Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Observer-based sliding mode mixed H_∞ and passive control for Markovian jump system with mode-dependent time-varying delay†

Huan Li | Yuechao Ma*

[1]School of Science, Yanshan University, Qinhuangdao Hebei, China. This work is partially supported by the National Natural Science Foundation of China, under Grant number 61273004, and the Natural Science Foundation of Hebei province No. F2018203099.

Correspondence
*Yuechao Ma, Yanshan University. Email: yuechaoma@ysu.edu.cn

Present Address
Qinhuangdao Hebei, P.R.China, 066004

Summary

This paper discusses mixed H_∞ and passive sliding mode control problem of uncertain Markovian jump system with one-sided Lipschitz non-linear and mode-dependent time-varying delay. The attractive point consist of the following. Firstly, by designing a suitable observer to estimate the unmeasurable state of the system. Secondly, based on a new mode-dependent Lyapunov-Krasovskii function, sufficient condition is established to ensure the stability of the closed-loop system. Thirdly, designing a suitable controller to guarantees reaches of predefined sliding mode surface. Finally, from the numerical examples, we can testify the effectiveness and less conservativeness of the theoretical method.

KEYWORDS:
Markovian jump, Observer, Time-varying delay, Mixed H_∞ and passive

1 | INTRODUCTION

In practical application, Markovian jump systems (MJSs) are widely concerned by scholars in many fields [1-4], such as electric power system, communication system, aircraft control, etc. Compared with traditional linear time-invariant systems, MJSs is better to modeling more complex systems that is inevitably affected by sudden changes in components, system internal structure and environmental disturbances [5-8]. Generally speaking, such systems contain multiple subsystems. Different subsystems describe the subsequent behavior of dynamic, while Markov chains describe the randomness of jump transfer among different subsystems. Especially control law design and stability analysis are important research contents of MJSs.

Time delay is a unavoidable factor which affects the stability of systems. It exists in various parts of the system, including system states time-delay [9], control signals time-delays [10,11], state derivative time-delays [12-14] and distributed time-delays [15,16]. In MJSs, time-delay divided into two forms: time-delay independence and time-delay dependence. The former has no relation with the switch signal, while the latter depends on the mode i. There are many scholars has been studied time-delay dependence[17,18]. In [10], researchers explored about delay-dependence stabilization of linear systems with time-varying state and input delays. In [12], this paper found novel robust stability criterion with mixed delays and non-linear perturbations. Then, delay-dependent robust stability with mixed delays was processed in [13]. In [14], a sliding mode approach to non-fragile observer-based control greatly improved the control of the system. Those references deal with nonlinear functions by using Lipschitz condition. It is worth

†This is an example for title footnote.

noting that the one-sided Lipscchitz (OSL) non-linear functions has fewer restrictions than Lipscchitz non-linearity which some parameters is positive. The first proposed the OSL non-linear in Hu [19]. The paper of [20,21] have less conservative than Hu [19]. Next, the observer devised for OSL non-linear systems. To our best knowledge, few studies focus on the stability of MJSs with time-delay dependence and OSL non-linear via sliding mode control. Hence this triggers us to study this paper.

In [26,27], the authors discussed the stability of MJSs. But, due to the influence of cost, measurement method and other factors, the state of the system can not be obtained directly in the real word, so it is necessary to construct an appropriate observer to estimate the state of the system. So it is necessary to construct an appropriate observer to estimate the state of the system. Therefore, the problem of observer design become great interest to scholars [4,28,29]. The robust observer-based finite-time for discrete-time implicit MJSs was studied in [4], the problem of robust observer based fault-tolerant control for MJSs were obtained in [28]. In [29], a observer-based sliding mode control for non-linear MJSs was discussed.

Sliding mode control (SMC), a kind of the variable structure control theory was proposed in 1960s, is an effective robust control strategies for systems with uncertainties or unknown models. Different from other control strategies, it has the feature of insensitivity, fast response and interference elimination. More recently, due to those features, SMC is proposed in the MJSs [30-32]. In general, the SMC method consists of two steps: (1) sliding phase synthesis. (2) arrival phase synthesis. Such as the SMC method being proposed for neutral-type stochastic systems and singular systems in [4,33]. In [34], T-S model-based sliding mode observer design for finite-time synthesis of MJSs was studied.

Owing to the influence of external disturbances, it is necessary to consider H_∞ and passivity performance. These performances are studied in many systems, such as hybrid [35], network control [36], T-S fuzzy [37] and random switching [38]. However, there are few discussions about mixed H_∞ and passivity for uncertain MJSs in the existing lierature.

This paper discusses mixed H_∞ /passive SMC problem of uncertain stochastic MJS with OSL non-linear and mode-dependent time-varying delay. The major contributions of this article are as follows.

1. Compared with the references [39,40], this paper which srudies the MJSs with mode-dependent time-varying and OSL non-linear function is more extensive. 2. A new Lyapunov-krasovskill is proposed, whose the parameters are mode-dependent in the integral term, which can reduce the conservativeness. 3. Our systems obeys one-sided Lipschitz non-linearity which is less conservative than the traditional Lipschitz.

Notation: $\langle \cdot,\cdot \rangle$ represent inner product in $\Re^n$,$\Re^n$ is the n-dimension Euclidean space, $\langle \phi_1, \phi_2 \rangle = {\phi_1}^T \phi_2, \forall \phi_1, \phi_2 \in \Re^n$ where ${\phi_1}^T$ is the transpose of vector ϕ_1, $\|\cdot\|$ denotes $L_2(0,\infty)$ norm, $*$ represent symmetric term. $(\lambda, \wp, p)$ is denote probability space, if there is no special explanation, matrix has suitable dimension and satisfies corresponding algebraic operation.

2 | PRELIMINARIES AND PROBLEM FORMULATION

Consider the following class of non-linear MJS

$$\begin{cases} \dot{x}(t) = (A(r_t) + \Delta A(r_t))\, x(t) + (A_d(r_t) + \Delta A_d(r_t))\, x(t - \Gamma(t, r_t)) + B(r_t)(u(t) + f(x(t), t, r_t)) \\ \qquad + F_a(r_t) f_a(t) + H(r_t)\varpi(t), \\ y(t) = C(r_t)\, x(t), \\ z(t) = C_x(r_t)\, x(t) + D_\varpi(r_t)\varpi(t), \\ x(\theta) = \varphi(\theta), \theta \in [-\Gamma_2, 0] \end{cases} \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $\varpi(t) \in \Re^q$, $y(t) \in \Re^p$, $z(t) \in \Re^q$, $f_a(t) \in \Re^{n_a}$ are stand for the system state, control input, disturbance input, measurement output, control output and actuator faults, respectively. $\varpi(t) \in \mathfrak{L}_2[0,\infty)$, and $f(x(t), t, r_t) \in \Re^m$ is continuity nonlinear function, $\varphi(\theta)$, $\theta \in [-\Gamma_2, 0]$ is initial function. Matrices $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $F_a(r_t)$, $H(r_t)$, $C_x(r_t)$, $C(r_t)$ and $D_\varpi(r_t)$ are given constant matrix, where matrix $B(r_t)$ is column full rank. $Rank\left(\begin{bmatrix} B_i & F_{ai} \end{bmatrix}\right) = Rank(B_i)$ and $\mathrm{F}_{ai} = B_i F_i$, F_i is constant matrix of suitable dimension.The uncertainties $\Delta A(r_t)$ and $\Delta A_d(r_t)$ are unknown time-varying matrix with norm bounded, $M(r_t)$, $N(r_t)$ and $N_d(r_t)$

are known matrices, and $\Xi(r_t,t)$ is satisfies

$$\Xi(r_t,t)\,\Xi(r_t,t)\leqslant \mathrm{I},\forall i\in S \tag{2}$$

$$\left[\Delta A(r_t)\ \ \Delta A_d(r_t)\right]=M(r_t)\,\Xi(r_t,t)\left[N(r_t)\ \ N_d(r_t)\right] \tag{3}$$

Let$\{r_t,t\geqslant 0\}$ is a Markov random jumping process which takes value in a finite set $S=\{1,2,...,s\}$ and relationships are shown as follows:

$$P_{\Upsilon}\left\{\Upsilon(t+\triangle)=j|\Upsilon(t)=i\right\}=\begin{cases}\pi_{ij}\triangle+\mathrm{o}(\triangle),i\neq j\\1+\pi_{ii}\triangle+\mathrm{o}(\triangle),i=j\end{cases} \tag{4}$$

where $\triangle>0$, $\lim\limits_{\triangle\to 0}\frac{o(\triangle)}{\triangle}=0$ and for $i\neq j,\ \forall\ i,\ j\in s,\ \pi_{ij}\geqslant 0,\ \pi_{ii}=-\sum\limits_{j=1,i\neq j}^{S}\pi_{ij}$.

Consider the transition probability $\prod=\hat{\prod}+\Delta\prod=\left(\hat{\prod}_{ij}\right)+(\Delta\pi_{ij})$, while $|\Delta\pi_{ij}|\leqslant\delta_{ij},\ \delta_{ij}\geqslant 0\,j\neq i,\ \delta_{ij}$ is known constant $\hat{\prod}=(\hat{\pi}_{ij})$ is known constant matrix. Introducing a new parameter $\lambda_{ij},\ \lambda_{ij}=\hat{\pi}_{ij}-\delta_{ij},\ \forall\ j\neq i$, obvious $\delta_{ii}=-\sum\limits_{j=1,j\neq i}^{S}\delta_{ij}$ and $\lambda_{ii}=-\sum\limits_{j=1,j\neq i}^{S}\lambda_{ij}$.

Let $A(r_t)=A_i,\ A_d(r_t)=A_{di},\ B(r_t)=B_i,\ F_a(r_t)=F_{ai},\ H(r_t)=H_i,\ \Delta A(r_t)=\Delta A_i,\ \Delta A_d(r_t)=\Delta A_{di},\ C(r_t)=C_i,\ C_x(r_t)=C_{xi},\ D_{\varpi}(r_t)=D_{\varpi i}$.

The (1) rewritten as follows

$$\begin{cases}\dot{x}(t)=(A_i+\Delta A_i)\,x(t)+(A_{di}+\Delta A_{di})\,x(t-\Gamma_i(t))+B_i\,(u(t)+f_i(x(t),t))\\\qquad +F_{ai}f_a(t)+\ H_i\varpi(t),\\y(t)=C_ix(t),\\z(t)=C_{xi}x(t)+D_{\varpi i}\varpi(t),\\x(\theta)=\varphi(\theta),\theta\in[-\Gamma_2,0]\end{cases} \tag{5}$$

Assumption 1. ([41]) $\Gamma_{1i}(t)\leqslant\Gamma_i(t)\leqslant\Gamma_{2i}(t)$, $\dot{\Gamma}_i(t)\leqslant\mu_i$, where $\mu_i\,(i=1,2)$ are the constants and $\Gamma_2=\max\Gamma_{2i}(t)$ for $\forall\ i\in S$.

Definition 1. ([42])(Lipschitz condition) The non-linear function $\varphi(\chi)$ satisfies the Lipschitz condition with Lipschitz constant σ_{0i} as

$$\|\varphi(\chi)-\varphi(\hat{\chi})\|\leqslant\sigma_{0i}\,\|\chi-\hat{\chi}\|\,. \tag{6}$$

Definition 2. ([16])(OSL condition) The non-linear function $F_i(\chi(t),t)$ is OSL, if exists constant $\sigma_{1i}\in\Re$, so that

$$\langle F_i(\chi(t),t)-F_i(\hat{\chi}(t),t),\chi(t)-\hat{\chi}(t)\rangle\leqslant\sigma_{1i}\|\chi(t)-\hat{\chi}(t)\|^2 \tag{7}$$

for $i\in S,\ \chi(t),\ \hat{\chi}(t)\in\Re^n$.

Definition 3. ([16])(quadratically inner bounded) The non-linear function $F_i(\chi(t),t)$ is quadratically inner bounded, if exists constant $\sigma_{2i},\ \sigma_{3i}\in\Re$, so that

$$\begin{aligned}\langle F_i(\chi(t),t)-F_i(\hat{\chi}(t),t),F_i(\chi(t),t)-F_i(\hat{\chi}(t),t)\rangle\leqslant\sigma_{2i}\|\chi(t)-\hat{\chi}(t)\|^2\\+\sigma_{3i}\,\langle\chi(t)-\hat{\chi}(t),F_i(\chi(t),t)-F_i(\hat{\chi}(t),t)\rangle\end{aligned} \tag{8}$$

for $i\in S,\ \chi(t),\ \hat{\chi}(t)\in\Re^n$.

Remark 1. The constants σ_{1i}, σ_{2i} and σ_{3i} in one-sided Lipschitz can be negative, positive or zero, but the constants of traditional Lipschitz must be positive. This is an advantage over the traditional Lipschitz function.

Remark 2. Generally speaking, the OSL represent a extensive family on practical non-linear possess and we can see that the OSL condition contion contains the transitional Lipschitz in Fig.1, which has great advantages over Lipschitz in conservativeness. Many non-linear forms satisfy the OSL continuity, but they do not satisfy the Lipschitz continuity.

Assumption 2. $f_a(t)\in\Re^{n_a}$ is a constant vector, $f_{ar}(t)$ is the r-th element of $f_a(t)$, and $f_{ar}(t)$ represents the r-th actuator failure of $f_a(t)$. And satisfy the following inequality

$$\left\|e^T(t)P_iF_{ai}\right\|-e^T(t)P_iF_{ai}\hat{f}_a(t)\leqslant 0$$

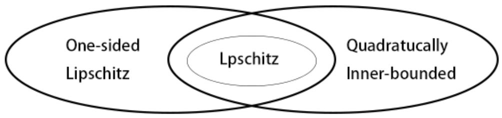

Fig 1 OSL, quadratically inner-bounded and Lipschitz function sets

where $\hat{f}_a(t) = \frac{\left\| e^T(t) P_i F_{ai} \right\|}{e^T(t) P_i F_{ai}} f_{Ma}(t)$, $f_{Ma}(t) = max\{f_a(t)\}$.

Lemma 1. ([43]) Matrices D, E with appropriate dimensions and for any $\varepsilon > 0$, $0 < P \in \Re^{n\times n}$, $\Gamma^T(t)\Gamma(t) \leqslant I$, the following non-equality holds

$$Q + D\Gamma(t)E + E^T\Gamma^T(t)D^T \leqslant Q + \varepsilon^{-1}D^TD + \varepsilon E^TE \tag{9}$$

$$\pm 2D^TE \leqslant D^TPD + E^TP^{-1}E. \tag{10}$$

Lemma 2. ([44]) For any symmetric $\mathfrak{M} > 0$, scalars $\partial < \beta$, vector function $x : [\partial, \beta] \to \Re^n$ the following non-equality holds

$$(\beta - \partial)\int_{\partial}^{\beta} x^T(\varrho)\mathfrak{M}x(\varrho)\,d\varrho \geqslant \left[\int_{\partial}^{\beta} x(\varrho)\,d\varrho\right]^T \mathfrak{M}\left[\int_{\partial}^{\beta} x(\varrho)\,d\varrho\right] \tag{11}$$

Definition 4. ([39]) Arbitrarily $\mathrm{T_p} \geqslant 0$ and any non-zero $\varpi(t) \in \mathfrak{L}_2[0,\infty)$, $\gamma_1\,(satisfies, \gamma_1 > 0)$ is a performance level of mixed $H_\infty/$ passive. When the initial value is zero, the following inequalities hold

$$\varepsilon\left\{\int_0^{T_p}\left[-\alpha Z^T(t)Z(t) + 2(1-\alpha)\gamma_1 Z^T(t)\varpi(t)\right]dt\right\} \geqslant -\gamma_1^2\varepsilon\left\{\int_0^{T_P}\left[\varpi^T(t)\varpi(t)\right]dt\right\} \tag{12}$$

Remark 3. Definition 4 contains H_∞ and passivity performance index [39], such as
(1) When index $\alpha = 1$, it is called H_∞ performance index;
(2) When index $\alpha = 0$, it is called passive performance index;
(3) When index $\alpha \in (0,1)$, it is called mixed passive performance index.

3 | MAIN RESULTS

We design a non-fragility obverse to evaluate the states which can not measured, the observer is designed as follow:

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + A_{di}\hat{x}(t - \Gamma_i(t)) + B_i(u(t) + f_i(\hat{x}(t), t)) + F_{ai}\hat{f}_a(t) + (L_i + \Delta L_i(t))(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases} \tag{13}$$

where $\hat{x}(t)$ is the estimation of the state, L_i is observer gains, $\Delta L_i(t)$ is a perturbed matrix and $\max\{\|\Delta L_i(t)\|\} \leqslant \eta$, $f_i(\hat{x}(t), t)$ is nonlinear estimation, $\hat{f}_a(t)$ is the observed value of actuator failure, $\hat{y}(t)$ is the measured output observation value.

Let $e(t) = x(t) - \hat{x}(t)$, $e(t)$ represents the estimation error.

According (1) and (13) we have

$$\left\{\begin{aligned}&\dot{e}(t) = (A_i - L_iC_i - \Delta L_i(t)C_i)e(t) + A_{di}e(t-\Gamma_i(t)) + \Delta A_i x(t) + \Delta Ad_i x(t-\Gamma_i(t))\\ &\qquad + B_i(f_i(x(t)) - f_i(\hat{x}(t))) + F_{ai}\left(f_a(t) - \hat{f}_a(t)\right) + H_i\varpi(t),\\ &e_y(t) = C_ie(t)\end{aligned}\right. \tag{14}$$

where $e_y(t)$ represents the measurement output error.

3.1 | Integral-type SMC design

A simple integral sliding surface is designed as follows:

$$S(t) = G_i(\hat{x}(t) - \hat{x}(0)) - \int_0^t G_i(A_i + B_iK_i)\hat{x}(t)\,ds \tag{15}$$

where K_i is a controller gains, $\hat{x}(0)$ is the initial value of the observer, when $S(t) = 0$ and $\dot{S}(t) = 0$, which means that the trajectory of the system can reach the sliding surface, thus an equivalent controller can be obtained.

$$U_{eq}(t) = K_i\hat{x}(t) - (G_iB_i)^{-1}G_i\left[A_{di}\hat{x}(t-\Gamma_i(t)) + (L_i + \Delta L_i(t))C_ie(t)\right] - F_if_a(t) - f_i(\hat{x}(t))$$

Take $U_{eq}(t)$ into (13) we get the following equation

$$\dot{\hat{x}}(t) = (A_i + B_iK_i)\hat{x}(t) + B_{Gi}A_{di}\hat{x}(t-\Gamma_i(t)) + B_{Gi}(L_i + \Delta L_i(t))(y(t) - \hat{y}(t)) \tag{16}$$

where $B_{Gi} = I - B_i(G_iB_i)^{-1}G_i$.

A new dynamic equation is obtained.

$$\left\{\begin{aligned}&\dot{\hat{x}}(t) = (A_i + B_iK_i)\hat{x}(t) + B_{Gi}A_{di}\hat{x}(t-\Gamma_i(t)) + B_{Gi}(L_i + \Delta L_i(t))(y(t) - \hat{y}(t))\\ &\dot{e}(t) = (A_i - L_iC_i - \Delta L_i(t)C_i)e(t) + A_{di}e(t-\Gamma_i(t)) + \Delta A_i x(t) + \Delta Ad_i x(t-\Gamma_i(t))\\ &\qquad + B_i(f_i(x(t)) - f_i(\hat{x}(t))) + F_{ai}\left(f_a(t) - \hat{f}_a(t)\right)\end{aligned}\right. \tag{17}$$

3.2 | Stability analysis

Theorem 1. For constants $\gamma > 0$, $\alpha > 0$, $0 < \Gamma_1 < \Gamma_2$ and $0 < \mu_i$, $(i = 1, 2)$, the closed-loop systems (17) are stochastically stable, if symmetric matrices $P_i > 0$ and $Q_\upsilon > 0$, $\upsilon = (1, 2, 3, 4, 5)$, W_i, S_i and exists positive scalars ε_{1i}, ε_{2i}, ε_{3i}, $\forall\, i \in S$ such that the following inequalities hold

$$\Omega_i = \begin{bmatrix}\Omega_{1i} & \Omega_{2i}\\ * & \Omega_{3i}\end{bmatrix} < 0 \tag{18}$$

$$P_j - P_i - W_i < 0,\ j \neq i \tag{19}$$

$$\begin{bmatrix}P_i - vI & G_i{}^T\\ * & G_iB_i\end{bmatrix} < 0 \tag{20}$$

where

$$\Omega_{1i} = \begin{bmatrix}\Omega_{11i} & 0 & 0 & P_iA_{di} & \Omega_{15i} & 0\\ * & \Omega_{22i} & 0 & 0 & 0 & 0\\ * & * & \Omega_{33i} & 0 & 0 & 0\\ * & * & * & \Omega_{44i} & 0 & 0\\ * & * & * & * & \Omega_{55i} & P_iA_{di}\\ * & * & * & * & * & \Omega_{66i}\end{bmatrix}, \Omega_{2i} = \begin{bmatrix}0 & \Omega_{1,8} & \Omega_{1,9} & \Omega_{1,10}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \Omega_{5,9} & \Omega_{5,10}\\ 0 & 0 & 0 & 0\end{bmatrix}, \Omega_{3i} = \begin{bmatrix}\Omega_{7,7} & 0 & 0 & 0\\ * & \Omega_{8,8} & \Omega_{8,9} & 0\\ * & * & \Omega_{9,9} & 0\\ * & * & * & \Omega_{10,10}\end{bmatrix}$$

$$\Omega_{1,1i} = P_i\left(A_i + B_iK_i\right) + \left(A_i + B_iK_i\right)^T {P_i}^T + 2{G_i}^T(G_iB_i)^{-1}G_i + Q_1 + Q_2 + Q_3 + \gamma\left(\Gamma_2 - \Gamma_1\right)Q_3$$
$$+ \varepsilon_{2i}{N_i}^T N_i + \rho_{2i}\sigma_{1i} + \rho_{4i}\sigma_{2i} + 0.25{\delta_{ii}}^2 S_i + \alpha {C_{xi}}^T C_{xi} + \sum_{j\neq i}^{S}\lambda_{ij}\left(P_j - P_i\right) - \delta_{ii}W_i + \eta^2 I + L_i{L_i}^T$$
$$\Omega_{5,5i} = P_i\left(A_i - L_iC_i\right) + \left(A_i - L_iC_i\right)^T {P_i}^T + {\varepsilon_{1i}}^{-1}P_i{P_i}^T + 2v^2{C_i}^2C_i + \varepsilon_{1i}\eta^2{C_i}^TC_i + 2{\varepsilon_{2i}}^{-1}P_iM_i{M_i}^TP_i$$
$$+ \varepsilon_{2i}{N_i}^TN_i + 2{\varepsilon_{3i}}^{-1}P_iM_i{M_i}^TP_i + Q_4 + \gamma\left(\Gamma_2 - \Gamma_1\right)Q_4 + {\Gamma_2}^2Q_5 + \frac{{\Gamma_2}^2 - {\Gamma_1}^2}{2}\Gamma_2\gamma Q_5 + P_i{P_i}^T$$
$$+ \rho_{1i}\sigma_{1i} + \rho_{3i}\sigma_{2i} + \sum_{j\neq i}^{S}\lambda_{ij}\left(P_j - P_i\right) - \delta_{ij}W_i + 0.25{\delta_{ii}}^2S_i$$

$\Omega_{2,2i} = -Q_1$, $\Omega_{3,3i} = -Q_2$, $\Omega_{4,4i} = -\left(1-\mu_i\right)Q_3 + {A_{di}}^TP_iA_{di} + \varepsilon_{3i}{N_{di}}^TN_{di}$
$\Omega_{6,6i} = \varepsilon_{ij}{N_{di}}^TN_{di} - \left(1-\mu_i\right)Q_4 + {A_{di}}^TA_{di}$, $\Omega_{1,5i} = P_i\left(L_i + \Delta L_i\left(t\right)\right)C_i$, $\Omega_{1,8i} = -\frac{\rho_{2i}}{2} + \frac{\rho_{4i}\sigma_{3i}}{2}$
$\Omega_{1,9i} = \frac{\rho_{2i}}{2} - \frac{\sigma_{3i}\rho_{4i}}{2}$, $\Omega_{1,10i} = \alpha{C_{xi}}^TD_{\varpi i} - \left(1-\alpha\right)\gamma_1{C_{xi}}^T$, $\Omega_{5,9i} = \frac{\rho_{3i}\sigma_{2i}}{2} - \frac{\rho_{1i}}{2}$, $\Omega_{5,10i} = P_iH_i$
$\Omega_{7,7i} = (\mu_i - 1)Q_5$, $\Omega_{8,8i} = -\rho_{4i}I$ $\Omega_{8,9i} = \rho_{4i}I$, $\Omega_{9,9i} = {B_i}^TB_i - \rho_{3i}I - \rho_{4i}I$, $\gamma = \max\left\{-\pi_{ii}\right\}$
$\Omega_{10,10i} = -\gamma_1^2I + \alpha{D_{\varpi i}}^TD_{\varpi i} - 2\left(1-\alpha\right)\gamma_1{D_{\varpi i}}^T$, $\tilde{f}_i\left(x\left(t\right)\right) = f_i\left(x\left(t\right)\right) - f_i\left(\hat{x}\left(t\right)\right)$, $\Gamma_1 = \min\Gamma_{1i}\left(t\right)$
Then the closed-loop system (17) are stochastically stable.

Proof: First, we choose a new mode-dependent Lyaunov function as follows:

$$V\left(\hat{x}, e, i\right) = V_1 + V_2$$

$$V_1 = \hat{x}^T\left(t\right)P_i\hat{x}\left(t\right) + \int\limits_{t-\Gamma_1}^{t}\hat{x}^T\left(s\right)Q_1\hat{x}\left(s\right)ds + \int\limits_{t-\Gamma_2}^{t}\hat{x}^T\left(s\right)Q_2\hat{x}\left(s\right)ds$$
$$+ \int\limits_{t-\Gamma_i(t)}^{t}\hat{x}^T\left(s\right)Q_3\hat{x}\left(s\right)ds + \gamma\int\limits_{-\Gamma_2}^{-\Gamma_1}\int\limits_{t+\theta}^{t}\hat{x}^T\left(s\right)Q_3\hat{x}\left(s\right)dsd\theta$$

$$V_2 = e^T\left(t\right)P_ie\left(t\right) + \int\limits_{t-\Gamma_i(t)}^{t}e^T\left(s\right)Q_4e\left(s\right)ds + \int\limits_{-\Gamma_2}^{-\Gamma_1}\int\limits_{t+\theta}^{t}e^T\left(s\right)Q_4e\left(s\right)dsd\theta$$
$$+ \Gamma_2\int\limits_{-\Gamma_i(t)}^{0}\int\limits_{t+\theta}^{t}e^T\left(s\right)Q_5e\left(s\right)dsd\theta + \Gamma_2\gamma\int\limits_{-\Gamma_2}^{-\Gamma_1}\int\limits_{\theta}^{0}\int\limits_{t+\lambda}^{t}e^T\left(s\right)Q_5e\left(s\right)dsd\lambda d\theta$$

From the above, we get

$$\begin{aligned}\ell V_1 \leqslant{}& 2\hat{x}^T\left(t\right)P_i\left[\left(A_i + B_iK_i\right)\hat{x}\left(t\right) + B_{Gi}A_{di}\hat{x}\left(t - \Gamma_i\left(t\right)\right) + B_{Gi}\left(L_i + \Delta L_i\left(t\right)\right)C_ie\left(t\right)\right]\\ &+ \hat{x}^T\left(t\right)\sum_{j=1}^{s}\pi_{ij}P_j\hat{x}\left(t\right) + \hat{x}^T\left(t\right)Q_1\hat{x}\left(t\right) - \hat{x}^T\left(t-\Gamma_1\right)Q_1\hat{x}\left(t-\Gamma_1\right) + \hat{x}^T\left(t\right)Q_2\hat{x}\left(t\right)\\ &- \hat{x}^T\left(t-\Gamma_2\right)Q_1\hat{x}\left(t-\Gamma_2\right) + \hat{x}^T\left(t\right)Q_3\hat{x}\left(t\right) - \left(1-\mu_i\right)\hat{x}^T\left(t-\Gamma_i\left(t\right)\right)Q_3\hat{x}\left(t-\Gamma_i\left(t\right)\right)\\ &+ \gamma\left(\Gamma_2 - \Gamma_1\right)\hat{x}^T\left(t\right)Q_3\hat{x}\left(t\right)\end{aligned} \tag{21}$$

Similarly, we have

$$\begin{aligned}
\ell V_2 \leqslant\ & 2e^T(t)\,P_i[(A_i - L_iC_i - \Delta L_i(t)\,C_i)\,e(t) + A_{di}e(t-\Gamma_i(t)) + \Delta A_i x(t) + \Delta Ad_i x(t-\Gamma_i(t)) \\
& + B_i\,(f_i(x(t)) - f_i(\hat{x}(t))) + F_{ai}\left(f_a(t) - \hat{f}_a(t)\right) + H_i\varpi(t)] + e^T(t)\sum_{j=1}^{s}\pi_{ij}P_je(t) \\
& + e^T(t)\,Q_4e(t) - (1-\mu_i(t))\,e^T(t-\Gamma_i(t))\,Q_4e(t-\Gamma_i(t)) + \gamma\,(\Gamma_2(t) - \Gamma_1(t))\,e^T(t)\,Q_4e(t) \\
& + \Gamma_2(t)\,(\mu_i(t)-1)\int_{t-\Gamma_i(t)}^{t} e^T(s)\,Q_5e(s)ds + \Gamma_2(t)\,\Gamma_2(t)\,e^T(t)\,Q_5e(t) \\
& + \Gamma_2(t)\,\gamma\frac{\Gamma^2{}_2(t) - \Gamma^2{}_1(t)}{2}e^T(t)\,Q_5e(t)
\end{aligned} \tag{22}$$

Noting that $\pi_{ij}\geqslant 0$ $(i \neq j)$ and $\pi_{ii}\leqslant 0$ we have

$$\begin{aligned}
& \sum_{j=1}^{s}\pi_{ij}\int_{t-\Gamma_j(t)}^{t}\hat{x}^T(t)\,Q_3\hat{x}(t)ds \\
& = \sum_{j\neq i}^{s}\pi_{ij}\int_{t-\Gamma_j(t)}^{t}\hat{x}^T(s)\,Q_3\hat{x}(s)ds + \pi_{ii}\int_{t-\Gamma_i(t)}^{t}\hat{x}^T(s)\,Q_3\hat{x}(s)ds \\
& \leqslant -\pi_{ii}\int_{t-\Gamma_2}^{t}\hat{x}^T(s)\,Q_3\hat{x}(s)ds + \pi_{ii}\int_{t-\Gamma_1}^{t}\hat{x}^T(s)\,Q_3\hat{x}(s)ds \\
& = \gamma\int_{t-\Gamma_2}^{t-\Gamma_1}\hat{x}^T(s)\,Q_3\hat{x}(s)ds
\end{aligned} \tag{23}$$

$$\begin{aligned}
& \sum_{j=1}^{S}\pi_{ij}\int_{-\Gamma_j(t)}^{0}\int_{t+\theta}^{t} e^T(s)\,Q_5e(s)dsd\theta \\
& \leqslant -\pi_{ii}\int_{-\Gamma_2}^{0}\int_{t+\theta}^{t} e^T(s)\,Q_5e(s)dsd\theta + \pi_{ii}\int_{-\Gamma_1}^{0}\int_{t+\theta}^{t} e^T(s)\,Q_5e(s)dsd\theta \\
& = \gamma\int_{-\Gamma_2}^{-\Gamma_1}\int_{t+\theta}^{t} e^T(s)\,Q_5e(s)dsd\theta
\end{aligned} \tag{24}$$

For $\sum\limits_{j=1}^{s}\pi_{ij}P_j$ in (21), (22), we have

$$\begin{aligned}
\sum_{j=1}^{s}\pi_{ij}P_j &= \sum_{j=1}^{s}(\hat{\pi}_{ij} + \Delta\pi_{ij})P_j - \sum_{j=1}^{s}(\Delta\pi_{ij} + \delta_{ij})W_i \\
&= \sum_{j\neq i}^{s}\lambda_{ij}\,(P_j - P_i) - \Delta\pi_{ii}W_i - \delta_{ii}W_i + \sum_{j\neq i}^{s}(\Delta\pi_{ij} + \delta_{ij})\,(P_j - P_i - W_i)
\end{aligned} \tag{25}$$

where $W_i = W_i{}^T$ according to (10) $\forall$ $S_i > 0$ we have

$$-\Delta\pi_{ii}W_i \leqslant 0.25\delta_{ii}{}^2S_i + W_iS_i{}^{-1}W_i \tag{26}$$

the upper formula can be rewritten as follows:

$$\sum_{j=1}^{s}\pi_{ij}P_j \leqslant \sum_{j\neq i}^{s}\lambda_{ij}\,(P_j - P_i) + 0.25\delta_{ii}{}^2S_i - \delta_{ii}W_i + W_iS_i{}^{-1}W{+}_i\sum_{j\neq i}^{s}(\Delta\pi_{ij} + \delta_{ij})\,(P_j - P_i - W_i) \tag{27}$$

According Lemma1, we have

$$\begin{aligned}2\hat{x}^T(t)\,P_iB_{Gi}A_{di}\hat{x}(t-\Gamma_i(t)) \leqslant{}& 2\hat{x}^T(t)\,P_iA_{di}\hat{x}(t-\Gamma_i(t))+\hat{x}^T(t)\,{G_i}^T(G_iB_i)^{-1}G_i\hat{x}(t)\\&+\hat{x}^T(t-\Gamma_i(t))\,{A_{di}}^TP_iA_{di}\hat{x}(t-\Gamma_i(t))\end{aligned} \tag{28}$$

$$2\hat{x}^T(t)\,P_iB_{Gi}(L_i+\Delta L(t))\,C_ie(t) \leqslant \hat{x}^T(t)\,L_i{L_i}^T\hat{x}(t)+\eta^2\hat{x}^T(t)\,\hat{x}(t)+2v^2e^T(t)\,{C_i}^TC_ie(t) \tag{29}$$

$$-2e^T(t)\,P_i\Delta L(t)\,C_ie(t) \leqslant {\varepsilon_{1i}}^{-1}e^T(t)\,P_i{P_i}^Te(t)+\varepsilon_{1i}\eta^2e^T(t)\,{C_i}^TC_ie(t) \tag{30}$$

$$\begin{aligned}2e^T(t)\,P_i\Delta A_ix(t) &= 2e^T(t)\,P_i\Delta A_i(\hat{x}(t)+e(t))\\&\leqslant 2{\varepsilon_{2i}}^{-1}e^T(t)\,P_iM_i{M_i}^TP_ie(t)+\varepsilon_{2i}\hat{x}^T(t)\,{N_i}^TN_i\hat{x}(t)+\varepsilon_{2i}e^T(t)\,N_i{N_i}^Te(t)\end{aligned} \tag{31}$$

$$\begin{aligned}2e^T(t)\,P_i\Delta A_{di}x(t-\Gamma_i(t)) &= 2e^T(t)\,P_i\Delta A_{di}(\hat{x}(t-\Gamma_i(t))+e(t-\Gamma_i(t)))\\&\leqslant 2{\varepsilon_{3i}}^{-1}e^T(t)\,P_iM_i{M_i}^TP_ie(t)+\varepsilon_{3i}\hat{x}(t-\Gamma_i(t))\,N_{di}^TN_{di}\hat{x}(t-\Gamma_i(t))\\&\quad+\varepsilon_{3i}e^T(t-\Gamma_i(t))\,N_{di}^TN_{di}e(t-\Gamma_i(t))\end{aligned} \tag{32}$$

$$2e^T(t)\,P_iB_i(f_i(x(t))-f_i(\hat{x}(t))) \leqslant e^T(t)\,P_i{P_i}^Te(t)+(f_i(x(t))-f_i(\hat{x}(t)))^T{B_i}^TB_i(f_i(x(t))-f_i(\hat{x}(t))) \tag{33}$$

from Definition 2, 3 we have

$$\rho_{1i}\sigma_{1i}e^T(t)\,e(t)-\rho_{1i}\tilde{f}(x(t),t)\,e(t) \geqslant 0 \tag{34}$$

$$\rho_{2i}\sigma_{1i}\hat{x}^T(t)\,\hat{x}(t)+\rho_{2i}\tilde{f}(x(t),t)\,\hat{x}(t)-\rho_{2i}f(x(t),t)\,\hat{x}(t) \geqslant 0 \tag{35}$$

from Definition 3 we have

$$\rho_{3i}\sigma_{2i}e^T(t)\,e(t)+\rho_{3i}e^T(t)\,\tilde{f}(x(t),t)-\rho_{3i}\tilde{f}^T(x(t),t)\,\tilde{f}(x(t),t) \geqslant 0 \tag{36}$$

$$\begin{aligned}&\rho_{4i}\sigma_{2i}\hat{x}^T(t)\,\hat{x}(t)-\rho_{4i}\sigma_{3i}\hat{x}^T(t)\,\tilde{f}(x(t),t)-\rho_{4i}\tilde{f}^T(x(t),t)\,\tilde{f}(x(t),t)\\&+\rho_{4i}\sigma_{3i}\hat{x}^T(t)\,f(x(t),t)+2\rho_{4i}\tilde{f}(x(t),t)\,f(x(t),t)-\rho_{4i}f^T(x(t),t)\,f(x(t),t) \geqslant 0\end{aligned} \tag{37}$$

we definite

$$J(t)=\varepsilon\left\{\int_0^{T_p}\left[\alpha Z^T(t)\,Z(t)-2(1-\alpha)\gamma_1Z^T(t)\,\varpi(t)-\gamma_1^2\varpi^T(t)\,\varpi(t)\right]dt\right\} \tag{38}$$

Lastly, bring (21-37) into (38), it is obvious that

$$\begin{aligned}J(t)&=\varepsilon\left\{\int_0^{T_p}\left[\alpha Z^T(t)\,Z(t)-2(1-\alpha)\gamma_1Z^T(t)\,\varpi(t)-\gamma_1^2\varpi^T(t)\,\varpi(t)+\ell V\right]dt\right\}\\&=\varepsilon\left\{\int_0^{T_p}\left[\alpha x^T(t)\,{C_{xi}}^T(t)\,C_{xi}(t)\,x(t)+2\alpha x^T(t)\,{C_{xi}}^T(t)\,D_{\varpi i}\varpi(t)+\varpi^T(t)\,{D_{\varpi i}}^TD_{\varpi i}\varpi(t)\right.\right.\\&\quad\left.-2(1-\alpha)\gamma_1x^T(t)\,{C_{xi}}^T(t)\,\varpi(t)-2(1-\alpha)\gamma_1\varpi^T(t)\,{D_{\varpi i}}^T\varpi(t)-\gamma_1^2\varpi^T(t)\,\varpi(t)+\ell V\right]dt\}\\&=\varepsilon\left\{\int_0^{T_p}{\xi_1}^T\Omega_i\xi_1dt\right\}\end{aligned}$$

where

$$\zeta_1=\left[\hat{x}^T(t)\;\;\hat{x}^T(t-\Gamma_1)\;\;\hat{x}^T(t-\Gamma_2)\;\;\hat{x}^T(t-\Gamma_i)\;\;e^T(t)\right]$$
$$\zeta_2=\left[e^T(t-\Gamma_i)\;\;\int_{t-\Gamma_i}^te^T(s)\,ds\;\;f_i(x(t))\;\;\tilde{f}_i(x(t))\;\;\varpi^T(t)\right],\xi_1=\left[\zeta_1\;\;\zeta_2\right]$$

From the above formula, we can get $J(t)\leqslant 0$ for any given $t>0$, the closed-loop systems (17) is mixed H_∞ and passive, The proof is completed. It is particularly noteworthy that when $\varpi(t)=0$, the closed-loop system is also stable. In the (29) formula, we assume $P_i+(G_iB_i)^{-1}G_i<vI$, as can be seem form the Schur lemma, it is equivalent to (20).

Remark 4. In ℓV, the integral term $\sum_{j=1}^{s} \pi_{ij} \int_{t-\Gamma_j(t)}^{t} \hat{x}^T(t) Q_3 \hat{x}(t) ds$, $\sum_{j=1}^{s} \pi_{ij} \int_{t-\Gamma_j(t)}^{t} e^T(t) Q_4(t) ds$, $\sum_{j=1}^{S} \pi_{ij} \int_{-\Gamma_j(t)}^{0} \int_{t+\theta}^{t} e^T(s) Q_5 e(s) ds d\theta$ are mode dependent, and using (23-24) can make it offset.

Theorem 2. For constants $\gamma > 0$, $\alpha > 0$, $0 < \Gamma_1 < \Gamma_2$ and $0 < \mu_i < 1$, $(i = 1, 2)$, the systemd (17) are stochastically stable, if symmetric matrices $P_i > 0$, $Q_v > 0$, $\upsilon = (1, 2, 3, 4, 5)$, W_i, S_i and exists positive scalars ε_{1i}, ε_{2i}, ε_{3i}, $\forall\, i \in S$ so that the following LMI hold.

$$\mho_i = \begin{bmatrix} \mho_{1i} & \mho_{2i} \\ \mho_{3i} & \mho_{4i} \end{bmatrix} \leqslant 0 \tag{39}$$

$$P_j - P_i - W_i < 0,\ j \neq i \tag{40}$$

$$\begin{bmatrix} P_i - vI & {G_i}^T \\ * & G_i B_i \end{bmatrix} < 0 \tag{41}$$

where

$$\mho_{1i} = \begin{bmatrix} \mho_{1,1i} & 0 & 0 & A_{di}{X_i}^T & 0 & 0 & 0 & 0 & 0 & \mho_{1,10i} \\ * & -R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \mho_{4,4i} & A_{di}{X_i}^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \mho_{5,5i} & 0 & 0 & 0 & 0 & H_i \\ * & * & * & * & * & \mho_{6,6i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \mho_{7,7i} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \mho_{8,8i} & 0 & 0 \\ * & * & * & * & * & * & * & * & \mho_{9,9i} & 0 \\ * & * & * & * & * & * & * & * & * & \mho_{10,10i} \end{bmatrix}$$

$$\mho_{6i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \psi_{1i} & \psi_{2i} & 0 & 0 & 0 \\ 0 & 0 & \psi_{3i} & \psi_{4i} & \psi_{5i} \end{bmatrix}, \mho_{7i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \psi_{6i} & \psi_{7i} & \psi_{8i} & 0 & 0 \end{bmatrix}, \mho_{8i} = \begin{bmatrix} 0 & 0 & 0 & \psi_{9i} & \psi_{10i} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mho_{4i} = \begin{bmatrix} \mho_{6i} & \mho_{7i} \\ 0 & \mho_{8i} \end{bmatrix}, \mho_{5i} = -diag\left\{ X_i \ X_i \ \varepsilon_{1i} \ I \ \rho_{1i}\delta_{1i} \ \rho_{3i}\delta_{2i} \ I \ \Xi \ I \ I \right\}$$

$$\mho_{1,1i} = A_i {X_i}^T + B_i T_i + X_i {A_i}^T + {T_i}^T {B_i}^T + R_1 + R_2 + R_3 + \gamma(\Gamma_2 - \Gamma_1) R_3 + 0.25{\delta_{ii}}^2 \bar{S}_i - \delta_{ii}\bar{W}_i$$

$$\begin{aligned} \mho_{5,5i} = {} & A_i {X_i}^T + X_i {A_i}^T + R_4 + {\Gamma_2}^2 R_5 + \gamma(\Gamma_2 - \Gamma_1) R_4 + \gamma\Gamma_2 \frac{{\Gamma_2}^2 - {\Gamma_1}^2}{2} R_5 - \delta_{ij} W_i + 0.25{\delta_{ii}}^2 S_i \\ & + I + 2{\varepsilon_{2i}}^{-1} M_i {M_i}^T + 2{\varepsilon_{3i}}^{-1} M_i {M_i}^T + {\varepsilon_{1i}}^{-1} I + 2\tilde{R} \end{aligned}$$

$\mho_{6,6i} = -(1-\mu_i) R_4$, $\mho_{1,4i} = A_{di}{X_i}^T$, $\mho_{7,7i} = -(1-\mu_i) R_5$, $\mho_{8,8i} = -\rho_{4i} I$, $\mho_{1,8i} = \frac{X_i(\rho_{4i}\delta_{3i} - \rho_{2i})}{2}$, $\mho_{5,10i} = H_i$, $\mho_{9,9i} = {B_i}^T B_i - \rho_{4i} I - \rho_{3i} I$, $\mho_{1,9i} = \frac{X_i(-\rho_{4i}\delta_{3i} + \rho_{2i})}{2}$, $\mho_{5,9i} = \frac{(\rho_{3i}\delta_{3i} - \rho_{1i})}{2} I$, $\mho_{8,9i} = \rho_{4i} I$, $\mho_{10,10i} = -\gamma_1^2 - 2(1-\alpha)\gamma_1 D_{\varpi i}$, $\mho_{1,10i} = \alpha X_i \left({C_{xi}}^T D_{\varpi i}\right) - 2(1-\alpha)\gamma_1 X_i {C_{xi}}^T$, $\Theta_i = \left\{ \sqrt{\lambda_{1i}} X_i \ \cdots \ \sqrt{\lambda_{i(i-1)}} X_i \ \sqrt{\lambda_{i(i+1)}} X_i \ \cdots \ \sqrt{\lambda_{is}} X_s \right\}$
$\Xi_i = -diag\left\{ X_1 \ \cdots \ X_{i-1} \ X_{i+1} \ \cdots \ X_s \right\}$, $\psi_1 = X_i {A_{di}}^T$, $\psi_2 = \varepsilon_{3i} X_i {N_{di}}^T$, $\psi_3 = \varepsilon_{1i}\eta X_i {C_i}^T$, $\psi_4 = \varepsilon_{2i} X_i {N_i}^T$
$\psi_5 = \rho_{1i}\delta_{1i} X_i$, $\psi_6 = \rho_{3i}\delta_{2i} X_i$, $\psi_7 = \sqrt{2} v X_i {N_i}^T$, $\psi_8 = \Theta_i$, $\psi_9 = \varepsilon_{3i} X_i {N_{di}}^T$, $\psi_{10} = X_i {N_{di}}^T$, $\xi_2 = \begin{bmatrix} \zeta_1 & \zeta_2 \end{bmatrix}$
$\zeta_1 = \begin{bmatrix} \hat{x}^T(t) & \hat{x}^T(t-\Gamma_1) & \hat{x}^T(t-\Gamma_2) & \hat{x}^T(t-\Gamma_i) & e^T(t) \end{bmatrix}$, $\zeta_2 = \begin{bmatrix} e^T(t-\Gamma_i) & \int_{t-\Gamma_i}^{t} e^T(s) ds & f_i(x(t)) & \tilde{f}_i(x(t)) & \varpi^T(t) \end{bmatrix}$

Proof: Let $X_i = {P_i}^{-1}$, $Y_i = P_i L_i$, $T_i = K_i X_i$, $X_i S_i {X_i}^T = \bar{S}_i$, $X_i Q_1 {X_i}^T = R_1$, $X_i Q_2 {X_i}^T = R_2$, $X_i Q_3 {X_i}^T = R_3$, $X_i Q_4 {X_i}^T = R_4$, $X_i Q_5 {X_i}^T = R_5$, $L_i C_i {X_i}^T = \widetilde{R}_i$, $X_i W_i {X_i}^T = \bar{W}_i$, By theorem 1 the Pre-and post-multiplying (19) by diag $\{X_i, X_i, X_i, X_i, X_i, X_i, X_i, I, I, I\}$ and its transpose, the $\mho_i \leqslant 0$, moreover $K_i = T_i {X_i}^{-1}$, $L_i = {P_i}^{-1} Y_i$. The proof is completed.

Remark 5. The processing of $\sum_{j=1}^{s} \pi_{ij} P_j$ is in the (27) form, where the range of transition probabilities is broader and easier to obtain in practice, and the feasibility solution is easier to derive in LMIs.

3.2.1 | SMC law design

Remark 6. Design a proper SMC law to enable its trajectory to reach the sliding mode surface $S(t) = 0$, the SMC law design as follows.

$$\begin{aligned} u(t) &= u_a(t) + u_b(t) \\ u_a(t) &= K_i \hat{x}(t) - F_i \hat{f}_a(t) - f_i(\hat{x}(t)) \\ u_b(t) &= -\bar{\lambda}_i(t)\, sign(S(t)) \end{aligned} \tag{42}$$

where

$$\bar{\lambda}_i(t) = h + \left\| (G_i B_i)^{-1} G_i A_{di} \right\| \left\| \hat{x}(t - \Gamma_i(t)) \right\| + \left\| (G_i B_i)^{-1} G_i (L_i + \Delta L_i(t)) C_i \right\| \left\| e(t) \right\|$$

Theorem 3. Presume the switching surface is given in (15) with K_i and L_i have been solved in Theorem 2, the state trajectories can reach the sliding mode surface $S(t) = 0$ with SMC (42) in finite time.
Proof: Choose Lyapunov function as

$$V_S = \frac{1}{2} S^T(t) S(t) \tag{43}$$

According to (43), we get

$$\begin{aligned} \ell V_S &= S^T(t) \dot{S}(t) \\ &= S^T(t) \left[G_i \dot{\hat{x}}(t) - G_i (Ai + B_i K_i) \hat{x}(t) \right] \\ &= S^T(t) \left\{ G_i \left[A_{di} \hat{x}(t - \Gamma_i(t)) + B_i (u(t) + f_i(\hat{x}(t))) + B_i F_i \hat{f}_a(t) - B_i K_i \hat{x}(t) \right] \right\} \\ &+ S^T(t) G_i (L_i + \Delta L_i(t)) C_i e(t) \\ &\leqslant \|S(t)\| \|G_i\| \left[\|A_{di} \hat{x}(t - \Gamma_i(t))\| + \|B_i (u(t) + f_i(\hat{x}(t)))\| + \|B_i\| \left\| \left(F_i \hat{f}_a(t) - K_i \hat{x}(t) \right) \right\| \right. \\ &+ \|L_i C_i e(t)\| + \eta \|C_i\| \|e(t)\| \end{aligned}$$

we have

$$\ell V_S \leqslant -h \|S(t)\| = -\sqrt{2} h V^{\frac{1}{2}}(t) \tag{44}$$

where $h > 0$, then, by determining$S(t_0, r_0) = S_0$, we have

$$\varepsilon[V_s(S(t), i, t) | S(t_0, r_0)] \leqslant -\frac{h}{2} t + {V_s}^{\frac{1}{2}}(S_0, i, t_0)$$

It can be known that there exist $t* \leqslant 2{V_s}^{\frac{1}{2}}(S_0, i, t_0)/h$, such that$[V_s(S(t), i, t) | S(t_0, r_0)]$, which implies the state strategies will reach the sliding mode surface in finite time. The proof of theorem 3 is completed.

Remark 7. In this paper, the SMC law includes observer gain L_i and control gain K_i for error system, which is different from other papers, such as [9,10,44]. According to the proof of Theorem 3, the state trajectories of system (1) can reach onto the predefined surface in finite time.

4 | EXAMPLES FOR ENUNCIATIONS

4.1 | example 1

In order to verify the validity of theorem 2, consider the jump of two modes. The parameters described by:
Mode 1

$$A_1 = \begin{bmatrix} -3 & 2 & -0.2 \\ -0.1 & -1 & -4 \\ 2 & 1 & -1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 0.1 & -0.1 & 0.01 \\ -0.2 & -0.1 & 0.1 \\ -0.1 & -0.1 & 0.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix},\ F_{a1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix},\ H_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.1 & 1.2 & -0.1 \end{bmatrix},$$

$$C_{x1} = \begin{bmatrix} 0.1 & -0.01 & 0.01 \end{bmatrix},\ M_1 = \begin{bmatrix} -0.1 \\ 0.3 \\ -0.1 \end{bmatrix},\ N_1 = \begin{bmatrix} 0.1 & 0.01 & -0.1 \end{bmatrix},\ N_{d1} = \begin{bmatrix} -0.1 & -0.1 & 0.1 \end{bmatrix},\ D_{\varpi 1} = 0.2$$

Mode2

$$A_2 = \begin{bmatrix} -3 & -2 & 2.5 \\ -0.1 & -3 & -2 \\ -4 & -1 & -3.5 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -0.1 & -0.1 & 0.1 \\ 0.3 & -0.1 & -0.1 \\ -0.1 & -0.2 & 0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix},\ F_{a2} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix},\ H_2 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.1 & 0.8 & -0.1 \end{bmatrix},$$

$$C_{x2} = \begin{bmatrix} -0.01 & 0.01 & -0.01 \end{bmatrix},\ M_2 = \begin{bmatrix} 0.1 \\ -0.2 \\ 0.3 \end{bmatrix},\ N_2 = \begin{bmatrix} 0.1 & 0.1 & -0.5 \end{bmatrix},\ N_{d2} = \begin{bmatrix} -0.1 & 0.1 & 0.01 \end{bmatrix},\ D_{\varpi 2} = 0.2$$

consider the known part of the transition probability matrix and the transition probability error matrix

$$\hat{\prod} = \begin{bmatrix} -0.6 & 0.6 \\ 0.9 & -0.9 \end{bmatrix},\ \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ 0.1 & -0.1 \end{bmatrix}$$

Next, the selected parameters are given. $\varepsilon_{11} = \varepsilon_{12} = 0.5$, $\varepsilon_{21} = \varepsilon_{22} = 0.1$, $\varepsilon_{31} = \varepsilon_{32} = 0.1$, $a = 0.5$ and $\gamma_1 = 1$, time delay and time delay derivatives are $d_1 = 0.1$, $d_2 = 0.6$, $\mu_1 = 0.01$, $\mu_2 = 0.6$, the Lipschitz constant is $\sigma_{11} = \sigma_{12} = 0.3$, $\sigma_{21} = \sigma_{22} = 0.01$ and $\sigma_{31} = \sigma_{32} = 0.7$. Use the same method as [28], assumption $f_i\left(x\left(t\right)\right) = 0.3\sin\left(x\left(t\right)\right)$, in definition 2,3, we can prove that ${f_i}^T(x(t))x(t) \leqslant 0.3\|x(t)\|^2$, ${f_i}^T(x(t))f_i(x(t)) \leqslant 0.09\|x(t)\|^2$. The remaining parameters are $\eta = 0.5$, $\rho_{11} = \rho_{12} = 0.15$, $\rho_{21} = \rho_{22} = 0.75$, $\rho_{31} = \rho_{32} = 0.2$ and $\rho_{41} = \rho_{42} = 0.7$. Exogenous input $\varpi\left(t\right) = 0.2 \cdot \exp\left(-100t\right)$.
based on the above discussion, solve the linear matrix inequality, and obtain the controller gain and observer gain.

$$K_1 = \begin{bmatrix} 3.6150 & -5.1410 & -7.3831 \end{bmatrix} \qquad K_2 = \begin{bmatrix} -6.9409 & -5.3619 & 8.8466 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -8.3244 \\ 28.4930 \\ -31.5833 \end{bmatrix} \qquad L_2 = \begin{bmatrix} -3.3630 \\ 14.7427 \\ -16.3453 \end{bmatrix}$$

The results are display in Figs.2-7. Fig.2 is the state of closed-loop system, Fig.3 and Fig.4 show the observational state trajectory and error state trajectory of the system, Fig.5 and Fig.6 represent the controller trajectory and sliding surface function, and the switching function is described in Fig.7.

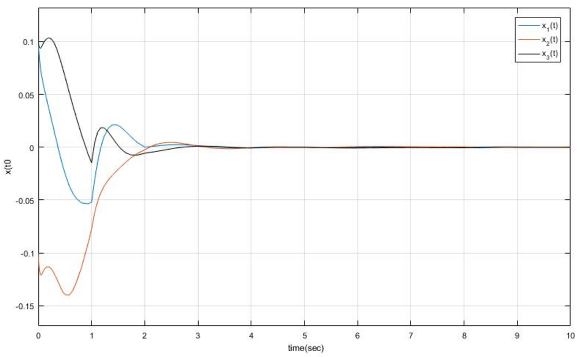

Fig 2 The state response $x(t)$

4.2 | example 2

Consider a RLC from ([39]) circuit where the current in the circuit is $i\left(t\right)$, L, R and C represent inductance, resistance and capacity, $u\left(t\right)$ is the driver, L, R and C voltages and quantities are represent by $u_L\left(t\right), u_R\left(t\right)\, and\, u_C\left(t\right)$. It is assumed that the switch have two places and switches from one place to another in a stochastic method, and is modeled by taking the value of Markov process in finite state space $S = \{1, 2\}$. Let $x_1\left(t\right) = u_C\left(t\right)$ and $x_2\left(t\right) = i_L\left(t\right)$, circuits can be designed as MJS (1) with the parameters as follows:

$$A_i = \begin{bmatrix} 0 & \frac{1}{C_i} \\ -\frac{1}{L_i} & -\frac{R}{L_i} \end{bmatrix} \qquad B_i = \begin{bmatrix} 0 \\ \frac{1}{L_i} \end{bmatrix}$$

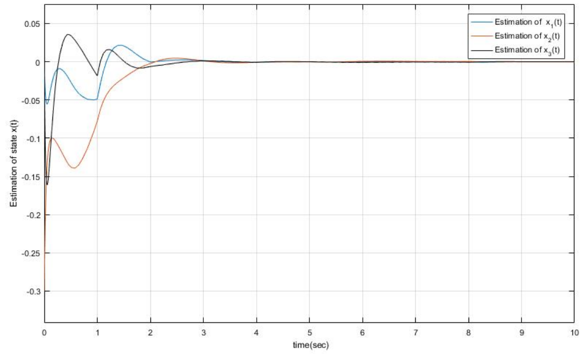

Fig 3 The estimation state $\hat{x}\left(t\right)$

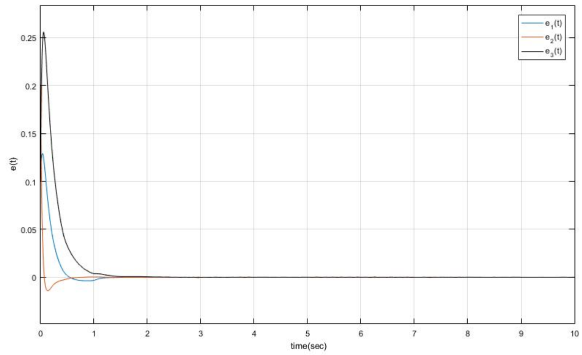

Fig 4 The estimation error $e\left(t\right)$

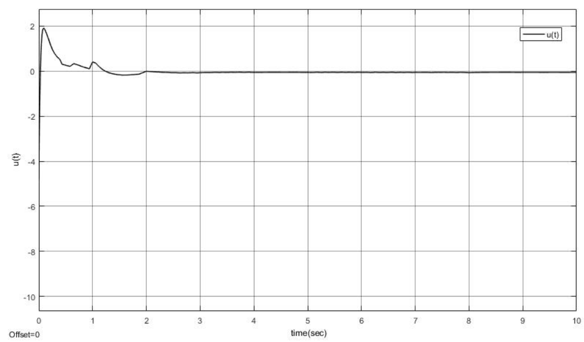

Fig 5 controller trajectory $u\left(t\right)$

where $R = 0.01\Omega$, $C_1 = 0.4F$, $C_2 = 0.6F$, $L_1 = 3H$, and $L_2 = 7H$. Next, some other parameters are given
Mode 1
$A_{d1} = \begin{bmatrix} -0.1 & 0.08 \\ -0.2 & -0.3 \end{bmatrix}$, $F_{a1} = \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix}$, $H_1 = \begin{bmatrix} -0.1 \\ 0.8 \end{bmatrix}$, $C_1 = \begin{bmatrix} -0.2 & -0.5 \end{bmatrix}$, $C_{x1} = \begin{bmatrix} 0.5 & -1 \end{bmatrix}$, $M_1 = \begin{bmatrix} -0.1 \\ -0.8 \end{bmatrix}$
$N_1 = \begin{bmatrix} 0.03 & 0.1 \end{bmatrix}$, $N_{d1} = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}$, $D_{\varpi 1} = 0.01$
Mode 2
$A_{d2} = \begin{bmatrix} 0.06 & -0.2 \\ 0.2 & -0.3 \end{bmatrix}$, $F_{a2} = \begin{bmatrix} 0 \\ \frac{1}{7} \end{bmatrix}$, $H_2 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}$, $C_2 = \begin{bmatrix} -0.42 & 0.1 \end{bmatrix}$, $C_{x2} = \begin{bmatrix} 0.1 & -0.012 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0.23 \\ 0.1 \end{bmatrix}$
$N_2 = \begin{bmatrix} -0.1 & 0.002 \end{bmatrix}$, $N_{d2} = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}$, $D_{\varpi 2} = 0.01$

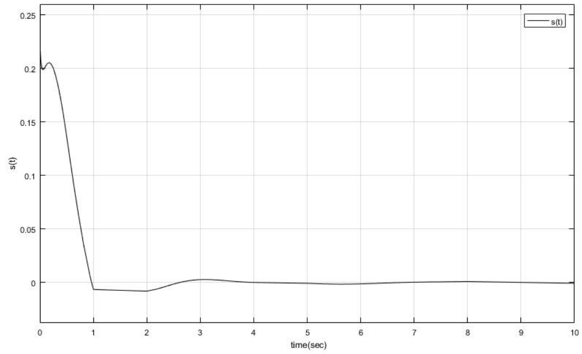

Fig 6 sliding mode $s\left(t\right)$

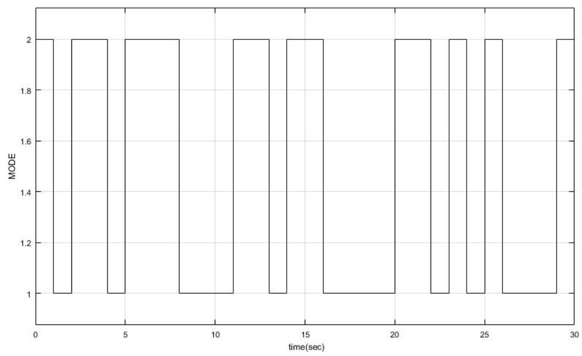

Fig 7 switching signal

Consider the known part of the transition probability matrix and the transition probability error matrix

$$\hat{\prod} = \begin{bmatrix} -0.11 & 0.11 \\ 0.11 & -0.11 \end{bmatrix}, \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}$$

Similarly, introducing a new parameter λ_{ij}, $\lambda_{ij} = \hat{\pi}_{ij} - \delta_{ij}$, $\forall\, j \neq i$ obvious $\delta_{ii} = -\sum\limits_{j=1, j \neq i}^{S} \delta_{ij}$ and $\lambda_{ii} = -\sum\limits_{j=1, j \neq i}^{S} \lambda_{ij}$. Next, given the selected parameters, $\varepsilon_{11} = \varepsilon_{12} = 0.5$, $\varepsilon_{21} = \varepsilon_{22} = 0.1$ and $\varepsilon_{31} = \varepsilon_{32} = 0.1$, $a = 0.5$, $\gamma_1 = 1$, time delay and time delay derivatives are $d_1 = 0.1$, $d_2 = 0.6$, $\mu_1 = 0.02$, $\mu_2 = 0.02$. The remaining parameters are $\eta = 0.11$, $\rho_{11} = \rho_{12} = 0.105$, $\rho_{21} = \rho_{22} = 0.77$, $\rho_{31} = \rho_{32} = 0.2$ and $\rho_{41} = \rho_{42} = 0.7$. The nonlinear function is the same sa example 1, exogenous input $\varpi\left(t\right) = 0.2 \cdot \exp\left(-100t\right)$.
Based on the above discussion, solve the linear matrix inequality, and obtain the controller gain and observer gain.

$$K_1 = \begin{bmatrix} -4.6195 & -8.3922 \end{bmatrix} \qquad K_2 = \begin{bmatrix} -3.4159 & -8.4352 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -4.4565 \\ -52.3160 \end{bmatrix} \qquad L_2 = \begin{bmatrix} -47.6393 \\ -67.3875 \end{bmatrix}$$

The results are display in Figs.8-13. Fig.8 is the state of closed-loop system, Fig.9 and Fig.10 show the observational state trajectory and error state trajectory of the system, Fig.11 and Fig.12 represent the controller trajectory and sliding surface function, and the switching function is described in Fig.13.

5 | CONCLUSIONS

This paper is about mixed H_∞ /passive SMC problem of uncertain stochastic MJSs, transition probabilities bound and unmeasured states. The considered non-linear system satisfies a class of generalized Lipshitz which is less

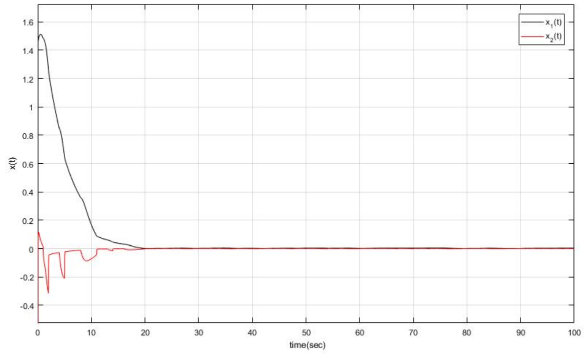

Fig 8 The state response $x(t)$

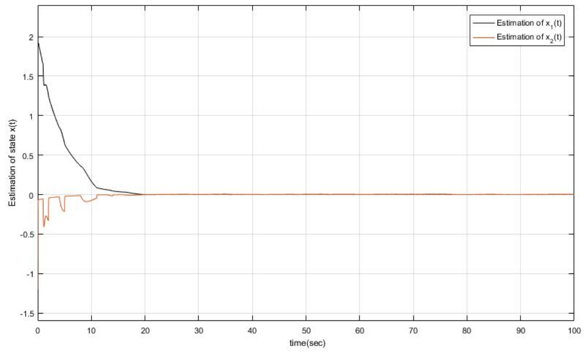

Fig 9 The estimation state $\hat{x}\,(t)$

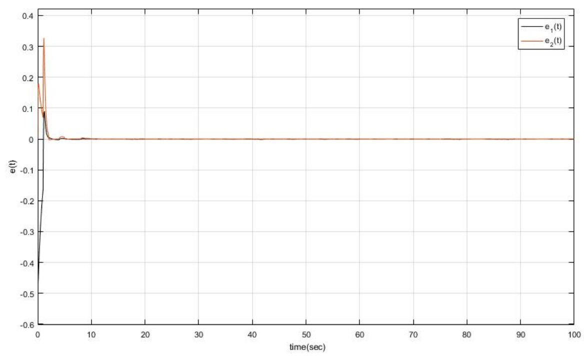

Fig 10 The estimation error $e\,(t)$

conservative than traditional Lipshitz. We designed parameters are mode-dependent in the integral term, which can reduce the conservativeness. The mixed H_∞ /passive performance index is more advanced. All of our results have been testified by numerical examples verifying the availability.

6 | ACKNOWLEDGEMENT

This work is partially supported by the National Natural Science Foundation of China No. 61273004, and the Natural Science Foundation of Hebei province No. F2018203099.

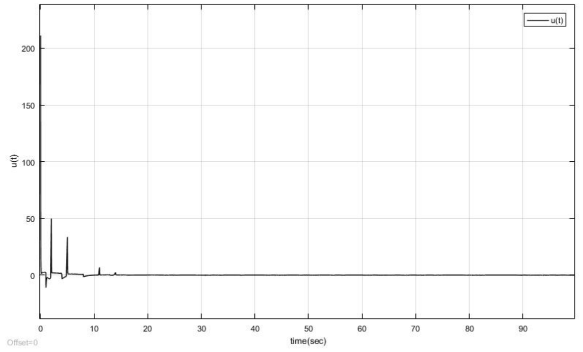

Fig 11 controller trajectory $u(t)$

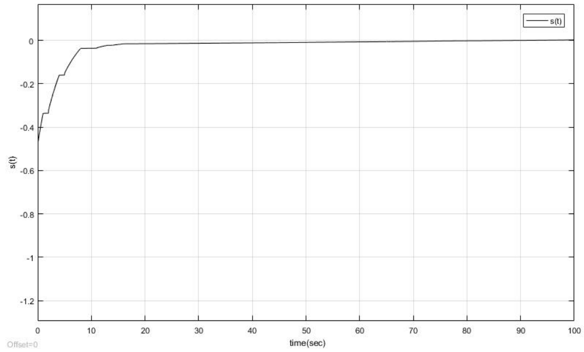

Fig 12 sliding mode $s(t)$

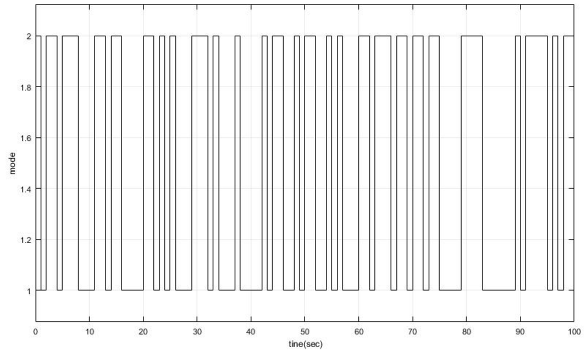

Fig 13 switching signal

References

1. Zheng D D, Hua M G, Chen J F, et al. Robust partially mode-dependent H_∞ filtering for discrete-time non-homogeneous Markovian jump neural networks with additive gain perturbations. Mathematical Methods in the Applied Sciences, 2019;42(3):982-998.

2. Li L, Qi W, Chen X, et al. Stability analysis and control synthesis for positive semi-Markov jump systems with time-varying delay. Applied Mathematics and Computation. 2018;332:363-375.

3. Nagamani G, Joo Y H, Radhika T. Delay-dependent dissipativity criteria for Markovian jump neural networks with random delays and incomplete transition probabilities. Nonlinear Dynamics. 2018;91(4):2503-2522.

4. Ma Y C, Jia X R, Liu D Y. Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates. Applied Mathematical Modelling. 2018;53:49-70

5. Chen W M, Wang L N. Delay-dependent H_∞ filtering of uncertain Markovian jump delay systems via delay-partitioning approach. Journal of the Franklin Institute. 2014;351(3):1431-1452.

6. Pan S Y, Ye Z Y, Zhou J. H_∞ robust control based on eventľsampling for hybrid systems with singular Markovian jump.?Mathematical Methods in the Applied Sciences. 2019;;42(3):790-805.

7. Cheng J, Ju H P, Karimi H R, et al. Static output feedback control of nonhomogeneous Markovian jump systems with asynchronous time delays. Information Sciences. 2017;399:219-238.

8. Tao J, Wu Z G, Su H Y, et al. Asynchronous and Resilient Filtering for Markovian Jump Neural Networks Subject to Extended Dissipativity. IEEE Transactions on Cybernetics. 2019;49(7):2504-2513.

9. Si W J, Qi L, Hou N, et al. Finite-time adaptive neural control for uncertain nonlinear time-delay systems with actuator delay and full-state constraints. International Journal of Systems Science. 2019;50(4):726-738.

10. Zhang X M, Min W, She J H, et al. Delay-dependent stabilization of linear systems with time-varying state and input delays. Automatica. 2005;41(8):1405-1412.

11. Zhao J J, Wang J, Park J H, et al. Memory feedback controller design for stochastic Markov jump distributed delay systems with input saturation and partially known transition rates. Nonlinear Analysis: Hybrid Systems. 2015;15:52-62.

12. Park J H. Novel robust stability criterion for a class of neutral systems with mixed delays and nonlinear perturbations. Applied Mathematics and Computation. 2005;161(2):413-421.

13. Zhang W A, Li Y U. Delay-dependent Robust Stability of Neutral Systems with Mixed Delays and Nonlinear Perturbations. Acta Automatica Sinica. 2007;33(8):863-866.

14. Kao Y G, Jing X, Wang C H, et al. A sliding mode approach to non-fragile observer-based control design for uncertain Markovian neutral-type stochastic systems. Automatica. 2015;52:218-226.

15. Pratap A, Raja R, Sowmiya C, et al. Global projective lag synchronization of fractional order memristor based BAM neural networks with mixed time varying delays. Asian Journal of Control. 2019. DOI: 10. 1002/asjc. 2075

16. Yao X, Chen Y C, Wen X L. Quantized feedback control scheme on coupled systems with time delay and distributed delay: A finite-time inner synchronization analysis. Applied Mathematics and Computation. 2018;337:315-328.

17. Ma L, Da F P, Wu L Y. Delayed-state-feedback exponential stabilization of stochastic Markovian jump systems with mode-dependent time-vary state delays. Acta Automatica Sinica. 2010;36(11):1601-1610.

18. Chen H B, Zhu C X, Hu P, et al. Delayed-state-feedback exponential stabilization for uncertain Markovian jump systems with mode-dependent time-varying state delays. Nonlinear Dynamics. 2012;69(3):1023-1039.

19. Hu, G.-D. Observers for one-sided Lipschitz non-linear systems. IMA Journal of Mathematical Control and Information. 2005;23(4):395-401.

20. Hu, G.-D. A note on observer for one-sided Lipschitz non-linear systems. IMA Journal of Mathematical Control and Information. 2007;25(3):297-303.

21. Zhao Y, Tao J, Shi N Z. A note on observer design for one-sided Lipschitz nonlinear systems. Systems and Control Letters. 2010;59(1):66-71.

22. Benallouch M, Boutayeb M, Zasadzinski M. Observer design for one-sided Lipschitz discrete-time systems. Systems and Control Letters. 2012;61(9):879-886.

23. Nguyen M C, Trinh H. Unknown input observer design for one-sided Lipschitz discrete-time systems subject to time-delay. Applied Mathematics and Computation. 2016;286:57-71.

24. Zhang W, Su H, Wang H, et al. Full-order and reduced-order observers for one-sided Lipschitz nonlinear systems using Riccati equations. Communications in Nonlinear Science and Numerical Simulation. 2012;17(12):4968-4977.

25. Zheng Q, Ling Y, Wei L, et al. Mixed H_∞ and passive control for linear switched systems via hybrid control approach. International Journal of Systems Science. 2018;49(9):1-14.

26. Xue X J, Xu H L, Xu L. Distributed output-feedback controllers design for Markovian jump systems interconnected over an undirected graph with time-varying delay. International Journal of Systems Science. 2019;50(1):563-584.

27. Li Y J, Huang Z W, Li J Z. H_∞ State Estimation for Stochastic Markovian Jumping Neural Network with Time-varying Delay and Leakage Delay. International Journal of Automation and Computing. 2016;(6):1-12.

28. Chen F F, Lu D K, Li X H. Robust observer based fault-tolerant control for one-sided Lipschitz Markocian jump systems with general uncertain transition rates. International Journal of Control Automation and Systems. 2019;17:1614-1625.

29. Li H Y, Shi P, Yao D Y, et al. Observer-based adaptive sliding mode control for nonlinear Markovian jump systems. Automatica. 2016;64:133-142.

30. Zhang D, Zhang Q L. Sliding mode control for TCS fuzzy singular semi-Markovian jump sustem. Nonlinear Analysis Hybrid Systems. 2018;30:72-91.

31. Colturato M. Sliding mode control for a diffuse interface tumor growth model coupling a Cahn-Hilliard equation with a reaction-diffusion equation. Mathematical Methods in the Applied Sciences. 2020;43(10):6598-6626.

32. Jiang B P, Karimi H R, Kao Y G, et al. A novel robust fuzzy integral sliding mode control for nonlinear semi-Markovian jump T-S fuzzy systems. IEEE Transactions on Fuzzy Systems. 2018;26(6):3594-3604.

33. Ma T, Li L. Stability analysis for uncertain nentral-type stochastic nonlinear systems with mixed time-varying delays. Journal of Inequalities and Applications. 2019, DOI 10.1186/s13660-019-2105-8.

34. Jiang B P, Karimi H R, Kao Y G, et al. Takagi-sugeno model-based sliding mode observer design for finite-time synthesis of semi-Markovian jump systems. IEEE Transactions Systems, Man, and Cybernetics: Systems. 2019;49:1505-1515.

35. Zheng Q, Ling Y, Wei L, et al. Mixed H_∞ and passive control for linear switched systems via hybrid control approach. International Journal of Systems Science. 2018;49(9):1-14.

36. Li N, Zheng W X. Passivity Analysis for Quaternion-Valued Memristor-Based Neural Networks With Time-Varying Delay. IEEE Transactions on Neural Networks and Learning Systems. 2020;31(2):639-650.

37. Chao M, Hong Q, Kang E. Mixed H_∞ and Passive Depth Control for Autonomous Underwater Vehicles with Fuzzy Memorized Sampled-Data Controller. International Journal of Fuzzy Systems. 2017;20(1):1-9.

38. Zheng Q, Zhang H, Ling Y, et al. Mixed H_∞ and passive control for a class of nonlinear switched systems with average dwell time via hybrid control approach. Journal of the Franklin Institute. 2018;355(3):1156-1175.

39. Xia W F, Li Y M, Chu Y M, et al. Observer-based mixed passive and H_∞ control for uncertain Markovian jump systems with time delays using quantized measurements. Nonlinear Analysis: Hybrid Systems. 2019;31:233-246.

40. Shi P, Zhang Y, Chadli M, et al. Mixed H-Infinity and Passive Filtering for Discrete Fuzzy Neural Networks With Stochastic Jumps and Time Delays. IEEE Trans Neural Netw Learn Syst. 2016;27(4):903-909.

41. Gao L, Wang D, Wu Y. Non-fragile observer-based sliding mode control for markovian jump systems with mixed mode-dependent time delays and input nonlinearity. Applied Mathematics and Computation. 2014;229(6):374-395.

42. Yang Y X, Lin C, Chen B, et al. Reduced-order observer design for a class of generalized Lipschitz nonlinear systems with time-varying delay. Applied Mathematics and Computation. 2018;337:267-280.

43. Kong C F, Ma Y C, Liu D Y. Observer-based quantized sliding mode dissipative control for singular semi-Markovian jump systems. Applied Mathematics and Computation. 2019;362:124539.

44. Zhao W Y, Ma Y C, Chen A H, et al. Robust sliding mode control for markovian jump singular systems with randomly changing structure. Applied Mathematics and Computation. 2019;349:81-96.

□