# On G continuity in neutrosophic soft spaces

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**Abstract.** In this paper, we introduce the concept of neutrosophic $G$-sequential continuity as a new tool to further studies presenting the definitions of neutrosophic soft sequence, neutrosophic soft quasi-coincidence, neutrosophic soft $q$-neighborhood, neutrosophic soft cluster point, neutrosophic soft boundary point, neutrosophic soft sequential closure, neutrosophic soft group, neutrosophic soft method, which constitute a base to define the concepts of neutrosophic soft $G$-sequential closure, neutrosophic soft $G$-sequential derived set, $G$-sequentially neutrosophic soft compactness of a subset of a neutrosophic soft topological space. Their characters are analyzed and some implications are given. A counterexample to each implication is also given.

Keywords: neutrosophic soft group, neutrosophic soft $G$-sequential continuity, neutrosophic soft quasi-coincidence, neutrosophic soft sequence.
2020 AMS Classification: 54A05, 54C10, 54D30, 54D10.

## 1. Introduction

By almost all Mathematicians, it is admitted that the concept of continuity plays a vital role in the world of Mathematics. Any notion connected with continuity constitutes an integral part in all disciplines that involve Mathematics such as computer science, information theory, biological science and dynamical systems. The notion of continuity has always been considered as a focus point in numerous investigations. In these investigations, various concepts related to it were introduced and these concepts were used by scholars to further studies. Sequential continuity has always been adopted as one of major concepts that are related to continuity. In [8], Connor and Grosse-Erdmann made the convergence of sequences gain a new identity by using the structure of sequential continuity. Furthermore, Çakallı [7] interpreted this identity by using topological group-valued sequences, offered theorems in this generalized setting some of which were not only new from the point of topological group setting, but also new from the point of the real case. In this paper, our goal is to make these ideas acquire new characters in neutrosophic soft topological spaces and investigate concepts related to them, specially achieve generalizations of the results obtained in [1] (see also [3], and [4]).

## 2. Preliminaries

In this section, some fundamental definitions related to neutrosophic set theory are reminded.

**Definition 1.** [12] *A neutrosophic set A on the universe set X is defined as:*

$$A = \{\langle x, T_A(x), _A(x), F_A x\rangle : x \in X\},$$

*where*

$$T,I,\ F : X \to ]^{-}0, 1^{+}[ \text{ and } ^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

**Definition 2.** [11] *Let $X$ be an initial universe, $E$ be a set of all parameters, and $P(X)$ denote the power set of $X$. A pair $(F,E)$ is called a soft set over $X$, where $F$ is a mapping given by $F : E \to P(X)$. In other words, the soft set is a parameterized family of subsets of the set $X$. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set $(F,E)$, or as the set of e-approximate elements of the soft set, i.e.*

$$(F,E) = \{(e,F(e)) : e \in E,\ F : E \to P(X)\}.$$

*After the neutrosophic soft set was defined by Maji* [10], *this concept was modified by Deli and Broumi* [9] *as given below:*

**Definition 3.** [9] *Let $X$ be an initial universe set and $E$ be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of $X$. Then a neutrosophic soft set $\left(\widetilde{F},E\right)$ over $X$ is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $\left(\widetilde{F},E\right)$. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:*

$$\left(\widetilde{F},E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

*where $T_{\widetilde{F}(e)}(x)$, $I_{\widetilde{F}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \in [0,1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. Since the supremum of each $T$, $I$, $F$ is 1, the inequality*

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

*is obvious.*

**Definition 4.** [6] *Let $\left(\widetilde{F},E\right)$ be a neutrosophic soft set over the universe set $X$. The complement of $\left(\widetilde{F},E\right)$ is denoted by $\left(\widetilde{F},E\right)^c$ and is defined by:*

$$\left(\widetilde{F},E\right)^c = \left\{\left(e,\ \left\langle x,\ F_{\widetilde{F}(e)}(x),\ 1 - I_{\widetilde{F}(e)}(x),\ T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

*It is obvious that $\left[\left(\widetilde{F},E\right)^c\right]^c = \left(\widetilde{F},E\right)$.*

**Definition 5.** [10] *Let $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ be two neutrosophic soft sets over the universe set $X$. $\left(\widetilde{F},E\right)$ is said to be a neutrosophic soft subset of $\left(\widetilde{G},E\right)$ if*

$$T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x),\ I_{\widetilde{F}(e)}(x) \leq I(x),\ F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x),\ \forall e \in E,\ \forall x \in X.$$

*It is denoted by $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)$. $\left(\widetilde{F},E\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{G},E\right)$ if $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)$ and $\left(\widetilde{G},E\right) \subseteq \left(\widetilde{F},E\right)$. It is denoted by $\left(\widetilde{F},E\right) = \left(\widetilde{G},E\right)$.*

**Definition 6.** [5] *Let $\left(\widetilde{F}_1,E\right)$ and $\left(\widetilde{F}_2,E\right)$ be two neutrosophic soft sets over the universe set $X$. Then their union is denoted by $\left(\widetilde{F}_1,E\right) \cup \left(\widetilde{F}_2,E\right) = \left(\widetilde{F}_3,E\right)$ and is defined by:*

$$\left(\widetilde{F}_3,E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}_3(e)}(x), I(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

*where*

$$T_{\widetilde{F}_3(e)}(x) = max\left\{T_{\widetilde{F}_1(e)}(x),\ T_{\widetilde{F}_2(e)}(x)\right\},$$
$$I_{\widetilde{F}_3(e)}(x) = max\left\{I_{\widetilde{F}_1(e)}(x),\ I_{\widetilde{F}_2(e)}(x)\right\},$$

$$F_{\widetilde{F}_3(e)}(x) = min\left\{F_{\widetilde{F}_1(e)}(x),\, F_{\widetilde{F}_2(e)}(x)\right\}.$$

**Definition 7.** [5] *Let* $\left(\widetilde{F}_1, E\right)$ *and* $\left(\widetilde{F}_2, E\right)$ *be two neutrosophic soft sets over the universe set* $X$*. Then their* intersection *is denoted by* $\left(\widetilde{F}_1, E\right) \cap \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_3, E\right)$ *and is defined by:*

$$\left(\widetilde{F}_3, E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}_3(e)}(x), I(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

*where*

$$T_{\widetilde{F}_3(e)}(x) = min\left\{T_{\widetilde{F}_1(e)}(x),\, T_{\widetilde{F}_2(e)}(x)\right\},$$
$$I_{\widetilde{F}_3(e)}(x) = min\left\{I_{\widetilde{F}_1(e)}(x),\, I_{\widetilde{F}_2(e)}(x)\right\},$$
$$F_{\widetilde{F}_3(e)}(x) = max\left\{F_{\widetilde{F}_1(e)}(x),\, F_{\widetilde{F}_2(e)}(x)\right\}.$$

**Definition 8.** [5] *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* $X$ *is said to be a null neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 0$, $I_{\widetilde{F}(e)}(x) = 0$, $F_{\widetilde{F}(e)}(x) = 1$; $\forall e \in E$, $\forall x \in X$*. It is denoted by* $0_{(X,E)}$.

**Definition 9.** [5] *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* $X$ *is said to be an absolute neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 1$, $I_{\widetilde{F}(e)}(x) = 1$, $F_{\widetilde{F}(e)}(x) = 0$; $\forall e \in E$, $\forall x \in X$ *It is denoted by* $1_{(X,E)}$.

*Clearly* $0^c_{(X,E)} = 1_{(X,E)}$ *and* $1^c_{(X,E)} = 0_{(X,E)}$.

**Definition 10.** [5] *Let* $NSS(X,E)$ *be the family of all neutrosophic soft sets over the universe set* $X$ *and* $\tau \subset NSS(X,E)$*. Then* $\tau$ *is said to be a neutrosophic soft topology on* $X$ *if:*

*1.* $0_{(X,E)}$ *and* $1_{(X,E)}$ *belong to* $\tau$,

*2. the union of any number of neutrosophic soft sets in* $\tau$ *belongs to* $\tau$,

*3. the intersection of a finite number of neutrosophic soft sets in* $\tau$ *belongs to* $\tau$.

*Then* $(X, \tau, E)$ *is said to be a neutrosophic soft topological space over* $X$*. Each member of* $\tau$ *is said tobe a neutrosophic soft open set* [5].

**Definition 11.** [5] *Let* $(X, \tau, E)$ *be a neutrosophic soft topological space over* $X$ *and* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over* $X$*. Then* $\left(\widetilde{F}, E\right)$ *is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.*

**Definition 12.** [5] *Let* $NSS(X,E)$ *be the family of all neutrosophic soft sets over the universe set* $X$*. Then neutrosophic soft set* $x^e_{(\alpha,\beta,\gamma)}$ *is called* a neutrosophic soft point *for every* $x \in X$, $0 < \alpha,\ \beta,\ \gamma \leq 1$, $e \in E$ *and is defined as follows:*

$$x^e_{(\alpha,\beta,\gamma)}\left(e'\right)(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } e' = e \text{ and } y = x \\ (0,0,1), & \text{if } e' \neq e \text{ or } y \neq x \end{cases}$$

*It is clear that every neutrosophic soft set is the union of its neutrosophic soft points.*

**Definition 13.** [5] *Let* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over the universe set* $X$*. We say that* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$ *read as belonging to the neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *whenever* $\alpha \leq T_{\widetilde{F}(e)}(x)$, $\beta \leq I_{\widetilde{F}(e)}(x)$ *and* $\gamma \geq F_{\widetilde{F}(e)}(x)$.

## 3. Some New Definitions

In the following, several new definitions are introduced that form the basis of next section.

**Definition 14.** *A neutrosophic soft point $x^e_{\alpha,\beta,\gamma}$ is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with $\left(\widetilde{F},E\right)$, denoted by $x^e_{\alpha,\beta,\gamma}q\left(\widetilde{F},E\right)$ if and only if $x^e_{\alpha,\beta,\gamma}\nsubseteq\left(\widetilde{F},E\right)^c$. If $x^e_{\alpha,\beta,\gamma}$ is not neutrosophic soft quasi-coincident with $\left(\widetilde{F},E\right)$, we denote by $x^e_{\alpha,\beta,\gamma}\tilde{q}\left(\widetilde{F},E\right)$.*

**Definition 15.** *A neutrosophic soft set $\left(\widetilde{F},E\right)$ in a neutrosophic soft topological space $(X,\tau,E)$ is said to be a neutrosophic soft q-neighborhood of a neutrosophic soft point $x^e_{\alpha,\beta,\gamma}$ if and only if there exists a neutrosophic soft open set $\left(\widetilde{G},E\right)$ such that $x^e_{\alpha,\beta,\gamma}q\left(\widetilde{G},E\right)\subset\left(\widetilde{F},E\right)$.*

**Definition 16.** *A neutrosophic soft set $\left(\widetilde{G},E\right)$ is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with $\left(\widetilde{F},E\right)$, denoted by $\left(\widetilde{G},E\right)q\left(\widetilde{F},E\right)$ if and only if $\left(\widetilde{G},E\right)\nsubseteq\left(\widetilde{F},E\right)^c$. If $\left(\widetilde{G},E\right)$ is not neutrosophic soft quasi-coincident with $\left(\widetilde{F},E\right)$, we denote by $\left(\widetilde{G},E\right)\tilde{q}\left(\widetilde{F},E\right)$.*

**Definition 17.** *A neutrosophic soft point $x^e_{\alpha,\beta,\gamma}$ is said to be a neurosophic soft cluster point of a neutrosophic soft set $\left(\widetilde{F},E\right)$ if and only if every neutrosophic soft open q-neighborhood $\left(\widetilde{G},E\right)$ of $x^e_{\alpha,\beta,\gamma}$ is q-coincident with $\left(\widetilde{F},E\right)$. The union of all neutrosophic soft cluster points of $\left(\widetilde{F},E\right)$ is called the neutrosophic soft closure of $\left(\widetilde{F},E\right)$ and denoted by $\overline{\left(\widetilde{F},E\right)}$.*

**Definition 18.** *A neutrosophic soft point $x^e_{\alpha,\beta,\gamma}$ is said to be a neurosophic soft boundary point of a neutrosophic soft set $\left(\widetilde{F},E\right)$ if and only if every neutrosophic soft open q-neighborhood $\left(\widetilde{G},E\right)$ of $x^e_{\alpha,\beta,\gamma}$ is q-coincident with $\left(\widetilde{F},E\right)$ and $\left(\widetilde{F},E\right)^c$. The union of all neutrosophic soft boundary points of $\left(\widetilde{F},E\right)$ is called the neutrosophic soft limit of $\left(\widetilde{F},E\right)$ and denoted by $\left(\widetilde{F},E\right)^b$.*

**Definition 19.** *A neutrosophic soft sequence in a neutrosophic soft topological space $(X,\tau,E)$ is a function $S:N\to(X,\tau,E)$, where $N$ is the set of naturel numbers. We write $\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ to denote the sequence of neutrosophic soft points in $(X,\tau,E)$ indexed by N.*

**Definition 20.** *A neutrosophic soft subsequence of a neutrosophic soft sequence $S:N\to(X,\tau,E)$ is a composition $S\circ P$, where $P:N\to N$ is an increasing cofinal function.That is,*

*a) $P(n_1)\le(n_2)$, whenever $n_1\le n_2$ (P is increasing),*

*b) For each $n_1\in N$, there exists a naturel number $n_2\in N$ such that $n_1\le P(n_2)$ (P is cofinal in N).*

*For $k\in N$, the neutrosophic point $(S\circ P)(k)$ will often be written $x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}}$.*

**Definition 21.** *Let $\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ be a neutrosophic soft sequence in a neutrosphic soft topological space $(X,\tau,E)$. Then, $\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ converges to a neutrosophic soft point $x^e_{\alpha,\beta,\gamma}$ in*

$(X,\tau,E)$ (written $x^{e_n}_{n_{r_n,t_n,s_n}} \to x^{e}_{\alpha,\beta,\gamma}$ provided that, for each neutrosophic soft q-neighbourhood $\left(\widetilde{U},E\right)$ of $x^{e}_{\alpha,\beta,\gamma}$ there exists $n_0 \in N$ such that $n \geq n_0$ implies $x^{e}_{\alpha,\beta,\gamma} q \left(\widetilde{U},E\right)$.

*We will use boldface letters* $\mathbf{x},\mathbf{y},\mathbf{z},\ldots$ *for neutrosophic soft sequences* $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$, $\mathbf{y} = \left\{y^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$, $\mathbf{z} = \left\{z^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$, $\ldots$ *of neutrosophic soft points in* $(X,\tau,E)$. *s(X) and c(X) denote the set of all neutrosophic soft sequences in* $(X,\tau,E)$ *and the set of all convergent neutrosophic soft sequences in* $(X,\tau,E)$*, respectively.*

**Definition 22.** *Let* $(X,\Delta,E)$ *is a neutrosophic soft group in a neutrosophic soft topological space* $(X,\tau,E)$*, where* $\Delta$ *is a binary operation defined on* $(X,\tau,E)$ *such that the following conditions hold:*

*a) Closure: For all neutrosophic soft points* $x^{e_1}_{1_{r_1,t_1,s_1}}$, $x^{e_2}_{2_{r_2,t_2,s_2}}$ *in* $(X,\tau,E)$, $x^{e_1}_{1_{r_1,t_1,s_1}} \Delta\, x^{e_2}_{2_{r_2,t_2,s_2}}$ *is a uniquely defined neutrosophic soft point in* $(X,\tau,E)$,

*b) Associativity: For all neutrosophic soft points* $x^{e_1}_{1_{r_1,t_1,s_1}}$, $x^{e_2}_{2_{r_2,t_2,s_2}}$, $x^{e_3}_{3_{r_3,t_3,s_3}}$ *in* $(X,\tau,E)$*, we have* $x^{e_1}_{1_{r_1,t_1,s_1}} \Delta \left(x^{e_2}_{2_{r_2,t_2,s_2}} \Delta\, x^{e_3}_{3_{r_3,t_3,s_3}}\right) = \left(x^{e_1}_{1_{r_1,t_1,s_1}} \Delta x^{e_2}_{2_{r_2,t_2,s_2}}\right) \Delta x^{e_3}_{3_{r_3,t_3,s_3}}$,

*c) Identity: There exists an identity neutrosophic soft point* $k^{e}_{\alpha,\beta,\gamma}$ *in* $(X,\tau)$ *such that* $x^{e_1}_{1_{r_1,t_1,s_1}} \Delta\, k^{e}_{\alpha,\beta,\gamma} = k^{e}_{\alpha,\beta,\gamma} \Delta\, x^{e_1}_{1_{r_1,t_1,s_1}} = x_{1_{r_1,t_1,s_1}}$ *for any neutrosophic soft point* $x^{e_1}_{1_{r_1,t_1,s_1}}$ *in* $(X,\tau,E)$,

*d) Inverses: For any neutrosophic soft point* $x^{e_1}_{1_{r_1,t_1,s_1}}$ *in* $(X,\tau,E)$*, there exists an inverse neutrosophic soft point* $\left(x^{e_1}_{1_{r_1,t_1,s_1}}\right)^{-1}$ *in* $(X,\tau,E)$ *such that* $x^{e_1}_{1_{r_1,t_1,s_1}} \Delta \left(x^{e_1}_{1_{r_1,t_1,s_1}}\right)^{-1} = k^{e}_{\alpha,\beta,\gamma}$ *and* $\left(x^{e_1}_{1_{r_1,t_1,s_1}}\right)^{-1} \Delta\, x^{e_1}_{1_{r_1,t_1,s_1}} = k^{e}_{\alpha,\beta,\gamma}$.

**Definition 23.** *Let* $(X,\Delta,E)$ *be a neutrosophic soft group in a neutrosophic soft topological space* $(X,\tau,E)$*. Consider two neutrosophic soft sets* $\left(\widetilde{A},E\right)$, $\left(\widetilde{B},E\right)$ *in* $(X,\tau,E)$*. Then,*

$$\left(\widetilde{A},E\right) \Delta \left(\widetilde{B},E\right) = \{x^{e_1}_{1_{r_1,t_1,s_1}} \mid x^{e_1}_{1_{r_1,t_1,s_1}} = x^{e_2}_{2_{r_2,t_2,s_2}} \Delta\, x^{e_3}_{3_{r_3,t_3,s_3}},\, x^{e_2}_{2_{r_2,t_2,s_2}} \in \left(\widetilde{A},E\right),\, x^{e_3}_{3_{r_3,t_3,s_3}} \in \left(\widetilde{B},E\right)\}$$

**Definition 24.** *Let* $(X,\Delta,E)$ *be a neutrosophic soft group in a neutrosophic soft topological space* $(X,\tau,E)$*. Consider a neutrosophic set* $\left(\widetilde{A},E\right)$ *in* $(X,\tau,E)$*. Then,* $\left(\widetilde{A},E\right)^{-1} = \left\{(x^{e}_{\alpha,\beta,\gamma})^{-1} \mid x^{e}_{\alpha,\beta,\gamma} \in \left(\widetilde{A},E\right)\right\}$.

**Definition 25.** *Let* $(s(X),^*)$ *be a group of neutrosophic soft sequences and* $(X,\Delta,E)$ *be a neutrosophic soft group in a neutrosophic soft topological space* $(X,\tau,E)$*. A neutrosophic soft method is a function G defined on a subgroup* $(c_G(X),^*)$ *of* $(s(X),^*)$ *into* $(X,\tau,E)$ *such that* $G(\mathbf{x}^*\mathbf{y}) = G(\mathbf{x})\, \Delta\, G(\mathbf{y})$ *for all neutrosophic soft sequences* $\mathbf{x}$, $\mathbf{y}$ *in* $(X,\tau,E)$.

**Definition 26.** *A neutrosophic soft sequence* $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ *is said to be G-convergent to* $x^{e}_{\alpha,\beta,\gamma}$, *if* $\mathbf{x} \in c_G(X)$ *and* $G(\mathbf{x}) = x^{e}_{\alpha,\beta,\gamma}$.

**Definition 27.** *A neutrosophic soft method G is called neutrosophic soft regular, if every convergent neutrosophic soft sequence* $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ *is G-convergent with* $G(\mathbf{x}) = x^{e}_{\alpha,\beta,\gamma}$, *where* $\mathbf{x}$ *converges to* $x^{e}_{\alpha,\beta,\gamma}$.

**Definition 28.** *A neutrosophic soft point* $x^{e}_{\alpha,\beta,\gamma}$ *is said to be a neurosophic soft sequential cluster point of a neutrosophic soft set* $\left(\widetilde{F},E\right)$ *if and only if there exists a neutrosophic soft sequence of* *neutrosophic soft points in* $\left(\widetilde{F},E\right)$ *that converges to* $x^e_{\alpha,\beta,\gamma}$. *The union of all neutrosophic soft sequntial cluster points of* $\left(\widetilde{F},E\right)$ *is called the neutrosophic soft sequential closure of* $\left(\widetilde{F},E\right)$ *and denoted by* $\overline{\left(\widetilde{F},E\right)}^{seq}$.

**Definition 29.** *Let* $(X,\tau,E)$, $(Y,\vartheta,K)$ *be two neutrosophic soft topological spaces and* $\Phi : X \to Y$, $\Psi : E \to K$ *be two functions. Then, the pair* $(\Phi,\Psi)$ *is called a neutrosophic soft mapping from* $(X,\tau,E)$ *to* $(Y,\vartheta,K)$ *and denoted by* $(\Phi,\Psi) : (X,\tau,E) \to (Y,\ vartheta,K)$. *The image of each neutrosophic soft set* $\left(\widetilde{F},E\right)$ *in* $(X,\tau,E)$ *under* $(\Phi,\Psi)$ *is denoted by* $(\widetilde{\Phi(F)},\Psi(E))$ *and the membership function, indeterminacy function and non-membership function of* $\widetilde{\Phi(F)}(\Psi(e))$, *for each* $e \in E$, *defined as*

$$T_{\widetilde{\Phi(F)}(\Psi(e))}(y) = \begin{cases} \sup_{z\in\Phi^{-1}(y)}\{T_A(z)\} & ,\textit{if } \Phi^{-1}(y) \textit{ is not empty}, \\ 0 & ,\textit{if } \Phi^{-1}(y) \textit{ is empty}, \end{cases}$$

$$I_{\widetilde{\Phi(F)}(\Psi(e))}(y) = \begin{cases} \sup_{z\in\Phi^{-1}(y)}\{I_A(z)\} & ,\textit{if } \Phi^{-1}(y) \textit{ is not empty}, \\ 0 & ,\textit{if } \Phi^{-1}(y) \textit{ is empty}, \end{cases}$$

$$F_{\widetilde{\Phi(F)}(\Psi(e))}(y) = \begin{cases} \inf_{z\in\Phi^{-1}(y)}\{F_A(z)\} & ,\textit{if } \Phi^{-1}(y) \textit{ is not empty}, \\ 1 & ,\textit{if } \Phi^{-1}(y) \textit{ is empty}, \end{cases}$$

*for all* $y$ *in* $Y$, *where* $\Phi^{-1}(y) = \{x : \Phi(x) = y\}$, *respectively.*

*Conversely, the inverse image of each neutrosophic set* $\left(\widetilde{G},K\right)$ *in* $(Y,\vartheta,K)$ *under* $(\Phi,\Psi)$ *is denoted by* $(\widetilde{\Phi^{-1}(G)},\Psi^{-1}(K))$ *and the membership function, indeterminacy function and non-membership function of* $\widetilde{\Phi^{-1}(G)}(\Psi^{-1}(K))$, *for each* $k \in K$ *defined as* $T_{\widetilde{\Phi^{-1}(G)}(\Psi^{-1}(K))}(\mathbf{x}) = T_{\widetilde{(G,K)}}(\Phi(\mathbf{x}))$, $I_{\widetilde{\Phi^{-1}(G)}(\Psi^{-1}(K))}(\mathbf{x}) = I_{\widetilde{(G,K)}}(\Phi(\mathbf{x}))$ *and* $F_{\widetilde{\Phi^{-1}(G)}(\Psi^{-1}(K))}(\mathbf{x}) = F_{\widetilde{(G,K)}}(\Phi(\mathbf{x}))$, *for all* $\mathbf{x} \in X$, *respectively.*

For convinience, the image of any neutrosophic soft sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ under any neutrosophic soft function $(\Phi,\Psi)$ will be denoted by $(\Phi,\Psi)(\mathbf{x})$ and the inverse image of any neutrosophic soft sequence $\mathbf{y} = \left\{y^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ under any neutrosophic soft function $(\Phi,\Psi)$ will be denoted by $(\Phi,\Psi)^{-1}(\mathbf{y})$. Also, the image of any neutrosophic soft point $u^e_{r,t,s}$, under any neutrosophic soft function $(\Phi,\Psi)$ will be denoted by $(\Phi,\Psi)(u^e_{r,t,s})$ and the inverse image of any neutrosophic soft point $u^e_{r,t,s}$, under any neutrosophic soft function $(\Phi,\Psi)$ will be denoted by $(\Phi,\Psi)^{-1}(u^e_{r,t,s})$.

## 4. Sequential Definitions of Continuity in Neutrosophic Soft Topological Space

In this section, we present new concepts and investigate their properties. Also, different counterexamples are given.

**Definition 30.** *Let $\left(\widetilde{A},E\right)$ be a neutrosophic soft set and $x^{e}_{\alpha,\beta,\gamma}$ be a neutrosophic soft point in $(X,\tau,E)$. Then, $x^{e}_{\alpha,\beta,\gamma}$ is in the neutrosophic soft G- sequential closure of $\left(\widetilde{A},E\right)$ (or neutrosophic soft G-hull of $\left(\widetilde{A},E\right)$), if there is a neutrosophic soft sequence $\mathbf{x}=\left\{x_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$ such that $G(\mathbf{x})=x^{e}_{\alpha,\beta,\gamma}$. We denote neutrosophic soft G-sequential closure of a neutrosophic soft set $\left(\widetilde{A},E\right)$ by $\overline{\left(\widetilde{A},E\right)}^{G}$. We say that a neutrosophic soft set is G-sequentially neutrosophic closed if it contains all the neutrosophic soft points in its neutrosophic soft G-closure. It is clear that $\overline{\emptyset}^{G}=\emptyset$ and $\overline{X}^{G}=X$. If G is a neutrosophic soft regular method, then $\left(\widetilde{A},E\right)\subset\overline{\left(\widetilde{A},E\right)}\subset\overline{\left(\widetilde{A},E\right)}^{G}$. Hence, $\left(\widetilde{A},E\right)$ is neutrosophic G-sequentially closed if and only if $\overline{\left((\widetilde{A},E)\right)}^{G}=\left((\widetilde{A},E)\right)$. The image of $\overline{(\widetilde{A},E}$ under neutrosophic soft function $(\Phi,\Psi)$ is denoted as $\left((\Phi\left(\widetilde{\overline{((\widetilde{A},E)}^{G}}\right),\Psi(E)\right))$.*

**Example 31.** *Let $(X,\tau,E)$ be any neutrosophic soft topological space, where $X=[0,1]$ and $E=\{e_1,e_2\}$. Consider be a neutrosophic soft method G defined as $G(\mathbf{x})=u^{e_1}_{0,5,0,5,0,5}$, where $u=\lim_{n\to\infty}\frac{x_n+x_{n+1}}{2}$, for any neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ in X. Consider a neutrosophic soft set $(\widetilde{A},E)=0^{e_1}_{0,5,0,5,0,5}\cup 1^{e_1}_{0,5,0,5,0,5}$. Then, $\overline{(\widetilde{A},E)}^{G}=0^{e_1}_{0,5,0,5,0,5}\cup 0,5^{e_1}_{0,5,0,5,0,5}\cup 1^{e_1}_{0,5,0,5,0,5}$ and $\overline{\left(\overline{(\widetilde{A},E)}^{G}\right)}^{G}=0^{e_1}_{0,5,0,5,0,5}\cup 0,25^{e_1}_{0,5,0,5,0,5}\cup 0,5_{0,5,0,5,0,5}\cup 0,75^{e_1}_{0,5,0,5,0,5}\cup 1^{e_1}_{0,5,0,5,0,5}$. So, $\overline{\left(\overline{(\widetilde{A},E)}^{G}\right)}^{G}\neq\overline{(\widetilde{A},E)}^{G}$.*

**Definition 32.** *A neutrosophic soft subset $\left(\widetilde{F},E\right)$ of X is called G -sequentially neutrsophic soft compact, if, for any neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ a of neutrosophic soft points in $\left(\widetilde{F},E\right)$, there is a subsequence $\mathbf{y}=\left\{x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}}\right\}_{n\in N}$ of $\mathbf{x}$ with $G(\mathbf{y})\in\left(\widetilde{F},E\right)$.*

**Definition 33.** *A neutrosophic soft function $(\Phi,\Psi):(X,\tau,E)\to(X,\tau,E)$ is neutrosophic soft G -sequentially continuous at a neutrosophic soft point $u^{e}_{r,t,s}$, if, for any given a sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in X , $G(\mathbf{x})=u^{e}_{r,t,s}$ implies that $G((\Phi,\Psi)(\mathbf{x}))=(\Phi,\Psi)(u^{e}_{r,t,s})$. For a neutrosophic soft subset $\left(\widetilde{D},E\right)$ of X, $(\Phi,\Psi)$ is called neutrosophic soft G-sequentially continuous on $\left(\widetilde{D},E\right)$, if it is neutrosophic G-sequentially continuous at every $u^{e}_{r,t,s}\in\left(\widetilde{D},E\right)$ and is neutrosophic soft G-sequentially continuous, if it is neutrosophic soft G-sequentially continuous on X.*

**Theorem 34.** *The image of any neutrosophic soft G-sequentially compact subset of X under a neutrosophic soft G-sequential continuous function is neutrosophic soft G-sequentially compact.*

*Proof.* Let $(\Phi,\Psi)$ be any neutrosophic soft $G$-sequentially continuous function on $X$ and $\left(\widetilde{F},E\right)$ be any neutrosophic soft $G$-sequentially compact subset of $X$. Take any neutrosophic soft sequence $\mathbf{y}=\left\{y^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}=\left\{(\Phi,\Psi)(x^{e_n}_{n_{r_n,t_n,s_n}})\right\}_{n\in N}$ of neutrosophic soft points in $(\widetilde{\Phi(F)},\Psi(E))$. Since $(\widetilde{F},E)$ is neutrosophic soft $G$-sequentially compact, there exists a subsequence $\mathbf{z}=x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}\,k\in N}$ of the neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ with $G(\mathbf{z})\in\left(\widetilde{F},E\right)$. Then, the sequence $(\Phi,\Psi)(\mathbf{z})=\left\{(\Phi,\Psi)(x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}})\right\}_{k\in N}$ is a subsequence of the sequence $\mathbf{y}$. Since $(\Phi,\Psi)$ is neutrosophic soft $G$- sequentially continuous, $G((\Phi,\Psi))(\mathbf{z})\in(\widetilde{\Phi(F)},\Psi(E))$. Thus, $(\widetilde{\Phi(F)},\Psi(E))$ is neutrosophic soft $G$-sequentially compact. $\square$

**Definition 35.** *A neutrosophic soft method G is called neutrosophic soft subsequential if, for any neutrosophic soft sequence* $\mathbf{x}$ *such that* $G(\mathbf{x})=x^e_{r,t,s}$, *there exists a subsequence* $\left\{x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}}\right\}_{k\in N}$ *of* $\mathbf{x}$ *that converges to* $x^e_{r,t,s}$.

**Definition 36.** *A neutrosophic soft point* $u^e_{r,t,s}$ *is in the neutrosophic soft sequential derived set of* $\left(\widetilde{A},E\right)$, *(or called a neutrosophic soft sequential accumulation point of* $\left(\widetilde{A},E\right)$ *if there exists a neutrosophic soft sequence* $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ *of neutrosophic soft points, where* $x^{e_n}_{n_{r_n,t_n,s_n}}q(u^e_{r,t,s})^c$ *for all* $n\in N$, *in* $\left(\widetilde{A},E\right)$ *such that* $x^{e_n}_{n_{r_n,t_n,s_n}}\to u^e_{r,t,s}$. *We denote neutrosophic soft sequential derived set (the set of all neutrosophic soft sequential accumulation points) of a neutrosophic soft set* $\left(\widetilde{A},E\right)$ *by* $\left(\widetilde{A},E\right)'$. *The image of* $\left(\widetilde{A},E\right)'$ *under neutrosophic soft function* $(\Phi,\Psi)$ *is denoted as* $\left(\widetilde{\Phi(\left(\widetilde{A},E\right)'}),\Psi(E)\right)$.

**Definition 37.** *A neutrosophic soft point* $u^e_{r,t,s}$ *is in the neutrosophic soft G -sequential derived set of* $\left(\widetilde{A},E\right)$ *(or called a neutrosophic soft G-sequential accumulation point of* $\left(\widetilde{A},E\right)$*) if there exists a neutrosophic soft sequence* $\mathbf{x}=\left\{x_{n_{r_n,t_n,s_n}}\right\}^{e_n}_{n\in N}$ *of neutrosophic soft points, where* $x^{e_n}_{n_{r_n,t_n,s_n}}q(u^e_{r,t,s})^c$ *for all* $n\in N$, *in* $\left(\widetilde{A},E\right)$ *such that* $G(\mathbf{x})=u^e_{r,t,s}$. *We denote neutrosophic soft G-sequential derived set (the set of all neutrosophic soft G-sequential accumulation points) of a neutrosophic soft set* $\left(\widetilde{A},E\right)$ *by* $(\left(\widetilde{A},E\right)')^G$. *The image of* $(\left(\widetilde{A},E\right)')^G$ *under neutrosophic soft function* $(\Phi,\Psi)$ *is denoted by* $\left(\widetilde{\Phi(\left(\widetilde{A},E\right)'G)},\Psi(E)\right)))$.

**Definition 38.** *A neutrosophic soft point* $x^e_{r,t,s}$ *of X is called a neutrosophic soft G-sequential boundary point of a neutrosophic soft set* $\left(\widetilde{A},E\right)$ *if* $x^e_{r,t,s}$ *lies in both neutrosophic soft G-sequential closure of* $\left(\widetilde{A},E\right)$ *and neutrosophic soft G-sequential closure of the complement of A. We denote neutrosophic soft G -sequential boundary set of A by* $((A)^b)^G$.

**Lemma 39.** *Let G be a neutrosophic soft regular method and* $\left(\widetilde{A},E\right)$ *be any neutrosophic soft subset in* $(X,\tau,E)$. *Then,* $\overline{\left(\widetilde{A},E\right)}^G=\overline{\left(\widetilde{A},E\right)}$ *if and only if G is a neutrosophic soft subsequential method, where* $\overline{\left(\widetilde{A},E\right)}$ *denotes the usual closure of the neutrosophic soft set* $\left(\widetilde{A},E\right)$.

**Theorem 40.** *Let G be a neutrosophic soft regular method. Then, G is a neutrosophic soft subsequential method if and only if* $\left(\widetilde{A},E\right)^{\prime}=(\left(\widetilde{A},E\right)^{\prime})^{G}$ *for every neutrosophic soft subset* $\left(\widetilde{A},E\right)$ *in* $(X,\tau,E)$.

*Proof.* First, suppose that $\left(\widetilde{A},E\right)^{\prime}=(\left(\widetilde{A},E\right)^{\prime})^{G}$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. Then $\overline{\left(\widetilde{A},E\right)}=\left(\widetilde{A},E\right)\cup\left(\widetilde{A},E\right)^{\prime}=\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{\prime})^{G}=\overline{\left(\widetilde{A},E\right)}^{G}$. It follows from Lemma 39 that $G$ is a neutrosophic soft subsequential method. Now suppose that $G$ is a neutrosophic soft subsequential method and take any soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. Let $u^{e}_{r,t,s}$ be any neutrosophic point in $\left(\widetilde{A},E\right)^{\prime}$. Then, there is a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$, where $x^{e_n}_{n_{r_n,t_n,s_n}}q(u^{e}_{r,t,s})^{c}$ for all $n\in N$, in $\left(\widetilde{A},E\right)$ such that $x^{e_n}_{n_{r_n,t_n,s_n}}\rightarrow u^{e}_{r,t,s}$. As $G$ is neutrosophic soft regular, $G(\mathbf{x})=u^{e}_{r,t,s}$. Hence $u^{e}_{r,t,s}\in(\left(\widetilde{A},E\right)^{\prime})^{G}$. To prove that $(\left(\widetilde{A},E\right)^{\prime})^{G}\subset\left(\widetilde{A},E\right)^{\prime}$, take any neutrosophic softt point $u^{e}_{r,t,s}$ in $(\left(\widetilde{A},E\right)^{\prime})^{G}$. Then, there is a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$, where $x^{e_n}_{n_{r_n,t_n,s_n}}q(u^{e}_{r,t,s})^{c}$ for all $n\in N$, such that $G(x)=u^{e}_{r,t,s}$. As G is a neutrosophic soft subsequential method, there is a neutrosophic soft subsequence $\left\{x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}}\right\}_{k\in N}$ of x with $x^{e_{n_k}}_{n_{k_{r_{n_k}},t_{n_k},s_{n_k}}}\rightarrow u^{e}_{r,t,s}$. Hence $u^{e}_{r,t,s}\in\left(\widetilde{A},E\right)^{\prime}$. This completes the proof. □

**Theorem 41.** *Let G be a neutrosophic soft regular method. Then G is a neutrosophic soft subsequential method if and only if* $\left(\widetilde{A},E\right)^{b}=(\left(\widetilde{A},E\right)^{b})^{G}$ *for every neutrosophic soft subset* $\left(\widetilde{A},E\right)$ *in* $(X,\tau,E)$.

*Proof.* Firstly suppose that $\left(\widetilde{A},E\right)^{b}=(\left(\widetilde{A},E\right)^{b})^{G}$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. Then $\overline{\left(\widetilde{A},E\right)}=\left(\widetilde{A},E\right)\cup\left(\widetilde{A},E\right)^{b}=\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{b})^{G}=\overline{\left(\widetilde{A},E\right)}^{G}$. It follows from Lemma 39 that $G$ is a neutrosophic soft subsequential method. Now suppose that $G$ is a neutrosophic soft subsequential method and take any neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. Let $u^{e}_{r,t,s}$ be any neutrosophic soft point in $\left(\widetilde{A},E\right)^{b}$. Then $u^{e}_{r,t,s}$ is both in $\overline{\left(\widetilde{A},E\right)}$ and $\overline{\left(\widetilde{A},E\right)^{c}}$, where $\left(\widetilde{A},E\right)^{c}$ denotes the complement of the neutrosophic soft set $\left(\widetilde{A},E\right)$. Hence, there exist a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$ and a neutrosophic soft sequence $\mathbf{y}=\left\{y^{e_n}_{n_{p_n,v_n,m_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)^{c}$ such that $x^{e_n}_{n_{r_n,t_n,s_n}}\rightarrow u^{e}_{r,t,s}$ and $y^{e_n}_{n_{p_n,v_n,m_n}}\rightarrow u^{e}_{r,t,s}$. As $G$ is neutrosophic soft regular, $G(x)=u^{e}_{r,t,s}$ and $G(y)=u^{e}_{r,t,s}$. Hence $u^{e}_{r,t,s}\in(\left(\widetilde{A},E\right)^{b})^{G}$.

To prove that $(\left(\widetilde{A},E\right)^{b})^{G}\subset\left(\widetilde{A},E\right)^{b}$, take any neutrosophic soft point $u^{e}_{r,t,s}$ of $(\left(\widetilde{A},E\right)^{b})^{G}$.

Then, $u^e_{r,t,s}$ is in both $\overline{\left(\widetilde{A},E\right)}^G$ and $\overline{\left(\widetilde{A},E\right)^c}^G$. Hence there exist a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$ and a neutrosophic soft sequence $\mathbf{y}=\left\{y^{e_n}_{n_{p_n,v_n,m_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)^c$ such that $G(x)=u^e_{r,t,s}$ and $G(y)=u^e_{r,t,s}$. As G is a neutrosophic soft subsequential method, there is a neutrosophic soft subsequence $\left\{x^{e_{n_k}}_{n_{k_{r_{n_k},t_{n_k},s_{n_k}}}}\right\}_{k\in N}$ of $x$ with $x^{e_{n_k}}_{n_{k_{r_{n_k},t_{n_k},s_{n_k}}}}\to u^e_{r,t,s}$ and a neutrosophic soft subsequence $\left\{y^{e_{n_k}}_{n_{k_{p_{n_k},v_{n_k},m_{n_k}}}}\right\}_{k\in N}$ of y with $y^{e_{n_k}}_{n_{k_{p_{n_k},v_{n_k},m_{n_k}}}}\to u^e_{r,t,s}$. Hence, $u^e_{r,t,s}\in\left(\widetilde{A},E\right)^b$. This completes the proof. $\square$

**Theorem 42.** *Let $(X,\tau,E)$ be a neutrosophic soft topological space and $(X,\Delta,E)$ be a neutrosophic soft group in $(X,\tau,E)$, whose identity neutrosophic soft point is $e^k_{\alpha,\beta,\gamma}$. Consider a neutrosophic soft regular method G and two neutrosophic soft sets $\left(\widetilde{A},E\right)$, $\left(\widetilde{B},E\right)$ in neutrosophic soft subsets in $(X,\tau,E)$. Then, the followings are satisfied:*

*i. If $\left(\widetilde{A},E\right)\subset\left(\widetilde{B},E\right)$ then $\overline{\left(\widetilde{A},E\right)}^G\subset\overline{\left(\widetilde{B},E\right)}^G$,*

*ii. $\overline{\left(\widetilde{A},E\right)}^G\cup\overline{\left(\widetilde{B},E\right)}^G\subset\overline{\left(\widetilde{A},E\right)\cup\left(\widetilde{B},E\right)}^G$,*

*iii. $\overline{\left(\widetilde{A},E\right)\cap\left(\widetilde{B},E\right)}^G\subset\overline{\left(\widetilde{A},E\right)}^G\cap\overline{\left(\widetilde{B},E\right)}^G$,*

*iv. $\overline{\left(\widetilde{A},E\right)}^G\Delta\overline{\left(\widetilde{B},E\right)}^G\subset\overline{\left(\widetilde{A},E\right)\Delta\left(\widetilde{B},E\right)}^G$,*

*v. $\overline{\left(\widetilde{A},E\right)^{-1}}^G=(\overline{\left(\widetilde{A},E\right)}^G)^{-1}$,*

*vi. Neutrosophic soft G-sequential closure of a neutrosophic soft subgroup is a neutrosophic subgroup,*

*vii. $\left(\widetilde{A},E\right)\Delta\overline{e^k_{\alpha,\beta,\gamma}}=\left(\widetilde{A},E\right)$, if $\left(\widetilde{A},E\right)$ is neutrosophic G-closed.*

*Proof.* i. The proof is omitted.

ii. Take any element $u^e_{r,t,s}$ of $\overline{\left(\widetilde{A},E\right)}^G\cup\overline{\left(\widetilde{B},E\right)}^G$. Then, $u^e_{r,t,s}$ is either in $\overline{\left(\widetilde{A},E\right)}^G$ or in $\overline{\left(\widetilde{B},E\right)}^G$. Then, there is a neutrosophic sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of all neutrosophic points in $\left(\widetilde{A},E\right)$ or all points in $\left(\widetilde{B},E\right)$ such that $G(x)=u^e_{r,t,s}$. Hence, $u_{r,t,s}$ is in $\overline{\left(\widetilde{A},E\right)\cup\left(\widetilde{B},E\right)}^G$.

iii. The proof is omitted.

iv. Let $u^e_{r,t,s}\in\overline{\left(\widetilde{A},E\right)}^G\Delta\overline{\left(\widetilde{B},E\right)}^G$. Hence there exist $u^{e_1}_{1_{r_1,t_1,s_1}}\in\overline{\left(\widetilde{A},E\right)}^G$ and $u^{e_2}_{2_{r_2,t_2,s_2}}\in\overline{\left(\widetilde{B},E\right)}^G$ such that $u^e_{r,t,s}=u^{e_1}_{1_{r_1,t_1,s_1}}\Delta u^{e_2}_{2_{r_2,t_2,s_2}}$. Then, there are neutrosophic sequences $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$ and $\mathbf{y}=\left\{y^{e_n}_{n_{p_n,v_n,m_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{B},E\right)$ such that $G(x)=u^{e_1}_{1_{r_1,t_1,s_1}}$ and $G(y)=u^{e_2}_{2_{r_2,t_2,s_2}}$. Now define a sequence $\mathbf{z}=\mathbf{x}\Delta\mathbf{y}$. From the additivity of $G$, we get $G(z)=G(x\Delta y)=G(x)\Delta G(y)=u^{e_1}_{1_{r_1,t_1,s_1}}\Delta u^{e_2}_{2_{r_2,t_2,s_2}}\in\overline{\left(\widetilde{A},E\right)\Delta\left(\widetilde{B},E\right)}^G$.

v. The proof is omitted.

vi. The proof is omitted.

vii. The proof is omitted.

Strictness of inclusions in (iii) and (iv) are obvious as they are same in ordinary closure. □

**Example 43.** *Take the same neutrosophic soft topological space $(X,\tau,E)$ and neutrosophic soft method G as in Example 31. Consider neutrosophic soft sets $\left(\widetilde{A},E\right)=0^{e_1}_{0.5,0.5,0.5}$ and $\left(\widetilde{B},E\right)=1^{e_1}_{0.5,0.5,0.5}$. Then, $\overline{\left(\widetilde{A},E\right)}^G\cup\overline{\left(\widetilde{B},E\right)}^G=0^{e_1}_{0.5,0.5,0.5}\cup 1^{e_1}_{0.5,0.5,0.5}$ and $\overline{A\cup B}^G=0^{e_1}_{0.5,0.5,0.5}\cup 0,5^{e_1}_{0.5,0.5,0.5}\cup 1^{e_1}_{0.5,0.5,0.5}$. This means that converse statement in Theorem 42(ii.) is not always true.*

**Example 44.** *Take any neutrosophic soft topological space $(X,\tau,E)$. Consider a neutrosophic soft method G defined as $G(\mathbf{x})=x^{e_1}_{1_{r_1,t_1,s_1}}$ for any neutrosophic sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ in $(X,\tau,E)$. Then, any neutrosophic subset of $(X,\tau,E)$ is neutrosophic G-sequentially closed. This shows that being in the neutrosophic soft G-sequential closure of a neutrosophic soft set does not imply being in the neutrosophic soft sequential closure of the set in general.*

**Theorem 45.** *Let G be a neutrosophic soft regular method and $\left\{\left(\widetilde{A}_i,E\right)\right\}$ be any collection of neutrosophic soft subsets in $(X,\tau,E)$, where I is any index set and $i\in I$. Then, the followings are satisfied:*

*i.* $\bigcup_{i\in I}\overline{\left(\widetilde{A}_i,E\right)}^G\subset\overline{\bigcup_{i\in I}\left(\widetilde{A}_i,E\right)_i}^G$,

*ii.* $\overline{\bigcap_{i\in I}\left(\widetilde{A}_i,E\right)_i}^G\subset\bigcap_{i\in I}\overline{\left(\widetilde{A}_i,E\right)}^G$,

*iii.* $\overline{\left(\widetilde{A_1},E\right)}^G\Delta\overline{\left(\widetilde{A_2},E\right)}^G\Delta\overline{\left(\widetilde{A_3},E\right)}^G\Delta\ldots\Delta\overline{\left(\widetilde{A}_k,E\right)}^G\Delta\ldots\subset\overline{\left(\widetilde{A_1},E\right)\Delta\left(\widetilde{A_2},E\right)\Delta\left(\widetilde{A_3},E\right)\Delta\ldots\Delta\left(\widetilde{A}_k,E\right)\Delta\ldots}$
*where $i_k\in I$ for any $k\in N$.*

*Proof.* Proofs are similar to those of the preceding theorem. □

**Theorem 46.** *Let G be a neutrosophic soft regular method and $\left(\widetilde{A},E\right)$ and $\left(\widetilde{B},E\right)$ neutrosophic soft subsets of X. Then the following are satisfied:*

*i. if* $\left(\widetilde{A},E\right)\subset\left(\widetilde{B},E\right)$ *then* $(\left(\widetilde{A},E\right)^{'})^G\subset(\left(\widetilde{B},E\right)^{'})^G$,

*ii.* $\overline{\left(\widetilde{A},E\right)}^G=\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{'})^G$,

*iii.* $\overline{\left(\widetilde{A},E\right)}^G=\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{b})^G$.

*Proof.* i. the proof is omitted.

ii. Take any neutrosophic soft point $u^e_{r,t,s}$ of $\overline{\left(\widetilde{A},E\right)}^G$. Then, $u^e_{r,t,s}$ is either in $\left(\widetilde{A},E\right)$ or not in $\left(\widetilde{A},E\right)$. If $u^e_{r,t,s}$ is in $\left(\widetilde{A},E\right)$, then it is in $\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{'})^G$. If $u^e_{r,t,s}$ is not in $\left(\widetilde{A},E\right)$, there exists a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points, where $x^{e_n}_{n_{r_n,t_n,s_n}}q\left(u^e_{r,t,s}\right)^c$ for all $n\in N$, in $\left(\widetilde{A},E\right)$ such that $G(\mathbf{x})=u^e_{r,t,s}$. Thus, $u^e_{r,t,s}\in(\left(\widetilde{A},E\right)^{'})^G$.
On the other hand, we get $\left(\widetilde{A},E\right)\cup(\left(\widetilde{A},E\right)^{'})^G\subset\overline{\left(\widetilde{A},E\right)}^G$ since $\left(\widetilde{A},E\right)\subset\overline{\left(\widetilde{A},E\right)}^G$ and $(\left(\widetilde{A},E\right)^{'})^G\subset\overline{\left(\widetilde{A},E\right)}^G$.

iii. Let $u^e_{r,t,s} \in \left(\widetilde{A},E\right) \cup (\left(\widetilde{A},E\right)^b)^G$. If $u^e_{r,t,s}$ is in $\left(\widetilde{A},E\right)$, there is nothing to prove since $\left(\widetilde{A},E\right) \subset \overline{\left(\widetilde{A},E\right)}^G$ for a neutrosophic soft regular method $G$. If $u^e_{r,t,s} \in (\left(\widetilde{A},E\right)^b)^G$ , then $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^G \cap \overline{(\left(\widetilde{A},E\right)^c)}^G$ , where $\left(\widetilde{A},E\right)^c$ denotes the complement of the neutrosophic soft set $\left(\widetilde{A},E\right)$. Hence, $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^G$ . Conversely, take any neutrosophic soft point $u^e_{r,t,s}$ of $\overline{\left(\widetilde{A},E\right)}^G$ . Thus, $u^e_{r,t,s}\tilde{q}\left(\widetilde{A},E\right)^c$ or $u^e_{r,t,s}q\left(\widetilde{A},E\right)^c$ . If $u^e_{r,t,s}\tilde{q}\left(\widetilde{A},E\right)^c$ , then $u^e_{r,t,s} \in \left(\widetilde{A},E\right)$ and there is nothing to prove. If $u^e_{r,t,s}q\left(\widetilde{A},E\right)^c$ , $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)^c}$. Then, there exists a neutrosophic soft sequence $\mathbf{y} = \left\{y^{e_n}_{n_{p_n,v_n,m_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)^c$ such that $y^{e_n}_{n_{p_n,v_n,m_n}} \to u^e_{r,t,s}$. As $G$ is neutrosophic soft regular, $G(\mathbf{y}) = u^e_{r,t,s}$. Hence $u^e_{r,t,s} \in (\left(\widetilde{A},E\right)^b)^G$.

For a neutrosophic soft regular method $G$, a neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$ is neutrosophic soft closed if and only if $(\left(\widetilde{A},E\right)^{\prime})^G \subset A$. But, we note that $\left(((A')^G)'\right)^G$ is not always a neutrosophic soft subset of $\left(\widetilde{A},E\right) \cup (\left(\widetilde{A},E\right)^{\prime})^G$. □

**Corollary 47.** *Let $G$ be a neutrosophic soft regular method. Then the intersection of any collection of neutrosophic soft $G$-sequentially closed subsets in $(X,\tau,E)$ is again a neutrosophic soft $G$-sequentially closed subset in $(X,\tau,E)$.*

*Proof.* The proof can be obtained easily, so is omitted.

Even for neutrosophic soft regular methods, the union of any two neutrosophic soft $G$-sequentially closed subsets of $(X,\tau,E)$ need not be a neutrosophic soft $G$-sequentially closed subset of $(X,\tau,E)$ as in Example 43 given after Theorem 42. □

**Theorem 48.** *Let $G$ be a neutrosophic soft regular method. If a neutrosophic soft function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous at a neutrosophic soft point $u^e_{r,t,s}$, then the statement that $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^G$ implies that $(\Phi,\Psi)(u^e_{r,t,s}) \in \overline{(\widetilde{\Phi(A)},\Psi(E))}^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.*

*Proof.* Suppose that $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous at a neutrosophic soft point $u^e_{r,t,s}$. Let $\left(\widetilde{A},E\right)$ be any neutrosophic soft subset in $(X,\tau,E)$ and $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^G$. Then, there is a neutrosophic soft sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in $\left(\widetilde{A},E\right)$ such that $G(\mathbf{x}) = u^e_{r,t,s}$ . Since $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous at the neutrosophic soft point $u^e_{r,t,s}$ , $G((\Phi,\Psi)(\mathbf{x})) = (\Phi,\Psi)(u^e_{r,t,s})$ . Thus $(\Phi,\Psi)(u^e_{r,t,s}) \in \overline{(\widetilde{\Phi(A)},\Psi(E))}^G$ . □

**Corollary 49.** *Let $G$ be a neutrosophic soft regular method. If a neutrosophic soft function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous, then $\left(\widetilde{\Phi(\overline{A}^G)},\Psi(E)\right) \subset \overline{(\widetilde{\Phi(A)},\Psi)(E)}^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.*

*Proof.* The proof follows from the preceding theorem. $\square$

For neutrosophic soft regular subsequential methods the converse of Theorem 48 is also valid so is the converse of Corollary 49 , i.e. a neutrosophic soft function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous at a neutrosophic soft point $u^e_{r,t,s}$ if and only if the statement $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^G$ implies that $(\Phi,\Psi)(u^e_{r,t,s}) \in \overline{(\widetilde{\Phi(A)},\Psi)(E)}^G$ and a neutrosophic soft function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous in $(X,\tau,E)$ if and only if $\left(\widetilde{\Phi(\overline{A}^G)},\Psi(E)\right) \subset \overline{(\widetilde{\Phi(A)},\Psi)(E)}^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.

**Corollary 50.** *Let $G$ be a neutrosophic soft regular method. If a one-to-one and onto function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous on $X$, then $\left(\widetilde{\Phi((A')^G)},\Psi(E)\right) \subset \left((\widetilde{\Phi(A)},\Psi(E))'\right)^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.*

*Proof.* Take a neutrosophic soft point $(\Phi,\Psi)(u^e_{r,t,s})$ in $\left(\widetilde{\Phi((A')^G)},\Psi(E)\right)$. This means that $u^e_{r,t,s} \in (\left(\widetilde{A},E\right)')^G$. So, there exists a neutrosophic sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points, where $x^{e_n}_{n_{r_n,t_n,s_n}} q \left(u^e_{r,t,s}\right)^c$ for all $n \in N$, in $\left(\widetilde{A},E\right)$ such that $G(\mathbf{x}) = u^e_{r,t,s}$ . Then, $(\Phi,\Psi)(\mathbf{x}) = \left\{(\Phi,\Psi)(x^{e_n}_{n_{r_n,t_n,s_n}})\right\}_{n\in N}$ is a neutrosophic sequnce of neutrosophic soft points in $(\widetilde{\Phi(A)},\Psi(E))$. Since $(\Phi,\Psi)$ is a one-to-one, onto function and neutrosophic soft $G$-sequentially continuous in $(X,\tau,E)$, $(\Phi,\Psi)(x^{e_n}_{n_{r_n,t_n,s_n}})q((\Phi,\Psi)(u^e_{r,t,s}))^c$ for all $n \in N$, in $(\widetilde{\Phi(A)},\Psi(E))$ such that $G((\Phi,\Psi)(\mathbf{x})) = (\Phi,\Psi)(u^e_{r,t,s})$. Hence, $(\Phi,\Psi)(u^e_{r,t,s}) \in \left((\widetilde{\Phi(A)},\Psi(E))'\right)^G$ $\square$

**Corollary 51.** *Let $G$ be a neutrosophic soft regular method. If a one-to-one and onto function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous on $X$, then $\left(\widetilde{\Phi((A^b)^G)},\Psi(E)\right) \subset \left((\widetilde{\Phi(A)},\Psi(E))^b\right)^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.*

*Proof.* The proof follows from Theorem 48.

For neutrosophic soft regular subsequential methods the converse of Corollaries 50 and 51 are also true, i.e. a one-to-one and onto function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous in $(X,\tau,E)$ if and only if $\left(\widetilde{\Phi((A')^G)},\Psi(E)\right) \subset \left((\widetilde{\Phi(A)},\Psi(E))'\right)^G$ and a one-to-one and onto function $(\Phi,\Psi)$ in $(X,\tau,E)$ is neutrosophic soft $G$-sequentially continuous if and only if $\left(\widetilde{\Phi((A^b)^G)},\Psi(E)\right) \subset \left((\widetilde{\Phi(A)},\Psi(E))^b\right)^G$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. $\square$

**Theorem 52.** *Let $G$ be a neutrosophic soft regular method. If a function $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially continuous in $(X,\tau,E)$, then inverse image of any neutrosophic soft $G$-sequentially closed subset in $(X,\tau,E)$ is neutrosophic soft $G$-sequentially closed i.e. $\left((\widetilde{\Phi^{-1}(U)},\Psi^{-1}(E)\right)$ is neutrosophic soft $G$-sequentially closed for every neutrosophic soft $G$-sequentially closed subset $\left(\widetilde{U},E\right)$ in $(X,\tau,E)$.*

*Proof.* Take a neutrosophic soft $G$-sequentially closed subset $\left(\widetilde{U},E\right)$ in $(X,\tau,E)$. Let $\left(\widetilde{V},E\right)=\left((\widetilde{\Phi^{-1}(U)},\Psi^{-1}(E)\right)$ and suppose that $u^{e_n}_{r,t,s}\in\overline{\left(\widetilde{V},E\right)}^{G}$. Then, there exists a neutrosophic soft sequence $\mathbf{x}=\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$, of neutrosophic soft points in $\left(\widetilde{V},E\right)$ such that $G(\mathbf{x})=u^{e}_{r,t,s}$. Since $G((\Phi,\Psi)(\mathbf{x}))=(\Phi,\Psi)(u^{e}_{r,t,s})$, $(\Phi,\Psi)(\mathbf{x})$ is a neutrosophic soft sequence of neutrosophic point is in $\left(\widetilde{U},E\right)$ and $\left(\widetilde{U},E\right)$ is neutrosophic soft $G$-closed. We obtain that $(\Phi,\Psi)(u^{e}_{r,t,s})\in\left(\widetilde{U},E\right)$. This implies that $u^{e}_{r,t,s}\in\left(\widetilde{V},E\right)$. Hence, $\overline{\left(\widetilde{V},E\right)}^{G}\subset\left(\widetilde{V},E\right)$. □

**Theorem 53.** *Let G be a neutrosophic soft regular subsequential method. Then every neutrosophic soft G-sequentially continuous function is neutrosophic soft continuous in the ordinary sense.*

*Proof.* The proof is easily seen, so is omitted. □

**Theorem 54.** *Let G be a neutrosophic soft regular method. If every continuous neutrosophic soft function is neutrosophic soft G-sequentially continuous, then G is a neutrosophic soft subsequential method.*

*Proof.* Suppose that $G$ is not a neutrosophic soft subsequential method. We are going to find a function which is neutrosophic soft continuous but not neutrosophic soft $G$-sequentially continuous. As $G$ is not neutrosophic soft subsequential there is a neutrosophic soft subset $\left(\widetilde{F},E\right)$ in $(X,\tau,E)$ whose closure is a proper neutrosophic soft subset of its neutrosophic soft $G$-sequential closure. Take $u^{e_1}_{1_{r_1,t_1,s_1}}\in\overline{\left(\widetilde{F},E\right)}^{G}\cap\overline{\left(\widetilde{F},E\right)}^{c}$ and $u^{e_2}_{2_{r_2,t_2,s_2}}\in\overline{\left(\widetilde{F},E\right)}$. Define a function $(\Phi,\Psi)$ as $(\Phi,\Psi)(x_{r,t,s})=u^{e_2}_{2_{r_2,t_2,s_2}}$ for all $x^{e}_{r,t,s}\in\overline{\left(\widetilde{F},E\right)}$ and $(\Phi,\Psi)(x)=u^{e_1}_{1_{r_1,t_1,s_1}}$ for $x^{e}_{r,t,s}\in\overline{\left(\widetilde{F},E\right)}^{c}$. It is clear that $(\Phi,\Psi)$ is not neutrosophic soft $G$-sequentially continuous and is neutrosophic soft continuous in the ordinary sense. This completes the proof. □

Now we give the following definition.

**Definition 55.** *A neutrosophic soft function $(\Phi,\Psi)$ is called neutrosophic soft G-sequentially closed if $\left((\widetilde{\Phi(F)},\Psi(E)\right)$ is neutrosophic G-sequentially closed for every neutrosophic G-sequentially closed neutrosophic subset $\left(\widetilde{F},E\right)$ in $(X,\tau,E)$.*

**Theorem 56.** *Let G be a neutrosophic soft regular method. A neutrosophic soft function $(\Phi,\Psi)$ is G-sequentially closed if and only if $\overline{(\widetilde{\Phi(A)},\Psi)(E)}^{G}\subset\left(\widetilde{\Phi(\overline{A}^{G})},\Psi(E)\right)$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$.*

*Proof.* From the neutrosophic soft regularity of $G$, we have $\left(\widetilde{A},E\right)\subset\overline{\left(\widetilde{A},E\right)}^{G}$. Since $(\Phi,\Psi)$ is neutrosophic soft $G$- sequentially closed, we deduce $\overline{(\widetilde{\Phi(A)},\Psi)(E)}^{G}\subset\left(\widetilde{\Phi(\overline{A}^{G})},\Psi(E)\right)$.

Now suppose that $\overline{(\widetilde{\Phi(A)},\Psi)(E)}^{G}\subset\left(\widetilde{\Phi(\overline{A}^{G})},\Psi(E)\right)$ for every neutrosophic soft subset $\left(\widetilde{A},E\right)$ in $(X,\tau,E)$. Let $\left(\widetilde{F},E\right)$ be any neutrosophic soft $G$-sequentially closed neutrosophic subset of $X$. Then $\overline{\widetilde{(\Phi(F)},\Psi)(E)}^{G} \subset \left(\widetilde{\Phi(\overline{F}^{G}}),\Psi(E)\right) = \widetilde{(\Phi(F)},\Psi)(E)$. Hence $(\Phi,\Psi)$ is neutrosophic soft $G$-sequentially closed. This completes the proof. $\square$

**Theorem 57.** *Let $(X,\Delta,E)$ be a neutrosophicsoft group in a neutrosophic soft topological space $(X,\tau,E)$. Let $G_1$ and $G_2$ be two neutrosophic soft methods of neutrosophic soft sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$. Then, $\overline{\left(\widetilde{A},E\right)}^{G_1\Delta G_2} \subset \overline{\left(\widetilde{A},E\right)}^{G_1} \Delta \overline{\left(\widetilde{A},E\right)}^{G_2}$.*

*Proof.* Let $u^e_{r,t,s} \in \overline{\left(\widetilde{A},E\right)}^{G_1\Delta G_2}$. Then there exists a sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ such that $(G_1\Delta G_2)(\mathbf{x}) = u^e_{r,t,s}$. Hence $G_1(\mathbf{x})\Delta G_2(\mathbf{x}) = u^e_{r,t,s}$. Write $G_1(\mathbf{x}) = u^{e_1}_{1_{r_1,t_1,s_1}}$ and $G_2(\mathbf{x}) = u^{e_2}_{2_{r_2,t_2,s_2}}$. Therefore $u^{e_1}_{1_{r_1,t_1,s_1}} \in \overline{\left(\widetilde{A},E\right)}^{G_1}$ and $u^{e_2}_{2_{r_2,t_2,s_2}} \in \overline{\left(\widetilde{A},E\right)}^{G_2}$. Hence $u^e_{r,t,s} = u^{e_1}_{1_{r_1,t_1,s_1}} \Delta u^{e_2}_{2_{r_2,t_2,s_2}} \in \overline{\left(\widetilde{A},E\right)}^{G_1} \Delta \overline{\left(\widetilde{A},E\right)}^{G_2}$. $\square$

**Corollary 58.** *Let $G_1$ and $G_2$ be two neutrosophic soft subsequential methods of sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$. Then $(\left(\widetilde{A},E\right)^{\prime})^{G_1+G_2} \subset (\left(\widetilde{A},E\right)^{\prime})^{G_1} + (\left(\widetilde{A},E\right)^{\prime})^{G_2}$.*

*Proof.* The proof follows from Theorem 57. $\square$

**Corollary 59.** *Let $G_1$ and $G_2$ be two neutrosophic soft subsequential methods of sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$. Then $(\left(\widetilde{A},E\right)^{b})^{G_1+G_2} \subset (\left(\widetilde{A},E\right)^{b})^{G_1} + (A^b)^{G_2}$.*

*Proof.* The proof follows from Theorem 57. $\square$

**Theorem 60.** *Let $(X,\Delta,E)$ be a neutrosophic soft group in a neutrosophic soft topological space $(X,\tau,E)$. A neutrosophic soft function $(\Phi,\Psi)$ on $(X,\tau,E)$ is additive if and only if for every neutrosophic soft methods $G_1$ and $G_2$ of neutrosophic soft sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$ , neutrosophic soft $G_1$-sequential continuity and neutrosophic soft $G_2$-sequential continuity of $(\Phi,\Psi)$ together imply neutrosophic soft $(G_1\Delta G_2)$-sequential continuity of $(\Phi,\Psi)$.*

*Proof.* Let $(G_1\Delta G_2)(\mathbf{x}) = u^e_{r,t,s}$. Write $G_1(\mathbf{x}) = u^{e_1}_{1_{r_1,t_1,s_1}}$ and $G_2(\mathbf{x}) = u^{e_2}_{2_{r_2,t_2,s_2}}$. As $(\Phi,\Psi)$ is $G_1$-continuous and $G_2$-continuous, $G_1(f(\mathbf{x})) = (\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}})$ and $G_2(f(\mathbf{x})) = (\Phi,\Psi)(u^{e_2}_{2_{r_2,t_2,s_2}})$. Hence $(G_1\Delta G_2)((\Phi,\Psi)(\mathbf{x})) = G_1((\Phi,\Psi)(\mathbf{x}))\Delta G_2(f(\mathbf{x})) = (\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}})\Delta f(u^{e_2}_{2_{r_2,t_2,s_2}}) = (\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}} \Delta u^{e_2}_{2_{r_2,t_2,s_2}}) =$ $(\Phi,\Psi)(u^e_{r,t,s})$. Now suppose that $(\Phi,\Psi)$ is not additive so that there are elements $u^{e_1}_{1_{r_1,t_1,s_1}}$ and $u^{e_2}_{2_{r_2,t_2,s_2}}$ with $(\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}} \Delta u^{e_2}_{2_{r_2,t_2,s_2}}) \neq (\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}})\Delta(\Phi,\Psi)(u^{e_2}_{2_{r_2,t_2,s_2}})$. Define $G_1(\mathbf{x}) = u^{e_1}_{1_{r_1,t_1,s_1}}$ and $G_2(\mathbf{x}) = u^{e_1}_{2_{r_2,t_2,s_2}}$, where $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$. Then $(G_1\Delta G_2)(\mathbf{x}) = G_1(\mathbf{x})\Delta G_2(\mathbf{x}) = u^{e_1}_{1_{r_1,t_1,s_1}} \Delta u^{e_2}_{2_{r_2,t_2,s_2}}$ but $(G_1\Delta G_2)((\Phi,\Psi)(\mathbf{x})) = (\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}})\Delta(\Phi,\Psi)(u^{e_2}_{2_{r_2,t_2,s_2}})$ which is different from $(\Phi,\Psi)(u^{e_1}_{1_{r_1,t_1,s_1}} \Delta u^{e_2}_{2_{r_2,t_2,s_2}})$. This completes the proof. $\square$

## 5. Conclusion

We have introduced the concept of neutrosophic $G$-sequential continuity as a new tool for further studies. The definitions of neutrosophic soft sequence, neutrosophic soft quasi-coincidence, neutrosophic soft $q$-neighborhood, neutrosophic soft cluster point, neutrosophic soft boundary point, neutrosophic soft sequential closure, neutrosophic soft group, neutrosophic soft method have been given to constitute a base to define the concepts of neutrosophic soft $G$-sequential closure, neutrosophic soft $G$-sequential derived set, and the concept of $G$-sequentially neutrosophic soft compactness of a subset of a neutrosophic soft topological space. Some characterizations and implications have been obtained. We have also given some counterexamples to show that the converse statements of the implications are not always true. Since topological structures of neutrosophic soft sets carry great importance for numerous mathematical problems, various terms related to the other types of topological spaces, which constitute advantageous situations in different fields, have been adapted to neutrosophic topological spaces. Our expectation is that many scientists will take advantage of using these detections to advance in their researches in not only mathematics but also different disciplines. We also hope that these findings may constitute a general framework for their applications in real life problems.

## Conflict of interests

The authors declare that there is no conflict of interests.

## Acknowledgements

The authors acknowledge that the statements of some results in this paper were presented at the Fourth International Conference of Mathematical Sciences (ICMS 2020), [2].

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