 0. \end{cases} \tag{4.9}$$

Here

$$A_2 = (1-\mu_1)e^{-(c+d_1)\tau} < 1,$$

$$B_2 = \frac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} > 0,$$

$$C_2 = (1-\mu_2)e^{-d_2\tau} < 1,$$

$$D_2 = \frac{(1-\mu_1)\frac{k\beta M_1\varepsilon}{1+\theta\varepsilon}(1-e^{-(c+d_1)\tau})}{c+d_1} + (1-\mu_1)pe^{-(c+d_1)(1-l)\tau} > 0,$$

and

$$m_1^{**} = \frac{\frac{k\beta M_1\varepsilon}{1+\theta\varepsilon}(1-e^{-(c+d_1)l\tau})}{c+d_1} + m_1^* e^{-(c+d_1)l\tau} + p.$$

Therefore, for any $\varepsilon_1 > 0$. there exists a $t_1, t > t_1$ such that

$$\widetilde{w_1(t)} - \varepsilon_1 < y_1(t) < \widetilde{m_1(t)} + \varepsilon_1,$$

and

$$\widetilde{w_2(t)} - \varepsilon_1 < y_2(t) < \widetilde{m_2(t)} + \varepsilon_1.$$

Let $\varepsilon \to 0$, so we have

$$\widetilde{y_1(t)} - \varepsilon_1 < y_1(t) < \widetilde{y_1(t)} + \varepsilon_1,$$

and

$$\widetilde{y_2(t)} - \varepsilon_1 < y_2(t) < \widetilde{y_2(t)} + \varepsilon_1,$$

for t large enough, which implies $y_1(t) \to \widetilde{y_1(t)}, y_2(t) \to \widetilde{y_2(t)}$ as $t \to \infty$. This completes the proof.

The next work is to investigate the permanence of system (2.1). Before starting this work, we should give the following definition.

Definition 4.2. System (2.1) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(x(t), y_1(t), y_2(t))$ with all initial values $x(0^+) > 0, y_1(0^+) > 0, y_2(0^+) > 0$, $m \le x(t) \le \frac{M}{k}, m \le y_1(t) \le M, m \le y_2(t) \le M$ holds for all $t \ge T_0$. Here T_0 may depend on the initial values $(x(0^+), y_1(0^+), y_2(0^+))$.

Theorem 4.3. If

$$\begin{aligned} \ln\frac{1}{1-\mu_1} &< a\tau \\ &-\frac{\beta c y_1^*}{c+d_1-d_2} \times (\frac{1-e^{-d_2\tau}}{d_2} - \frac{1-e^{-(c+d_1)\tau}}{c+d_1}) - \frac{y_2^*}{d_2} \times (1-e^{-d_2\tau}), \end{aligned} \tag{4.10}$$

holds, System (2.1) is permanent. Where y_1^* and y_2^* are defined as (3.13).

Proof. Let $(x(t), y_1(t), y_2(t))$ be a solution of (2.1) with $x(0) > 0, y_1(0) > 0, y_2(0) > 0$. By Lemma 3.6, we have proved there exists a constant $M > 0$ such that $x(t) \le \frac{M}{k}, y_1(t) \le M, y_2(t) \le M$ for t large enough. We may assume $x(t) \le \frac{M}{k}, y_1(t) \le M, y_2(t) \le M$ and $M > \sqrt{\frac{a}{\beta}}$ for $t \ge 0$. From remark 3.5, we know $y_1(t) > \rho_1, y_2(t) > \rho_2$

for all t large enough. Thus, we only need to find $m_1 > 0$ such that $x(t) \geq m_1$ for t large enough.

By the condition of Theorem 4.1., we can select $m_3 > 0$, and $\varepsilon_1 > 0$ small enough such that

$$\sigma = \ln\frac{1}{1-\mu_1} - a\tau + bm_3\tau + \frac{\beta c z_1^*}{c+d_1-d_2} \times \left(\frac{1-e^{-d_2\tau}}{d_2} - \frac{1-e^{-(c+d_1)\tau}}{c+d_1}\right) - \frac{z_2^*}{d_2} \times (1-e^{-d_2\tau}) > 0,$$

where z_1^* is defined as same as z_1^* in (4.14).

We will prove that $x(t) < m_3$ can not hold for $t \geq 0$. Otherwise,

$$\begin{cases} \left.\begin{aligned} \frac{dy_1(t)}{dt} &\leq \frac{k\beta M_1 m_3}{1+\theta m_3} - (c+d_1)y_1(t), \\ \frac{dy_2(t)}{dt} &= cy_1(t) - d_2 y_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle y_1(t) &= p, \\ \triangle y_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle y_1(t) &= -\mu_1 y_1(t), \\ \triangle y_2(t) &= -\mu_2 y_2(t), \end{aligned}\right\} t = (n+1)\tau. \end{cases} \tag{4.11}$$

By Lemmas 3.2., we have $y_1(t) \leq z_1(t), y_2(t) \leq z_2(t)$ and $z_1(t) \to \overline{z_1(t)}, z_2(t) \to \overline{z_2(t)}, t \to \infty$, where $(z_1(t), z_2(t))$ is the solution of

$$\begin{cases} \left.\begin{aligned} \frac{dz_1(t)}{dt} &= \frac{k\beta M_1 m_3}{1+\theta m_3} - (c+d_1)z_1(t), \\ \frac{dz_2(t)}{dt} &= cz_1(t) - d_2 z_2(t)), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle z_1(t) &= p, \\ \triangle z_2(t) &= 0, \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle z_1(t) &= -\mu_1 z_1(t), \\ \triangle z_2(t) &= -\mu_1 z_2(t), \end{aligned}\right\} t = (n+1)\tau, \end{cases} \tag{4.12}$$

and

$$\begin{cases} \overline{z_1(t)} = \begin{cases} \dfrac{\frac{k\beta M_1 m_3}{1+\theta m_3}(1-e^{-(c+d_1)(t-n\tau)})}{c+d_1} \\ \qquad + z_1^* e^{-(c+d_1)(t-n\tau)}, t \in (n\tau, (n+l)\tau], \\ \dfrac{\frac{k\beta M_1 m_3}{1+\theta m_3}(1-e^{-(c+d_1)(t-(n+l)\tau)})}{c+d_1} \\ \qquad + z_1^{**} e^{-(c+d_1)(t-(n+l)\tau)}, t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ \overline{z_2(t)} = e^{-d_2(t-n\tau)}[\dfrac{c(1-e^{-(c+d_1-d_2)(t-n\tau)})}{c+d_1-d_2} \times z_1^* \\ \qquad + z_2^*], t \in (n\tau, (n+1)\tau]. \end{cases} \tag{4.13}$$

where z_1^*, z_2^* are determined as

$$\begin{cases} z_1^* = \dfrac{D_3}{1-A_3} > 0, \\ z_2^* = \dfrac{B_3}{1-C_3} \times z_1^* > 0. \end{cases} \tag{4.14}$$

Here

$$\begin{aligned} A_3 &= (1-\mu_1)e^{-(c+d_1)\tau} < 1, \\ B_3 &= \frac{(1-\mu_2)ce^{-d_2\tau}(1-e^{-(c+d_1-d_2)\tau})}{c+d_1-d_2} > 0, \\ C_3 &= (1-\mu_2)e^{-d_2\tau} < 1, \\ D_3 &= \frac{(1-\mu_1)\frac{k\beta M_1 m_3}{1+\theta m_3}(1-e^{-(c+d_1)\tau})}{c+d_1} + (1-\mu_1)pe^{-(c+d_1)(1-l)\tau} > 0, \end{aligned}$$

and

$$z_1^{**} = \frac{\frac{k\beta M_1 m_3}{1+\theta m_3}(1-e^{-(c+d_1)l\tau})}{c+d_1} + m_1^* e^{-(c+d_1)l\tau} + p.$$

Therefore, there exists a $T_1 > 0$ such that

$$\begin{cases} y_1(t) \le z_1(t) \le \overline{z_1(t)} + \varepsilon_1, \\ y_2(t) \le z_2(t) \le \overline{z_2(t)} + \varepsilon_1, \end{cases} \tag{4.15}$$

and

$$\begin{cases} \dfrac{dx(t)}{dt} \ge x(t)(a - bm_3 - \beta(\overline{z_2(t)} + \varepsilon_1)), t \ne n\tau, \\ \triangle x(t) = -\mu x(t), t \ne n\tau, \end{cases} \tag{4.16}$$

for $t \ge T_1$. Let $N_1 \in N$ and $N_1\tau > T_1$, integrating (4.16) on $(n\tau, (n+1)\tau), n \ge N_1$, we have

$$\begin{aligned} x((n+1)\tau) &\ge (1-\mu)x(n\tau^+)\exp\{\int_{n\tau}^{(n+1)\tau}[a - bm_3 - \beta(\overline{z_2(t)} + \varepsilon_1)]dt\} \\ &= x((n+l-1)\tau)e^{\sigma}, \end{aligned}$$

then, $x((N_1+k+1)\tau) \ge (1-\mu)^k x((N_1+1)\tau)e^{k\sigma} \to \infty$, as $k \to \infty$, which is a contradiction to the boundedness of $x(t)$. Hence, there exist $t_1 > 0$ and $m_1 > 0$ such that $x(t) \ge m_1$ for $t \ge t_1$. This completes the proof.

5 Discussion

In this paper, we propose an IPM pest-predator model with impulses and stage structure on predator population, where the predator population is divided into two stages, a juvenile stage and a mature stage. The mature predator's predation conversion for production of new predators. This kind of stage-structured pest-predator model has been omitted in the mathematical models for integrated pest management. The dynamical properties for the pest-extinction solution and permanence of system (2.1) are established. It is assumed that $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.9, \mu = 0.75, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, then the pest-extinction periodic solution of system (2.1) is globally asymptotically stable (see Fig1.). It is assumed that $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.5, \mu = 0.75, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, then system (2.1) is permanent (see Fig2.). From simulations in Fig 1. and Fig2., we can deduce that there exist a threshold p^*. If $p > p^*$, the pest-extinction periodic solution of system (2.1) is globally asymptotically stable. If $p < p^*$, the pest-extinction periodic solution of system (2.1) is permanent. Furthermore, It is also assumed that $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.5, \mu = 0.8, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, then the pest-extinction periodic solution of system (2.1) is globally asymptotically stable (see Fig3.). From simulations in Fig 2. and Fig3., we can deduce that there exist a threshold μ^*. If $\mu < \mu^*$, the pest-extinction periodic solution of system (2.1) is permanent. If $\mu > \mu^*$, the pest-extinction periodic solution of system (2.1) is globally asymptotically stable. Our results show that the releasing juvenile predator population and spraying pesticides play important roles in integrated pest management. From our results, we can provide a good balance between the biological control and chemical control. Selecting biological pesticides and releasing juvenile predator population, which can destroys pest rapidly but causes low environmental loss, and is beneficial for economics of integrated pest management.

Declaration of interests statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Ghosh, S., Bhattacharya, D.K., Optimization in microbial pest control: an integrated approach, *Appl. Math. Model.,* **34** (2010) 1382-1395.

[2] Fabrice Vinatier, Francoise Lescourret, Pierre-Francois Duyck, Philippe Tixier, From IBM to IPM: Using individual-based models to design the spatial arrangement of traps and crops in integrated pest management strategies, *Agriculture, Ecosystems and Environment,* **146** (2012) 52-59.

[3] Francois Rebaudo, Olivier Dangles, An agent-based modeling framework for integrated pest management dissemination programs, *Environmental Modelling and Software,* **45** (2013) 141-149.

[4] Rohollah Rezaeia, Leila Safa, Christos A. Damalas, Mohammad Mahdi Ganjkhanloo, Drivers of farmers' intention to use integrated pest management: Integrating theory of planned behavior and norm activation model, *Journal of Environmental Management,* **236** (2019) 328-339.

[5] J. C. Van Lanteren, Integrated pest managemant in protected crops, in: D. Dent (Ed.), *I Integrated Pest Magangement, Chapman and Hall, London,* (1995).

[6] H. J. Barclay, Models for pest control using predator release, habitat management and pesticide release in combineation, *J. Appl. Ecol..* **19** (1982), 337-348.

[7] Bing Liu,Yujuan Zhang, Lansun Chen, Dynamical behavior of a Lotka- Volterra predator-prey model concerning integrated pest management, *Chaos, Solits and Fractals,* **6** (2004), 123-134.

[8] Lansun Chen, Xinzhu Meng, Jianjun Jiao, Biological Dynamics, *Scientific Press, Beijing,* (2009) (in Chinese).

[9] Jianjun Jiao, lansun Chen, A pest management SI model with periodic biological and chemical control concern,*Applied Mathematics and Computation,* **1183**(2006) 1018-1026.

[10] Jianjun Jiao, Lansun Chen, Guilie Luo, An appropriate pest management SI model with biological and chemical control concern, *Applied Mathematics and Computation,* **196**(2008) 285-293.

[11] Kaibiao Sun, Tonghua Zhang, Yuan Tian, Dynamics analysis and control optimization of a pest management predator-prey model with an integrated control strategy, *Applied Mathematics and Computation,* **292** (2017) 253-271.

[12] Olcay Akman, Timothy Comar, Miranda Henderson, An analysis of an impulsive stage structured integrated pest management model with refuge effect,*Chaos, Solits and Fractals,* **111** (2018), 44-54.

[13] V. Lakshmikantham, D. D. Bainov, P. Simeonov, Theory of impulsive differential equations, *World scientific, Singapor,* (1989).

[14] D. Bainov, P. Simeonov, Impulsive differential equations: periodic solutions and applications, *Pitman mongraphs and surveys in pure and applied mathematics,* **66** (1993).

[15] Weimin Liu, S. A. Levin, Y. Lwasa, Influence of nonlinear incidence rates upon the behavior of $SIRS$ Epidemiological models, *J.Math.Biol.,* **25** (1987) 359–380.

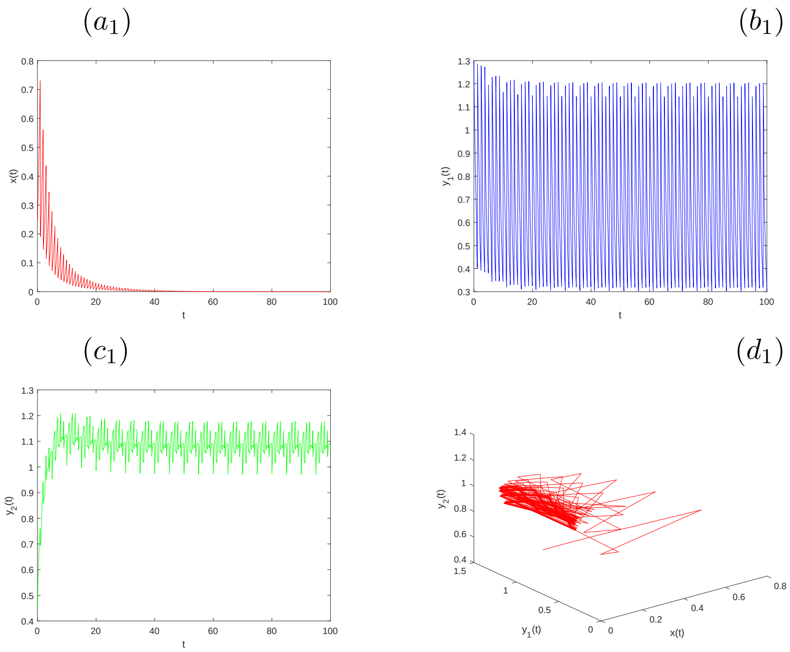

Fig.1. Globally asymptotically stable pest-extinction periodic solution of System (2.1) with $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.9, \mu = 0.75, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, (a_1) Time-series of $x(t)$; (b_1) time-series of $y_1(t)$; (c_1) time-series of $y_2(t)$; (d_1) The phase portrait of the globally asymptotically stable pest-extinction periodic solution of system (2.1).

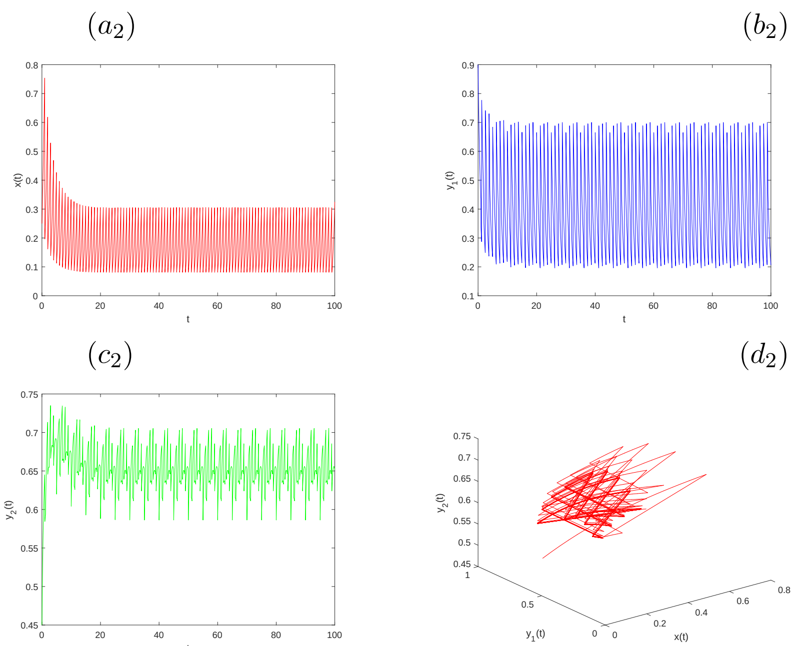

Fig.2. Permanence of system (2.1) with $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.5, \mu = 0.75, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, (a_2) Time-series of $x(t)$; (b_2) time-series of $y_1(t)$; (c_2) time-series of $y_2(t)$; (d_2) The phase portrait of the permanence of System (2.1).

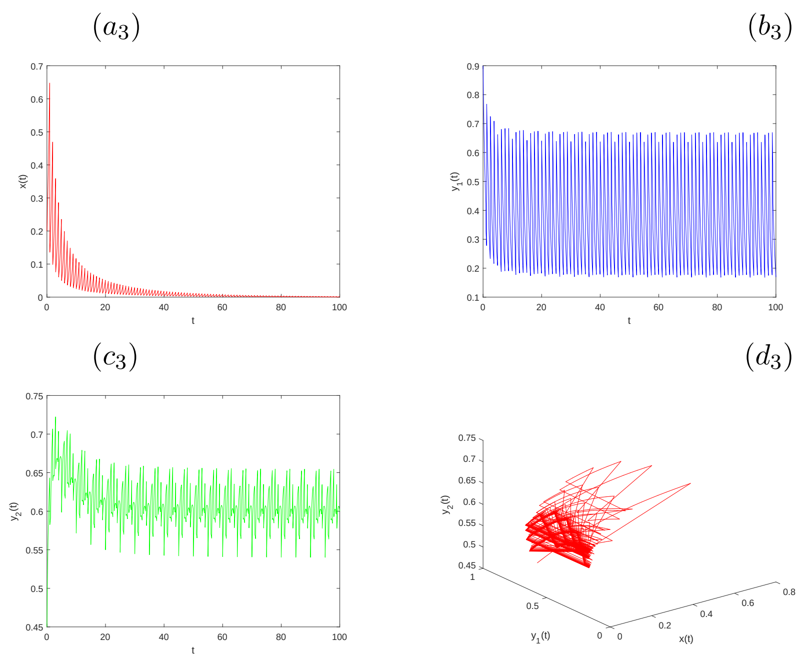

Fig.3. Globally asymptotically stable pest-extinction periodic solution of System (2.1) with $x(0) = 1, y_1(0) = 0.5, y_2(0) = 0.5, a = 2, b = 1, c = 0.5, \beta = 0.6, k = 0.5, d_1 = 0.4, d_2 = 0.2, p = 0.5, \mu = 0.8, \mu_1 = 0.2, \mu_2 = 0.1, \tau = 1, l = 0.25$, (a_3) Time-series of $x(t)$; (b_3) time-series of $y_1(t)$; (c_3) time-series of $y_2(t)$; (d_3) The phase portrait of the globally asymptotically stable pest-extinction periodic solution of system (2.1).