Received 26 April 2016; | Revised 6 June 2016; | Accepted 6 June 2016

DOI: xxx/xxxx

ARTICLE TYPE

Complexity analysis of local behaviors of a new nonlinear differential dynamic system

Yong-Li Sun[1,2] | Wen-Xiu Ma[3,4,5,6,7] | Jian-Ping Yu[8]

[1]Department of Mathematics, Beijing University of Chemical Technology, Beijing 100029, China
[2]Department of Mathematics and statistics, Brock University, ON L2S 3A1, Canada
[3]Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA
[4]Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, China
[5]Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
[6]School of Mathematics, South China University of Technology, Guangzhou 510640, China
[7]International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, South Africa
[8]Department of Applied Mathematics, University of Science and Technology Beijing, Beijing 100083, China

Correspondence
Jian-Ping Yu, Department of Applied Mathematics, University of Science and Technology Beijing, Beijing 100083, China. Email: jpyu@ustb.edu.cn

Present Address
Department of Applied Mathematics, University of Science and Technology Beijing, Beijing 100083, China

Summary

In this paper, we propose and study a (3+1)-dimensional generalized Hirota-Satsu-Ito equation, which is an important physical model. Here, by using the Hirota bilinear method, we derive its lump-type solutions, which are almost rationally localized in all spatial directions. The interaction solutions play an important roel in studying nonlinear phnoemnon, such as nonlinear optics. Thus, three kind of localized interaction solutions are constructed, respectively. In order to study the dynamic behaviours, numerical simulations are implemented, which show that there are two interesting physical phenomen: one is that fission and fusion ohenoenon happen during the collision; the other is that rogue wave phenomena is triggered by the interaction between a lump-type wave and a soliton wave (see Figure 2). The proposed (3+1)-dimensional model and results obtained can be applyied to the research on other nonlinear localized waves.

KEYWORDS:
Integrable system; Partial differential equations; Complexity analysis; Local behaviors; Nonlinear differential complex system.

1 | INTRODUCTION

As an important part of soliton theory, which has been applied in many fields such as nonlinear optics, plasmas, solid-state physics and so forth, searching for exact solutions of nonlinear evolution equations (NLEEs) has attracted a lot of attention and has been one of hot topics [1]-[13]. Such as the construction of rational solutions [14] and other exact solutions [16]-[21]. In the recent decades, many systematic and effective methods have been proposed for constructing exact solutions, for example, the Painlevé analysis method[22], the Darboux transformation method[23], the Bäcklund transformation method[24], the Hirota bilinear method and the generalized bilinear approach[25]-[28], the inverse scattering method[29]. Among the aforementioned methods, the Hirota direct method is a direct and effective method for constructing exact solutions, which will be used in our research. For example, by using the Hirota direct method, lump and hybrid solutions of NLEEs have been studied by many researchers[30]-[39].

Zhou, Manukure and Ma[40] applied the Hirota direct method to construct lump and interaction solutions to the (2+1)-dimensional Hirota-Satsuma-Ito (HSI) equation. Moreover, Ma[41] and Liu, Wen and Wang[42] constructed the Lump and lump-soliton interaction solutions of an (2+1)-dimensional extension of the HSI equation. Thsese research motivates us to generalize he (2+1)-dimensional cases to the following (3+1)-dimensional case, which can be applied to study shallow water:

$$w_t = u_{xxt} + 3uu_t - 3u_x v_t + \alpha u_x, w_x = -u_y - u_z, v_x = -u. \tag{1}$$

To the best of our knowlege, the equation (1) is the first time to be propsed and there are no reference about the equation (1).

The aim of this paper is to construct lump-type and interaction slutions of the equation (1) by using the Hirota bilinear method. In the meanwhile, numerical simulaitons are performed via the corresponding graphs, which show that choosing parameters have big impacts on the types and dynamic properties of the solutions. Since the structure of the (3+1)-dimensional equation (1) is more complex and complicated than the (2+1)-dimensional models, then obtained results within this manuscript are significantly richer than those derived about the (2+1)-dimensional equations[41] and Liu[42]

This paper is organized as follows. In Section 2, localized lump-type solutions are given to the (3+1)-dimensional equation (1), wherein the dyanmic beharours are graphically analyzed in detail. In Section 3, Section 4 and Section 5, three kinds of interaction solutions are constructed: the first consisting of a lump-type and a stripe soltion waves, the second is the combination of a lump-type and a periodic waves, the last is the interaction between a lump-type and a pair of kink waves. Due to the complex structure of (3+1)-dimensional models and the tedious compuations, we only provide three kinds of interaction solutions. And the numerical simulations are given through the corresponding graphs. Finally, some discussions are given in Section 6.

2 | LUMP-TYPE SOLUTION

In order to construct the lump-type solutions of Eq.(1), by using the logarithmic transformaton $u = 2(\ln f(x, y, z, t))_{xx}$, we firstly turn Eq.(1) into the following Hirota bilinear form:

$$(D_t D_x^3 + D_t D_y + D_t D_z + \alpha D_x^2)(f \cdot f) = 0, \tag{2}$$

where $\alpha \neq 0$ is a real constant and D_x, D_t, D_y, D_z are the Hirota derivatives defined by

$$\begin{aligned} & D_x^m D_y^n D_z^l a(x, y, z) \cdot b(x, y, z) \\ & = (\tfrac{\partial^m}{\partial s^m} \tfrac{\partial^n}{\partial t^n} \tfrac{\partial^l}{\partial r^l}) a(x+s, y+t, z+r) b(x-s, y-t, z-r)|_{s=0,t=0,r=0}, \\ & m, n, l = 0, 1, 2, 3, \cdots. \end{aligned} \tag{3}$$

According to the quadratic function method[31], function f in Eq. (2) as the following is

$$f = g^2 + h^2 + a_{11} \tag{4}$$

with

$$g = a_1 x + a_2 y + a_3 z + a_4 t + a_5, h = a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \tag{5}$$

where the a_i's are real parameters to be determined later. Now, plugging (4) into (2) and setting the coefficients of x, y, z, t to zero yield a solution of parameters a_i's as follows

$$\begin{aligned} a_2 &= -\frac{(\alpha a_1^2 a_4 + 2\alpha a_1 a_6 a_9 - \alpha a_4 a_6^2 + a_3 a_4^2 + a_3 a_9^2)}{(a_4^2 + a_9^2)}, \\ a_7 &= \frac{\alpha a_1^2 a_9 - 2\alpha a_1 a_4 a_6 - \alpha a_6^2 a_9 - a_4^2 a_8 - a_8 a_9^2}{a_4^2 + a_9^2}, \\ a_{11} &= \frac{-3(a_1^3 a_4^3 + a_1^3 a_4 a_9^2 + a_1^2 a_4^2 a_6 a_9 + a_1^2 a_6 a_9^3 + a_1 a_4^3 a_6^2 + a_1 a_4 a_6^2 a_9^2 + a_4^2 a_6^3 a_9 + a_6^3 a_9^3)}{\alpha (a_1 a_9 - a_4 a_6)^2}, \end{aligned} \tag{6}$$

which satisfy the following constraint conditions

$$a_1 a_9 - a_4 a_6 \neq 0, a_4^2 + a_9^2 \neq 0, \tag{7}$$

which make sure that $u = 2(\ln f(x, y, z, t))_{xx}$ is well-defined. The above-mentioned solution of a_i's in (6) leads to a positive quadratic function solution of (2), which, in turn, yields a lump-type solution of (1) given by

$$u = 2(\ln f)_{xx} = \frac{4((a_1^2 + a_4^2)f - 2(a_1 g + a_4 h)^2)}{f^2} \tag{8}$$

with functions g and h defined in (5). As a matter of fact, the solution (8) is a lump solution if and only if $u \to 0$ while $x^2 + y^2 + z^2 \to \infty$. But the solution (8) does not go to zero along any direction in the space of x, y, z because of the special and complicate characters of (3+1)-dimensions in the resulting solution. For example, when $x^2 + y^2 + z^2 \to \infty$ in the direction of the intersection line of surfaces $g = 0$ and $h = 0$, the solution (8) goes to a nonzero constant $\frac{4(a_1^2+a_4^2)}{a_{11}} \neq 0$ for $a_1^2 + a_4^2 \neq 0$ and $a_{11} \neq 0$. Hence, it is only a lump-type solution not a lump solution.

The corresponding numerical simulations are given graphically by choosing specific parameters $\alpha = -1, t = 0, z = 1, a_1 = 2, a_3 = 1, a_4 = 1, a_5 = 1, a_6 = 1, a_8 = 1, a_9 = 1, a_{10} = 1$. By the tedious computaions in Maple, we see that the solution u in (8) has one maximum point $(-\frac{2}{7}, \frac{4}{7}, 1)$ and two minimum points $(-\frac{12}{7} \pm \frac{\sqrt{6}}{3}, \frac{4}{7}, 1)$. Therefore, there is one peak corresponding to the maximum point and two valleys corresponding to the two minimum points. Moreover, this solution u in (8) is a bright lump-type solution since the height of the peak is bigger than the depth of the two valley bottoms.

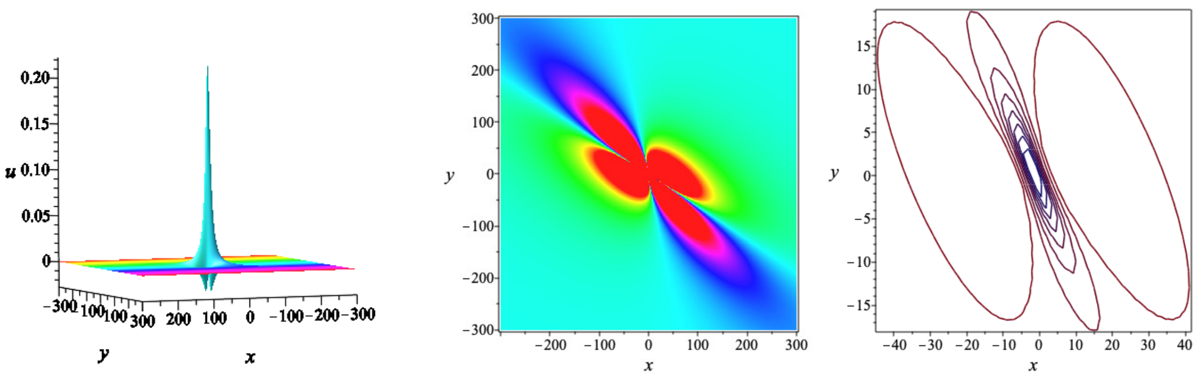


FIGURE 1 Profile of the bright lump-type solution u in (8) with $t = 0, z = 1$ at different time. The specific parameters are $\alpha = -1, a_1 = 2, a_3 = 1, a_4 = 1, a_5 = 1, a_6 = 1, a_8 = 1, a_9 = 1, a_{10} = 1$.(a) 3D plot; (b) Density plot ;(c) Contour plot.

3 | INTERACTION BETWEEN A LUMP-TYPE SOLUTION AND A STRIPE SOLITON SOLUTION

In recent years, based on the extensive investigation of exact solutions of NLEEs, the study of interaction solutions among nonlinear excitations of integrable or nonintegrable systems has been paid a lot of attention since interaction solutions have more interesting features and important applications. To construct the hybrid solution between a lump-type and a soliton solution, funciton f is taken in the form

$$f = g^2 + h^2 + ke^{\beta} + a_{15} \tag{9}$$

with

$$\begin{aligned} g &= a_1 x + a_2 y + a_3 z + a_4 t + a_5, \\ h &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \\ \beta &= a_{11} x + a_{12} y + a_{13} z + a_{14} t, \end{aligned} \tag{10}$$

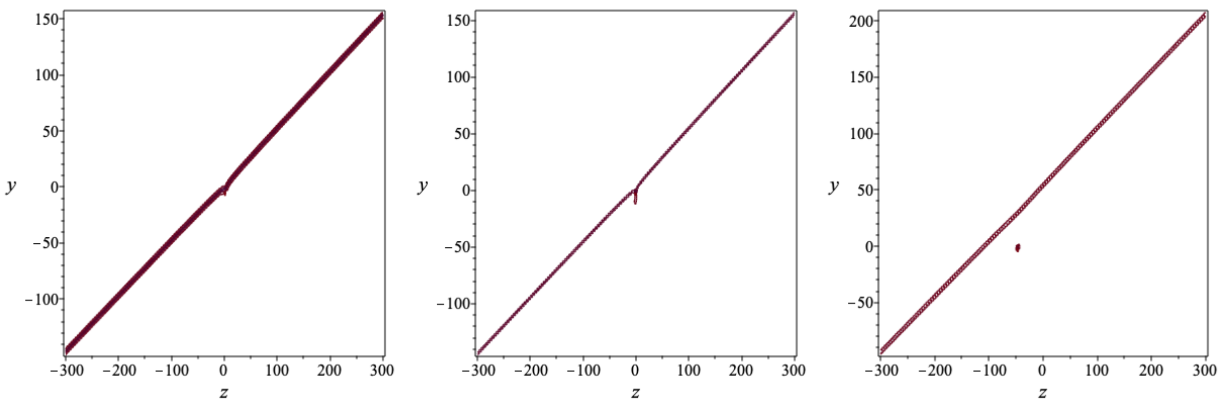
where a_i's and k are real parameters to be determined. By substituting (9) into (2), the solution of these parameters are obtained as follows:

$$\begin{aligned} a_1 &= \frac{3a_9a_{11}^2}{2\alpha}, a_2 = -\frac{3a_6a_{11}^2}{2}, a_3 = 0, a_4 = -\frac{2\alpha a_6}{3a_{11}^2}, a_7 = \frac{9a_9a_{11}^4}{4\alpha}, \\ a_8 &= 0, a_{10} = -\frac{3a_5a_9a_{11}^2}{2\alpha a_6}, a_{12} = \frac{1}{2}a_{11}^3 - a_{13}, a_{14} = -\frac{2\alpha}{3a_{11}}, a_{15} = 0 \end{aligned} \tag{11}$$

satisfying the condition $a_6a_{11} \neq 0$, which is a sufficient and necessary conditon for the solution u to be well-defined. Without loss of generality, selecting appropriate parameters, $\alpha = 1, k = 1, a_5 = 1, a_6 = -1, a_7 = -1, a_9 = -1, a_{11} = -1, a_{13} = -1, a_{14} = 1$, yields the corresponding solution u with $t = 0$ and $z = 1$

$$\begin{aligned} u &= 2(\ln(g + h + ke^{\beta} + a_{15}))_{xx} \\ &= \frac{2(g_{xx}+h_{xx}+(ke^{\beta}_{xx}))-2(g_x+h_x+(ke^{\beta})_x)^2}{(g+h+ke^{\beta})^2}, \end{aligned} \tag{12}$$

Then the interaction pheonmena between a lump-type wave and a strip soliton wave is shown by Fig. 2. It is note that the



(a) (b) (c)

FIGURE 2 Profile of the bright lump-type solution u in (8) with $z = 1$ at different time. The specific parameters are $\alpha = -1, a_1 = 2, a_3 = 1, a_4 = 1, a_5 = 1, a_6 = 1, a_8 = 1, a_9 = 1, a_{10} = 1$.(a) $t = -5$; (b) $t = 0$;(c) $t = 100$.

lump-type wave is firstly hidden in the stripe soliton wave (see Fig. 2(a)). From Fig. 2(a), since it travels faster than the stripe soliton wave with slow velocity, we can see that the stripe solition begins to split up into one lump-type wave, which implies that the fission phenomenon happens. By Fig. 2(c), it is noted that it has totally run away from the stripe soltion wave and keep propagating along the negative direction of the y-axis. The mathematical reason is that the solution u consists of two parts: a polynomial funciton and an exponential function, which plays more important role than the polynomial part. Moreover, we found that the polynomial part of u has nothing to do with spatial variable z.

4 | INTERACTION BETWEEN A LUMP-TYPE SOLUTION AND A PERIODIC SOLUTION

In this section, to obtain the interaction solution between a lump-type and a periodic solution, the form of function f in (2) as a combination of a periodic function and a positive quadratic polynomial function as the following is

$$f = g^2 + h^2 + k\cos\beta + a_{15} \tag{13}$$

with

$$\begin{aligned} g &= a_1x + a_2y + a_3z + a_4t + a_5, \\ h &= a_6x + a_7y + a_8z + a_9t + a_{10}, \\ \beta &= a_{11}x + a_{12}y + a13z + a_{14}t, \end{aligned} \tag{14}$$

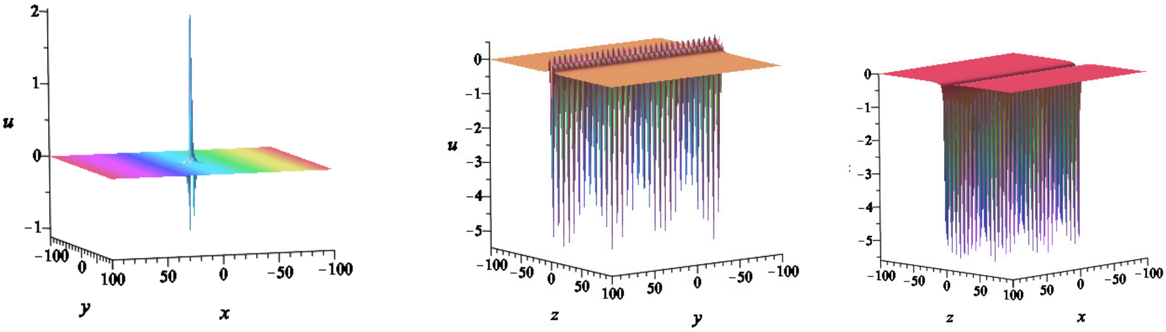
where a_i's and k are all real parameters to be determined. Substituting (3)into the bilinear form (2) leads to the constraint conditions among the parameters as follows

$$\begin{aligned} a_1 &= -\frac{3a_9a_{11}^2}{2\alpha}, a_2 = \frac{3a_6a_{11}^2}{2}, a_3 = 0, a_4 = \frac{2\alpha a_6}{3a_{11}^2}, \\ a_7 &= \frac{9a_9a_{11}^4}{4\alpha}, a_8 = 0, a_{10} = \frac{3a_5a_9a_{11}^2}{2\alpha a_6}, a_{12} = -\frac{a_{11}^3}{2} - a_{13}, \\ a_{14} &= \frac{2\alpha}{3a_{11}}, a_{15} = -\frac{2\alpha^2k^2a_{11}^2}{9a_9^2a_{11}^4+4\alpha^2a_6^2} \end{aligned} \tag{15}$$

satisfying $a_{11}a_6 \neq 0$, which is a guarantee that the solution u of (1) is well-defined. If we take the above-mentioned parameters as $\alpha = 1, k = 1, a_5 = 1, a_6 = -1, a_7 = -1, a_9 = -1, a_{11} = -1, a_{13} = -1, a_{14} = 1$, the solution at $t = 0$ and $y = 1$ is given as follows

$$\begin{aligned} u &= 2(\ln(g + h + ke^{\beta} + a_{15}))_{xx} \\ &= \frac{2(g_{xx}+h_{xx}+(k\cos(\beta))_{xx})-2(g_x+h_x+(k\cos(\beta))_x)^2}{(g+h+k\cosh(\beta))^2}. \end{aligned} \tag{16}$$

From (15) and (16), the polynomial part of the solution u in (16) has noting to do with the spatial variable z but the coefficient of the spatial variable z in the periodic part is not equal to zero. The interaction phenomena between a lump-type and a strip soliton wave is illustrated by Fig. 3. From Fig. 3, at time of $t = 0$, we see that the solution u in (16) has different shapes on different



(a) (b) (c)

FIGURE 3 Evolution profile of the interaction solution u in (16) with $t = 0$. The specific parameters are $\alpha = 1, k = 1, a_5 = 1, a_6 = -1, a_7 = -1, a_9 = -1, a_{11} = -1, a_{13} = -1, a_{14} = 1$.(a) 3D plot on xOy plane; (b) 3D plot on yOz plane;(c) 3D plot on xOz plane.

coordinate planes, for example, the shape of u on the xOy plane is a lump wave, but the shapes of u on other two coordinate planes looks like the complex dark period solitons.

5 | INTERACTION BETWEEN A LUMP-TYPE SOLUTION AND A PAIR OF KINK SOLUTIONS

In order to study the interaction solution between a lump-type solution and a pair of kink soluions,function f in (2) is taken to be a combination of a hyperbolic cosine function and a positive quadratic polynomial function as follows:

$$f = g^2 + h^2 + k\cosh\beta + a_{15} \tag{17}$$

with

$$\begin{aligned} g &= a_1x + a_2y + a_3z + a_4t + a_5, \\ h &= a_6x + a_7y + a_8z + a_9t + a_{10}, \\ \beta &= a_{11}x + a_{12}y + a_{13}z + a_{14}t, \end{aligned} \tag{18}$$

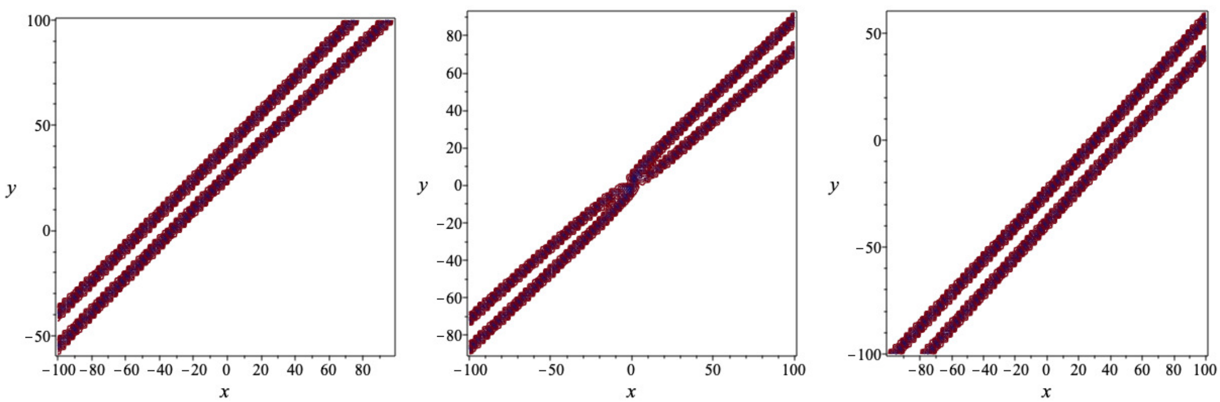
where parameters a_i's and k are real to be determined. By putting (17)into the bilinear equation (2), the corresponding relations as the following are:

$$\begin{aligned} &k = \frac{9a_9^2 a_{11}^4 + 16\alpha^2 a_1^2}{8a_{11}^2 \alpha^2}, a_2 = -\frac{3a_{11}^2 (9a_9^2 a_{11}^4 + 16\alpha^2 a_1^2 + 16\alpha a_7 a_9}{64 a_1 \alpha^2}, a_3 = \frac{3(9a_9 a_{11}^4 + 16\alpha a_7)) a_9 a_{11}^2}{64 a_1 \alpha^2}, \\ &a_4 = \frac{4\alpha a_1}{3a_{11}^2}, a_6 = \frac{3a_9 a_{11}^2}{4\alpha}, a_8 = -\frac{9a_9 a_{11}^4 + 16\alpha a_7}{16\alpha}, a_{12} = -\frac{1}{4}a_{11}^3 - a_{13}, a_{14} = -\frac{4\alpha}{3a_{11}} \end{aligned} \tag{19}$$

satisfying $a_{11} \neq 0$, which guarantees the solution u of (1) be well-defined. Now, choosing the parameters $\alpha = 1, a_1 = 1, a_5 = 1, a_7 = 1, a_9 = 1, a_{10} = 1, a_{11} = 1, a_{13} = 1, a_{15} = 1$, we have the solution u at time $t = 0$ and $z = 1$ as follows

$$\begin{aligned} u &= 2(\ln(g + h + k\cosh(\beta) + a_{15}))_{xx} \\ &= \frac{2(g_{xx} + h_{xx} + (k\cosh(\beta))_{xx}) - 2(g_x + h_x + (k\cosh(\beta))_x)^2}{(g + h + k\cosh(\beta))^2}. \end{aligned} \tag{20}$$

The numericla simulation of Eq. (20) is given by Fig. 4. During the interaction progress, Figure 4(a) appears only a pair of kink



(a) (b) (c)

FIGURE 4 Evolution profile of the interaction solution u in (1) with $t = 0, z = 1$. The specific parameters are $\alpha = 1, a_1 = 1, a_5 = 1, a_7 = 1, a_9 = 1, a_{10} = 1, a_{11} = 1, a_{13} = 1, a_{15} = 1$.(a) Contour plot at $t = -30$; (b) Contour plot at $t = 0$; (c) Contour plot at $t = 30$.

waves exist, as a matter of fact, the lump-type solution hiding itself in one of this pair of kink waves. By Fig. 4(b), the collisions happened among the lump-type solution and the kink waves since the kink wave in which the lump-type wave is hiding itself travels faster than the other kink wave. Figure 4(c) shows that the kink wave which carries the lump-type wave and the other kink wave are seperated from each other and keep travling forward.

6 | DISCUSSIONS

The generalized (3+1)-dimensional HSI euqation (1) is a physical model of very importance, since it has richer dynamic properties due to its complex structure. In this paper, by using the Hirota direct method, lump-type solutions are constructed. And then three kinds of localized interaction solutions are constructed. Moreover, numerical analysis is implemented on the dynamic behavours and propagation properties of the solutions obtained in this research. All the numerical simulations show that the dynamic behavours are much richer and the types of parameters chosen have a great impact on the propagation properties, which can be seen from Figs. 2, 3 and 4. Due to the complexity of selecting the parameters and the complex structure of the (3+1)-dimensional model, it is very hard to study the ineraction solutions, however, they are worthy of being investigated since they play a key role in physics. Therefore, in the future, we will study other interaction solutions, such as the hybrid solutions between a high-order breather and a line-soltion, And how the related parameters affect the complex dynamic behavours of the interaction solutions. The method used in this paper can be used to construct solutions of other nonlinear physical models. Moreover, the results obtained in this paper may provide a potential technique to study other localized waves.

ACKNOWLEDGEMENTS

The work was supported in part by National Natural Science Foundation of China under the grants 11971067 and 11101029.

References

1. D. S. Wang, J. Liu, Integrability aspects of some two-component KdV systems, Appl. Math. Letters 79 (2018) 211-219.
2. X. Z. Hao, Y. P. Liu, X. Y. Tang, Z. B. Li and W. X. Ma, Nonlocal symmetries and the nth finite symmetry transformation or AKNS system, Mod. Phys. Lett. B, Vol.32(2018), no.27, 1850332.
3. X. L. Yong, Y. J. Fan, Y. H. Huang, W. X. Ma, J. Tian, Darboux transformation and solitons for an integrable nonautonomous nonlinear integro-differential Schrödinger equation, Modern Physics Letters B, 31(2017), no. 30, 1750276, 14 pp.
4. B. Ren, X.P. Cheng, and J. Lin, The (2+1)-dimensional Konopelchenko-Dubrovsky equation: nonlocal symmetries and interaction solutions, Nonlinear Dyn. 86(2016), 1855-1862.
5. J. P. Yu, F. D. Wang, W. X. Ma, Y. L. Sun, C. M. Khalique, Multiple-soliton solutions and lumps of a (3+1)-dimensional generalized KP equation, Nonlinear Dyn. 25 (2019) 1687-1692.
6. H. Wang, Y. H. Wang, W. X. Ma and C. Temuer, Lump solutions of a new extended (2+1)-dimensional Boussinesq equation, Modern Physics Letters B, Vol.32(2018), no. 31, 1850376, 8 pp.
7. J. P. Yu, W. X. Ma, B. Ren, Y. L. Sun, C. M. Khalique, Diversity of Interaction Solutions of a Shallow Water Wave Equation, Complexity, Volume 2019(2019), Article ID 5874904, 6 pp.
8. J. P. Yu, Y. L. Sun, Study of lump solutions to dimensionally reduced generalized KP equations, Nonlinear Dyn (2017) 87:2755-2763.
9. D. S. Wang, X. L. Wang, Long-time asymptotics and the bright N-soliton solutions of the KunduâĂŞEckhaus equation via the Riemann-Hilbert approach, Nonlinear Analysis: Real World Applications 41 (2018) 334-361.
10. J. P. Yu, W. X. Ma, Y. L. Sun and C. M. Khalique, N-fold Darboux transformation and conservation laws of the modified Volterra lattice, Modern Physics Letters B, Vol.32(2018), no. 33, 1850409, 10 pp.
11. B. Ren, Interaction solutions for mKP equation with nonlocal symmetry reductions and CTE method, Phys. Scr. 90, 065206, 2015. http://iopscience.iop.org/article/ 10.1088/0031-8949/90/6/065206.
12. D. S. Wang, B. L. Guo, X. L. Wang, Long-time asymptotics of the focusing Kundu-Eckhaus equation with nonzero boundary conditions, J. Diff. Equa. 266 (2019) 5209-5253.
13. Y. L. Sun, W. X. Ma, J. P. Yu and C. M. Khalique, Exact solutions of the Rosenau Hyman equation, coupled KdV system and Burgers Huxley equation using modified transformed rational function method, Modern Physics Letters B, Vol.32(2018), no.24, 1850282, 12 pp.
14. B. Ren, W. X. Ma, J. Yu, Characteristics and interactions of solitary and lump waves of a (2 + 1)-dimensional coupled nonlinear partial differential equation, Nonlinear Dyn. 96 (2019), 717-727.
15. Y. L. Sun, W. X. Ma, J. P. Yu, Lump solutions of the 2D Toda equation, Mathematical Methods in the Applied Sciences, 2020, https://doi.org/10.1002/mma.6370.
16. S. J. Chen, Y. H. Yin, W. X. Ma, X. Lü, Abundant exact solutions and interaction phenomena of the (2+1)-dimensional YTSF equation, Analy. Math. Phys. 9(2019), 2329-2344.
17. H. N. Xu, W. Y. Ruan, Y. Zhang, X. L{u}, Multi-exponential wave solutions to two extended Jimbo-Miwa equations and the resonance behavior, Appl. Math. Lett. 99(2020), 105976.

18. Y. F. Hua, B. L. Guo, W. X. Ma, X. Lü, Interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation for nonlinear waves, Appl. Math. Model. 74 (2019), 184-198.

19. X. Lü, W. X. Ma, Study of lump dynamics based on a dimensionally reduced Hirota bilinear equation, Nonli. Dyn. 85 (2016), 1217-1222.

20. Y. H, Yin, W. X. Ma, J. G. Liu, X. Lü, Diversity of exact solutions to a (3+1)-dimensional nonlinear evolution equation and its reduction, Comput. Math. Appl. 76 (2018), 1275-1283.

21. S. J. Chen, W. X. Ma, Lü, Bäcklund transformation, exact solutions and interaction behaviour of the (3+1)-dimensional Hirota-Satsuma-Ito-like equation, Commun. Nonli. Scien. Numer. Simula. 83(2020), 105135.

22. J. Weiss, M. Tabor, G. Carnevale,The Painlevé property for partial differential equations, J. Math. Phys. 24(1983) 522-526.

23. V. B. Matveev, M. A. Salle, Darboux transformations and solitons, Springer-Verlag, Berlin, 1991.

24. D. J. Kaup, The lump solutions and the bäcklund transformation for the three-dimensional three-wave resonant interaction, J. Math. Phys. 22(1981) 1176-1181.

25. R. Hirota, The direct method in soliton theory, Cambridge University Press, New York, 2004.

26. B. Ren, J. Lin, Z. W. Lou, Consistent Riccati expansion and rational solutions of the Drinfel'd-Sokolov-Wilson equation, Appl. Math. Lett. 105 (2020), 106326.

27. R. Hirota, Exact soliton of the Korteweg-deVries equation fo rmultiple collisions of solitons, Phys. Rev. Lett. 27(1971) 1192-1194.

28. W. X. Ma, Generalized Bilinear Differential Equations, Stud. Nonlinear Scie. 2(2011) 140-144.

29. M. J. Ablowitz, P. A. Clarkson, Solitons, Nonlinear evolution equations and inverse scattering, Cambridge University Press, NewYork, 1991.

30. R. S. Johnson, S. Thompson, A solution of the inverse scattering problem for the Kadomtsev-Petviashvili equation by the method ofseparation of variables, Phys. Lett. A 66 (1978) 278-281.

31. W. X. Ma, Lump solutions to the Kadomtsev-Petviashvili equation, Phys. Lett. A 379(2015) 1975-1978.

32. W.X. Ma, Y. Zhou, Lump solutions to nonlinear partial differential equations via Hirota bilinear forms, J. Differ. Equations 264 (2018) 2633-2659.

33. X. Lü, W.X.Ma, Study of lump dynamics based on a dimensionally reduced Hirota bilinear equation, Nonlinear Dyn. 85(2016) 1217-1222.

34. J.G. Liu, Y. He, Abundant lump and lump-kink solutions for the new (3+1)-dimensional generalized Kadomtsev-Petviashvili equation, Nonlinear Dyn. 92 (2018) 1103-1108.

35. J. P. Yu, Y. L. Sun, Study of lump solutions to dimensionally reduced generalized KP equations, Nonlinear Dyn. 87(2017) 2755-2763.

36. W. X. Ma, X. L. Yong, H. Q. Zhang, Diversity of interaction solutions to the(2+1)-dimensional Ito equation, Comput. Math .Appl. 75 (2018) 289-295.

37. J. P. Yu, Y. L. Sun, Lump solutions to dimensionally reduced Kadomtsev-Petviashvili-like equations, Nonlinear Dyn. 87 (2017) 1405-1412.

38. J. P. Yu, J. Jing, Y. L. Sun, S. P. Wu, (n+1)-Dimensional reduced differential transform method for solving partial differential equations, Appl. Math. Computa. 273(2016) 697-705.

39. W. X. Ma, A search for lump solutions to a combined fourth-order nonlinear PDE in (2+1)-dimensions, J. Appl. Anal. Comput. 9 (2019) 1319-1332.

40. Y. Zhou, S. Manukure, W. X. Ma, Lump and lump-soliton solu- tions to the Hirota-Satsuma-Ito equation, Commu. Nonlinear Scie. Numer. Simula. (2018) doi: https://doi.org/10.1016/j.cnsns.2018.07.038.

41. W. X. Ma, Interaction solutions to the Hirota-Satsuma-Ito equation in (2+1)-dimensions, Front. Math. China 14 (2019) 619-629.

42. Y. Q. Liu, X. Y. Wen, D. S. Wang, The N-soliton solution and localized wave interaction solutions of the (2+1) -dimensional generalized Hirota-Satsuma-Ito equation, Comput. Math. Appl. 77 (2019), 947-966.

$\square$