Complex compound-combination multi switching anti-synchronization of fractional-order complex chaotic systems and integer-order complex chaotic systems

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Based on three fractional-order complex chaotic systems and two integer-order complex chaotic systems, we propose a novel synchronization scenario of complex compound-combination multi switching anti-synchronization (CCCMSAS), which is first of this kind. The CCCMSAS states are completed between three leader and two follower systems by adopting the nonlinear control method and choosing suitable Lyapunov function on the basis of the complex space. Eventually, two typical examples have served as illustrations to show the validity and maneuverability of the proposed scheme.

KEYWORDS
complex compound-combination multi switching anti-synchronization, fractional-order complex chaotic system, integer-order complex chaotic system

1 | INTRODUCTION

Chaos is a kind of seemingly random motion in the deterministic system, which is extremely dependent on the initial value, so researchers have always thought that chaos can be difficult to anticipate and control. After a long period of research and exploration, however, Ott[1] proposed the OGY method, reached the purpose of controlling chaos in 1990. In the same year, Pecora and Carroll[2] realized the synchronization of chaotic systems and introduced the new conception of drive-response, laying the foundation for the development of this unfathomable field. As a pivotal of nonlinear dynamics, chaotic synchronization has captured great attention among scholars in assorted fields. There are many popular research directions in chaotic synchronization. For example, different synchronization types including projective synchronization,[3-5] generalized synchronization,[6-8] etc.; various methods to achieve synchronization state, like sliding mode control method,[9-10]adaptive control,[11-15] etc., and some important interdisciplinary applications, viz. communication security,[16] data encryption, and brain activity modeling,[17] etc. Yet in addition to these tasks, there is still a lot of room for development. For instance, the efforts have been devoted to the synchronization of multiple leaders-followers dynamic systems and multiple switching in the past fifteen years.

Two-leader and single-follower systems are introduced into the combination synchronization scheme by Runzi et al.[18] This scheme could break the situation that chaotic synchronization was limited to the single-leader and single-follower systems. Subsequently, the researchers put forward the synchronization method of combination-combination synchronization,[19-21] compound-combination synchronization,[22-24] dual combination synchronization,[25, 26]combination-combination phase synchronization [27] etc. , which are not single-leader/single-follower systems. It deserves to be specially noted that in safety communication, the strength of having multiple leader-follower systems is that the signals being transmitted can be divided into several different sections, and each section is loaded into different drive systems, which can greatly enhance the anti-attack ability. Sun et al[19,22] studied the combination-combination synchronization of four integer chaotic systems and compound-combination synchronization of five integer chaotic systems.

The multiple switching synchronization scheme proposed by Ucar et al[28] in 2008 is a bold and interesting synchronization scenario. The special feature of multi-switch synchronization is that the selection of leader and follower state variables is arbitrary, and thence there are multiple possible synchronization directions. These kinds of researches are crucial to communication security and only a handful of studies exist for such topic. Vincent et al[29] extended the multi-switch synchronization from the single-leader/single-follower systems to the two-leader/single-follower systems. Shahzad[30] discussed the multi switching synchronization for different orders of integer order leader and follower systems. Khan et al[31,32] investigated combination-combination multi switching synchronization of four integer hyperchaotic systems in 2017 and achieved dual combination multi switching synchronization state of six integer time-delay chaotic systems in 2018. Aysha[33] explored compound-compound multi switching anti-synchronization of six integer hyperchaotic real systems. It's important to note that anti-synchronization is an interesting phenomenon that has been focused greater attention on many physical systems. Additionally, the integer-order systems and homologous fractional-order systems are fundamentally distinct or different in kind. Most results or features of integer order systems can't simply extend to the fractional order cases. From what we can learn, there is no report about multi-switch anti synchronization between integer-order and fractional-order chaotic systems.

Complex chaotic systems, which first introduced by Fowler et al,[34] have captured great attention owing to their wide application foreground in the secure communication domain. The complex variables can double the number of variables, thereby increasing the amount of information transmitted and enhancing the security of communication. Vijay et al[27] designed the appropriate control functions to achieve combination-combination phase synchronization among four different fractional-order complex chaotic systems. All scaling factors in the above multiswitching synchronization are real matrices. We have learned about the complex scale factor is harder to predict than the real scale factor. Therefore, choosing the scale matrix as a complex matrix will greatly increase the variability and complexity of synchronization. From an application point of view, different synchronization types with complex scale matrix may provide better security and greater versatility. So far as I know, up to this date, multi-switching complex synchronization about complex variables has rarely been reported in view of the in fact that it is difficult to construct Lyapunov function based on complex space. Therefore, how to effectively carry out multi-switch anti synchronization between fractional and integer order complex chaotic systems are particularly important in secure communication.

Inspired by the preceding discussion, in the present article, we put forward a new synchronization scheme of complex compound-combination multi switching anti-synchronization among three fractional-order complex chaotic leader systems and two integer-order complex follower systems. This scheme can radically enhance functionality at the anti-attack during the communication process due to its complexity, and for all we know, so far, there are no results about this topic. By designing suitable complex controllers without separating the real parts and the imaginary parts, and choosing proper Lyapunov function of complex variables, CCCMSAS has been achieved. In this paper, two examples are brilliantly finished between different 3D fractional-order complex chaotic leader systems and 3D integer-order complex chaotic follower systems as well as hyperchaotic complex fractional-order chaotic systems and hyperchaotic integer-order complex chaotic systems, respectively.

The rest of this article is arranged as below. We start with the definition of multiswitching compound-combination complex anti-synchronization and put forward a universal control mechanism of CCCMSAS among fractional order complex leader systems and integer-order complex follower systems. Then, Section 3 introduces the CCCMSAS among the three different fractional-order leader systems and two different integer-order follower systems with complex variables. The CCCMSAS of three identical fractional-order hyperchaotic complex leader systems and two identical integer-order hyperchaotic complex follower systems was achieved in Section 4. Finally, Section 5 generalizes a conclusion.

Notation $\mathbb{C}^n$ represents n -dimensional complex vector space. If $x_1 \in \mathbb{C}^n$,then $x_1 = x_1^r + jx_1^i$, j is the square root of -1, superscripts r and i deputize for the real part and imaginary part of complex vector x_1, x_1^T is the transpose of x_1, x_1^H stands for the conjugate transpose of x_1,and $\|x_1\|$ is matrix norm.

2 | DEFINITIONS AND PROBLEM FORMULATION

Suppose the three fractional order complex chaotic leader systems are

$$\frac{d^\alpha x_1}{dt^\alpha} = f_1(x_1), \tag{1}$$

$$\frac{d^\alpha x_2}{dt^\alpha} = f_2(x_2), \tag{2}$$

$$\frac{d^\alpha x_3}{dt^\alpha} = f_3(x_3), \tag{3}$$

where $0 < \alpha < 1$, $x_k = (x_{k1}, x_{k2}, \cdots, x_{kn})^T \in \mathbb{C}^n$, $k = 1,2,3$ is a complex state vector, $f_1 = (f_{11}, f_{12}, \cdots, f_{1n})$, $f_2 = (f_{21}, f_{22}, \cdots, f_{2n})$, $f_3 = (f_{31}, f_{32}, \cdots, f_{3n})$ are three n-dimensional complex continuous vector functions. Suppose $X_1 = diag(x_{11}, x_{12}, \cdots, x_{1n})$, $X_2 = diag(x_{21}, x_{22}, \cdots, x_{2n})$, $X_3 = diag(x_{31}, x_{32}, \cdots, x_{3n})$.

The scaling compound signal of three fractional order complex chaotic leader systems is described by

$$S = CX_1(PX_2 + QX_3), \tag{4}$$

where $C, P, Q \in \mathbb{C}^{n\times n}$ are complex scaling diagonal matrices, we assume $C = diag(c_1, c_2, \ldots, c_n)$, $P = diag(p_1, p_2, \ldots, p_n)$, $Q = diag(q_1, q_2, \ldots, q_n)$.

Suppose the two integer order complex chaotic follower systems with the controllers are depicted as

$$\dot{y}_1 = g_1(y_1) + u_1, \tag{5}$$

$$\dot{y}_2 = g_2(y_2) + u_2, \tag{6}$$

where $y_k = (y_{k1}, y_{k2}, \ldots, y_{kn})^T \in \mathbb{C}^n$, $k = 1,2$ is the complex state vector, $g_1, g_2 : \mathbb{C}^n \to \mathbb{C}^n$ are n-dimensional complex continuous vector functions and u_1, u_2 are two controllers of two follower systems (5),(6). Suppose $Y_1 = diag(y_{11}, y_{12}, \ldots, y_{1n})$, $Y_2 = diag(y_{21}, y_{22}, \ldots, y_{2n})$.

The error system in matrix form is depicted as

$$e = AY_1 + BY_2 + S \tag{7}$$

where $A,B\in\mathbb{C}^{n\times n}$ are complex scaling diagonal matrices, and scaling matrices are taken as $A=diag(a_1,a_2,\cdots,a_n),\ B=diag(b_1,b_2,\cdots,b_n)$. Substituting Equation (4) into (7) and the matrix form of the error is represented as

$$e=AY_1+BY_2+CX_1\left(PX_2+QX_3\right), \tag{8}$$

where $e=diag\left(e_1,e_2,\cdots,e_n\right)$.

Definition 1. For given leader and follower systems, if there exist five complex constant nonzero diagonal matrices $A,B,C,P,Q\in\mathbb{C}^{n\times n}$ such that

$$\lim_{t\to\infty}\|e\|=\lim_{t\to\infty}\left\|AY_1+BY_2+CX_1\left(PX_2+QX_3\right)\right\|=0, \tag{9}$$

then the systems (1)-(3)and the systems (5),(6) are said to achieve complex compound-combination anti-synchronization (CCCAS).

Remark 1. By multiplying both sides of Equation (8) by vector$\left(1,1,\cdots,1\right)_{n\times1}^{T}$, the matrix form of the error is transformed into the vector form $e^*=(e_1,e_2,\cdots,e_n)^T=e\bullet\left(1,1,\cdots,1\right)_{n\times1}^{T}=\left[AY_1+BY_2+CX_1\left(PX_2+QX_3\right)\right]\bullet\left(1,1,\cdots,1\right)_{n\times1}^{T}$.

Remark 2. The compound-combination complex anti-synchronization will be achieved if the requirement is attained:

$$\lim_{t\to\infty}e_k=\lim_{t\to\infty}\left[a_ky_{1k}+b_ky_{2k}+c_kx_{1k}\left(p_kx_{2k}+q_kx_{3k}\right)\right]=0, \tag{10}$$

where $k=1,2,\cdots,n$, $e=diag(e_1,e_2,\cdots,e_n)$.

Remark 3. Let us redefined the component of e_k as

$$e_{klmsg}=a_ky_{1k}+b_ly_{2l}+c_mx_{1m}\left(p_sx_{2s}+q_gx_{3g}\right), \tag{11}$$

where $k,l,m,s,g\in\left(1,2,\cdots,n\right)$ and subscript $\left(klmsg\right)$ denotes k^{th} component of y_1, l^{th} component of y_2, m^{th} component of x_1, s^{th} component of x_2, and g^{th} component of x_3. About Definition 1, the error elements are strictly selected to satisfy $k=l=m=s=g$.

Definition 2. Based on Definition 1, the error states redeployed such that

$k = l = m = s = g$ is not satisfied, where $k, l, m, s, g \in (1, 2, \cdots, n)$ and

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \left\| AY_1 + BY_2 + CX_1 \left(PX_2 + QX_3 \right) \right\| = 0, \tag{12}$$

in that case, the leader systems (1)-(3) are called accomplished complex compound-combination multi switching anti-synchronization (CCCMSAS) with the follower systems (5),(6).

Remark 4. If $A = 0$, or $B = 0$ then the complex compound-combination multi switching anti-synchronization problem will be turned into the complex compound multi switching anti-synchronization of four complex chaotic systems.

Remark 5. If $X_1 = aI, a \neq 0$, then complex compound-combination multi switching anti synchronization changes to complex combination-combination multi switching anti synchronization.

Remark 6. If $X_1 = aI, a \neq 0,$ and $A = 0$ or $B = 0$ then the complex compound-combination multi switching anti synchronization problem will be turned into complex combination multi switching anti-synchronization of three complex chaotic systems.

Remark 7. If $C = 0$ then the complex multi switching anti synchronization could be fully realized for complex chaotic systems (5), (6).

Remark 8. If $B = 0, C = 0$, then the synchronization problem will degenerate into the control problem of the complex system (5).

Remark 9. If $A = 0, C = 0$, then the synchronization problem will degenerate into the control problem of the complex system (6).

Theorem 1. *The leader complex systems (1)-(3) will achieve CCCMSAS with follower complex chaotic systems (5), (6) if the complex controllers are selected as*

$$\begin{aligned} \sigma_k = & -a_k \left(g_{1k}(y_1) + y_{1k} \right) - b_l \left(g_{2l}(y_2) + y_{2l} \right) - c_m \dot{x}_{1m} \left(p_s x_{2s} + q_g x_{3g} \right) \\ & - c_m x_{1m} \left(p_s \dot{x}_{2s} + q_g \dot{x}_{3g} + p_s x_{2s} + q_g x_{3g} \right) \end{aligned} \tag{13}$$

where $\sigma_k = a_k u_{1k} + b_l u_{2l}, (k, l = 1, 2, \cdots, n)$.

Proff: The derivative of error components are

$$\dot{e}_{klmsg} = a_k \dot{y}_{1k} + b_l \dot{y}_{2l} + c_m \dot{x}_{1m} \left(p_s x_{2s} + q_g x_{3g} \right) + c_m x_{1m} \left(p_s \dot{x}_{2s} + q_g \dot{x}_{3g} \right), \tag{14}$$

Insertion of (5), (6) into (11) gets

$$\begin{aligned}\dot{e}_{klmsg} &= a_k\left(g_{1k}\left(y_1\right)+u_{1k}\right)+b_l\left(g_{2l}\left(y_2\right)+u_{2l}\right)+c_m\dot{x}_{1m}\left(p_s x_{2s}+q_g x_{3g}\right)+c_m x_{1m}\left(p_s\dot{x}_{2s}+q_g\dot{x}_{3g}\right)\\ &= a_k g_{1k}\left(y_1\right)+b_l g_{2l}\left(y_2\right)+c_m\dot{x}_{1m}\left(p_s x_{2s}+q_g x_{3g}\right)+c_m x_{1m}\left(p_s\dot{x}_{2s}+q_g\dot{x}_{3g}\right)+a_k u_{1k}+b_l u_{2l}\end{aligned} \tag{15}$$

Let $\sigma_k = a_k u_{1k}+b_l u_{2l},\left(l,k=1,2,\cdots,n\right)$, by using (13)

$$\dot{e}_{klmsg} = -a_k y_{1k}-b_l y_{2l}-c_m x_{1m}\left(p_s x_{2s}+q_g x_{3g}\right) = -e_{klmsg} \tag{16}$$

The derivative of the error system in matrix form is depicted as

$$\begin{aligned}\dot{e} &= A\dot{Y}_1+B\dot{Y}_2+C\dot{X}_1\left(PX_2+QX_3\right)+CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)\\ &= A\left(G_1\left(y_1\right)+U_1\right)+B\left(G_2\left(y_2\right)+U_2\right)+C\dot{X}_1\left(PX_2+QX_3\right)+CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)\\ &= AG_1\left(y_1\right)+BG_2\left(y_2\right)+C\dot{X}_1\left(PX_2+QX_3\right)+CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)+AU_1+BU_2,\end{aligned} \tag{17}$$

where $G_1\left(y_1\right) = diag\left(g_{11}(y_1), g_{12}(y_1),\cdots,g_{1n}(y_1)\right)$, $U_1 = diag\left(u_{11},u_{12},\cdots,u_{1n}\right)$,

$G_2\left(y_2\right) = diag\left(g_{21}(y_2), g_{22}(y_2),\cdots,g_{2n}(y_2)\right), U_2 = diag\left(u_{21},u_{22},\cdots,u_{2n}\right)$,

$u_1 = \left(u_{11},u_{12},\cdots,u_{1n}\right)^T$, $u_2 = \left(u_{21},u_{22},\cdots,u_{2n}\right)^T$, $g_1\left(y_1\right) = \left(g_{11}(y_1), g_{12}(y_1),\cdots,g_{1n}(y_1)\right)^T$

$g_2\left(y_2\right) = \left(g_{21}(y_2), g_{22}(y_2),\cdots,g_{2n}(y_2)\right)^T$.

We find that AU_1+BU_2 and $diag\left(\sigma_1,\sigma_2,\cdots,\sigma_n\right)$ are equal and AU_1+BU_2 can be expressed as

$$AU_1+BU_2 = -AG_1\left(y_1\right)-BG_2\left(y_2\right)-C\dot{X}_1\left(PX_2+QX_3\right)-CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)-e. \tag{18}$$

Define the Lyapunov function with complex variables as

$$V = \frac{1}{2}\left(\left(e^*\right)^H e^*\right). \tag{19}$$

The derivative of V is

$$\begin{aligned}\dot{V} &= \frac{1}{2}\left[\left(\dot{e}^*\right)^H e^* + \left(e^*\right)^H \dot{e}^*\right]\\ &= \frac{1}{2}\left[\left(A\left(G_1\left(y_1\right)+U_1\right)+B\left(G_2\left(y_2\right)+U_2\right)+C\dot{X}_1\left(PX_2+QX_3\right)+CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)\right)\bullet\left(1,1,\cdots,1\right)_{n\times1}^T\right]^H e^*\\ &\quad+\frac{1}{2}\left(e^*\right)^H\left[\left(A\left(G_1\left(y_1\right)+U_1\right)+B\left(G_2\left(y_2\right)+U_2\right)+C\dot{X}_1\left(PX_2+QX_3\right)+CX_1\left(P\dot{X}_2+Q\dot{X}_3\right)\right)\bullet\left(1,1,\cdots,1\right)_{n\times1}^T\right]\\ &= \frac{1}{2}\left(-e^*\right)^H e^* + \frac{1}{2}\left(e^*\right)^H\left(-e^*\right)\\ &= -e^{*H}e^* < 0,\end{aligned}$$

(20)

which is negative definite. By using the Lyapunov stability theory, we obtain $\lim_{t\to\infty}\|e\|=0$. This means that the CCCMSAS will be achieved for the leader complex fractional-order chaotic systems (1)-(3) and follower complex integer-order chaotic systems (5), (6).

3 | CCCMSAS AMONG FIVE 3D CHAOTIC SYSTEMS

Throughout this section, the nonlinear complex controllers are designed to realize CCCMSAS between three different 3D fractional-order complex chaotic leader systems and two different 3D integer-order complex chaotic follower systems.

The three fractional-order complex chaotic leader systems are defined as follows:

$$\begin{cases} \dfrac{d^{0.9}x_{11}}{dt^{0.9}} = 10\left(x_{12} - x_{11}\right) \\ \dfrac{d^{0.9}x_{12}}{dt^{0.9}} = 180x_{11} - x_{12} - x_{11}x_{13} \\ \dfrac{d^{0.9}x_{13}}{dt^{0.9}} = \dfrac{1}{2}\left(\overline{x}_{11}x_{12} + x_{11}\overline{x}_{12}\right) - x_{13} \end{cases} \tag{21}$$

$$\begin{cases} \dfrac{d^{0.9}x_{21}}{dt^{0.9}} = 35\left(x_{22} - x_{21}\right) \\ \dfrac{d^{0.9}x_{22}}{dt^{0.9}} = -7x_{21} + 28x_{22} - x_{21}x_{23} \\ \dfrac{d^{0.9}x_{23}}{dt^{0.9}} = \dfrac{1}{2}\left(\overline{x}_{21}x_{22} + x_{21}\overline{x}_{22}\right) - 3x_{23} \end{cases} \tag{22}$$

$$\begin{cases} \dfrac{d^{0.9}x_{31}}{dt^{0.9}} = 2.1\left(x_{32} - x_{31}\right) \\ \dfrac{d^{0.9}x_{32}}{dt^{0.9}} = 27.9x_{31} - 2.1x_{31}x_{33} \\ \dfrac{d^{0.9}x_{33}}{dt^{0.9}} = \dfrac{1}{2}\left(\overline{x}_{31}x_{32} + x_{31}\overline{x}_{32}\right) - 0.6x_{33} \end{cases} \tag{23}$$

where $x_{11} = x_{11}^r + jx_{11}^i$, $x_{12} = x_{12}^r + jx_{12}^i$, $x_{21} = x_{21}^r + jx_{21}^i$, $x_{22} = x_{22}^r + jx_{22}^i$, $x_{31} = x_{31}^r + jx_{31}^i$ $x_{32} = x_{32}^r + jx_{32}^i$ are complex variables, x_{13}, x_{23} , x_{33} are real variables. The leader systems are chaotic when $\left(x_{11}, x_{12}, x_{13}\right)^T = \left(2+3j, 5+6j, 9\right)^T$,

$(x_{21}, x_{22}, x_{23})^T = (6+9j, 5+7j, 12)^T$, $(x_{31}, x_{32}, x_{33})^T = (8+7j, 6+8j, 7)^T$ see Figs.1-3, respectively.

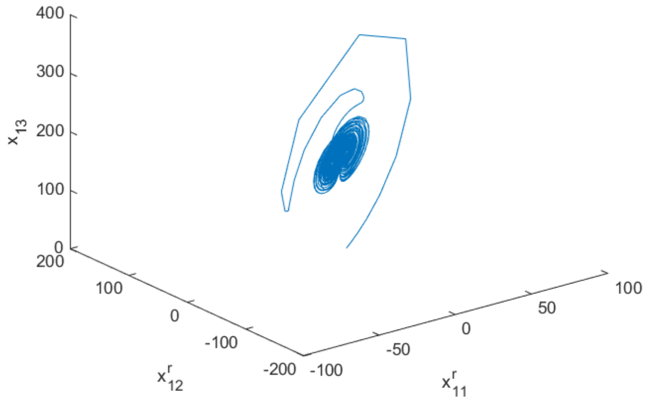

FIGURE1 Chaotic attractor for the system (21).

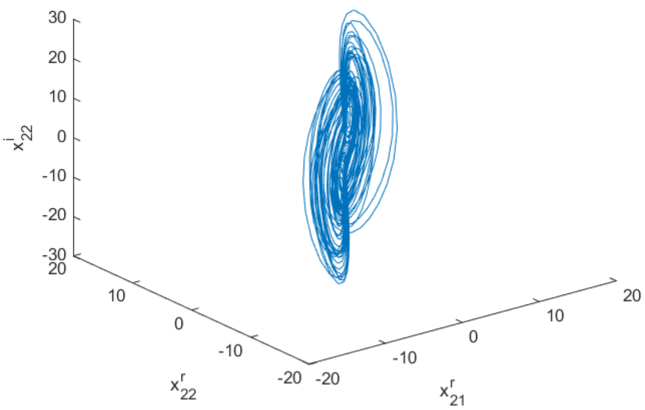

FIGURE 2 Chaotic attractor for the system (22).

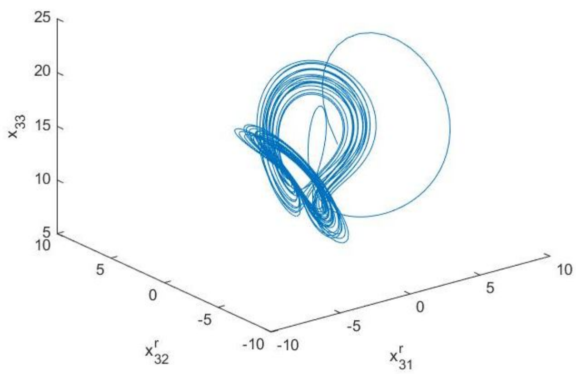

FIGURE 3 Chaotic attractor for the system (23).

The two integer order complex chaotic follower systems with nonlinear controllers are given by:

$$\begin{cases} \dot{y}_{11} = 40(y_{12} - y_{11}) + u_{11} \\ \dot{y}_{12} = 22y_{12} - y_{11}y_{13} + u_{12} \\ \dot{y}_{13} = 0.5(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}) - 5y_{13} + u_{13} \end{cases} \tag{24}$$

$$\begin{cases} \dot{y}_{21} = 14(y_{22} - y_{21}) + u_{21} \\ \dot{y}_{22} = 35y_{21} - y_{21}y_{23} - y_{22} + u_{22} \\ \dot{y}_{23} = 0.5(\overline{y}_{21}y_{22} + y_{21}\overline{y}_{22}) - 3.7y_{23} + u_{23} \end{cases} \tag{25}$$

where $y_{11} = y_{11}^r + jy_{11}^i$, $y_{12} = y_{12}^r + jy_{12}^i$, $y_{21} = y_{21}^r + jy_{21}^i$, $y_{22} = y_{22}^r + jy_{22}^i$ are complex variables, y_{13}, y_{23} are real variables. The integer-order complex systems are chaotic when $(y_{11}, y_{12}, y_{13})^T = (1+2j, 3+4j, 5)^T$, $(y_{21}, y_{22}, y_{23})^T = (3+6j, -3+j, 3)^T$, see Figs.4-5.

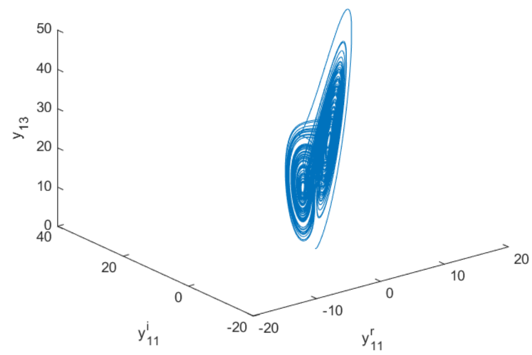

FIGURE 4 Chaotic attractor for the system (24).

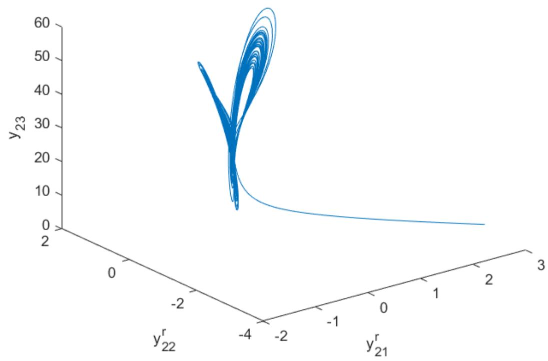

FIGURE 5 Chaotic attractor for the system (25).

Arbitrary switched error states are chosen as below:

$$\begin{cases} e_{12312} = a_1y_{11} + b_2y_{22} + c_3x_{13}(p_1x_{21} + q_2x_{32}) \\ e_{23321} = a_2y_{12} + b_3y_{23} + c_3x_{13}(p_2x_{22} + q_1x_{31}) \\ e_{31232} = a_3y_{13} + b_1y_{21} + c_2x_{12}(p_3x_{23} + q_2x_{32}) \end{cases} \tag{26}$$

Theorem 2. *The drive systems (21)-(23) and response systems (24)-(25) could achieve CCCMSAS for the complex nonlinear controllers (27):*

$$\begin{cases}\sigma_1 = a_1u_{11} + b_2u_{22} = -a_1\left(40\left(y_{12}-y_{11}\right)+y_{11}\right) - b_2\left(35y_{21} - y_{21}y_{23} - y_{22} + y_{22}\right)\\ \quad -c_3\dot{x}_{13}\left(p_1x_{21}+q_2x_{32}\right) - c_3x_{13}\left(p_1\dot{x}_{21}+q_2\dot{x}_{32}+p_1x_{21}+q_2x_{32}\right)\\ \sigma_2 = a_2u_{12} + b_3u_{23} = -a_2\left(22y_{12} - y_{11}y_{13} + y_{12}\right) - b_3\left(0.5\left(\overline{y}_{21}y_{22} + y_{21}\overline{y}_{22}\right) - 3.7y_{23} + y_{23}\right)\\ \quad -c_3\dot{x}_{13}\left(p_2x_{22}+q_1x_{31}\right) - c_3x_{13}\left(p_2\dot{x}_{22}+q_1\dot{x}_{31}+p_2x_{22}+q_1x_{31}\right)\\ \sigma_3 = a_3u_{13} + b_1u_{21} = -a_3\left(0.5\left(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}\right) - 5y_{13} + y_{13}\right) - b_1\left(14\left(y_{22}-y_{21}\right)+y_{21}\right)\\ \quad -c_2\dot{x}_{12}\left(p_3x_{23}+q_2x_{32}\right) - c_2x_{12}\left(p_3\dot{x}_{23}+q_2\dot{x}_{32}+p_3x_{23}+q_2x_{32}\right)\end{cases}$$

(27)

Proff: The error dynamical system is

$$\begin{cases}\dot{e}_{12312} = a_1\dot{y}_{11} + b_2\dot{y}_{22} + c_3\dot{x}_{13}\left(p_1x_{21}+q_2x_{32}\right) + c_3x_{13}\left(p_1\dot{x}_{21}+q_2\dot{x}_{32}\right)\\ \dot{e}_{23321} = a_2\dot{y}_{12} + b_3\dot{y}_{23} + c_3\dot{x}_{13}\left(p_2x_{22}+q_1x_{31}\right) + c_3x_{13}\left(p_2\dot{x}_{22}+q_1\dot{x}_{31}\right)\\ \dot{e}_{31232} = a_3\dot{y}_{13} + b_1\dot{y}_{21} + c_2\dot{x}_{12}\left(p_3x_{23}+q_2x_{32}\right) + c_2x_{12}\left(p_3\dot{x}_{23}+q_2\dot{x}_{32}\right)\end{cases} \tag{28}$$

By substituting Eqs. (24) - (25) into (28), the error dynamical systems of (21)-(25) are translated into the formula, as described below:

$$\begin{cases}\dot{e}_{12312} = a_1\left(40\left(y_{12}-y_{11}\right)+u_{11}\right) + b_2\left(35y_{21} - y_{21}y_{23} - y_{22} + u_{22}\right)\\ \quad +c_3\dot{x}_{13}\left(p_1x_{21}+q_2x_{32}\right) + c_3x_{13}\left(p_1\dot{x}_{21}+q_2\dot{x}_{32}\right)\\ \dot{e}_{23321} = a_2\left(22y_{12} - y_{11}y_{13} + u_{12}\right) + b_3\left(0.5\left(\overline{y}_{21}y_{22} + y_{21}\overline{y}_{22}\right) - 3.7y_{23} + u_{23}\right)\\ \quad +c_3\dot{x}_{13}\left(p_2x_{22}+q_1x_{31}\right) + c_3x_{13}\left(p_2\dot{x}_{22}+q_1\dot{x}_{31}\right)\\ \dot{e}_{31232} = a_3\left(0.5\left(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}\right) - 5y_{13} + u_{13}\right) + b_1\left(14\left(y_{22}-y_{21}\right)+u_{21}\right)\\ \quad +c_2\dot{x}_{12}\left(p_3x_{23}+q_2x_{32}\right) + c_2x_{12}\left(p_3\dot{x}_{23}+q_2\dot{x}_{32}\right)\end{cases} \tag{29}$$

Denote

$$\begin{cases}\sigma_1 = a_1u_{11} + b_2u_{22}\\ \sigma_2 = a_2u_{12} + b_3u_{23}\\ \sigma_3 = a_3u_{13} + b_1u_{21}\end{cases} \tag{30}$$

From (29) and (30), we get

$$\begin{cases}\dot{e}_{12312} = a_1\left(40\left(y_{12}-y_{11}\right)\right) + b_2\left(35y_{21} - y_{21}y_{23} - y_{22}\right)\\ \quad +c_3\dot{x}_{13}\left(p_1x_{21}+q_2x_{32}\right) + c_3x_{13}\left(p_1\dot{x}_{21}+q_2\dot{x}_{32}\right)+\sigma_1\\ \dot{e}_{23321} = a_2\left(22y_{12} - y_{11}y_{13}\right) + b_3\left(0.5\left(\overline{y}_{21}y_{22} + y_{21}\overline{y}_{22}\right) - 3.7y_{23}\right)\\ \quad +c_3\dot{x}_{13}\left(p_2x_{22}+q_1x_{31}\right) + c_3x_{13}\left(p_2\dot{x}_{22}+q_1\dot{x}_{31}\right)+\sigma_2\\ \dot{e}_{31232} = a_3\left(0.5\left(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}\right) - 5y_{13}\right) + b_1\left(14\left(y_{22}-y_{21}\right)\right)\\ \quad +c_2\dot{x}_{12}\left(p_3x_{23}+q_2x_{32}\right) + c_2x_{12}\left(p_3\dot{x}_{23}+q_2\dot{x}_{32}\right)+\sigma_3\end{cases} \tag{31}$$

The Lyapunov function with complex variables is depicted as

$$V=\frac{1}{2}\left(\left(e^{*}\right)^{H} e^{*}\right), \tag{32}$$

where $e^{*}=\left(e_{12312}, e_{23321}, e_{31232}\right)^{T}$.

The derivative of V is

$$\begin{aligned}
\dot{V}= & \frac{1}{2}\left[\left(\dot{e}^{*}\right)^{H} e^{*}+\left(e^{*}\right)^{H} \dot{e}^{*}\right] \\
= & \frac{1}{2}\left(\dot{\bar{e}}_{12312} e_{12312}+\dot{\bar{e}}_{23321} e_{23321}+\dot{\bar{e}}_{31232} e_{31232}+\bar{e}_{12312} \dot{e}_{12312}+\bar{e}_{23321} \dot{e}_{23321}+\bar{e}_{31232} \dot{e}_{31232}\right) \\
= & \frac{1}{2}\left[\left(\bar{a}_{1}\left(40\left(\bar{y}_{12}-\bar{y}_{11}\right)\right)+\bar{b}_{2}\left(35 \bar{y}_{21}-\bar{y}_{21} \bar{y}_{23}-\bar{y}_{22}\right)+\bar{c}_{3} \dot{\bar{x}}_{13}\left(\bar{p}_{1} \bar{x}_{21}+\bar{q}_{2} \bar{x}_{32}\right)+\bar{c}_{3} \bar{x}_{13}\left(\bar{p}_{1} \dot{\bar{x}}_{21}+\bar{q}_{2} \dot{\bar{x}}_{32}\right)+\bar{\sigma}_{1}\right) e_{12312}\right] \\
& +\frac{1}{2}\left[\left(\bar{a}_{2}\left(22 \bar{y}_{12}-\bar{y}_{11} \bar{y}_{13}\right)+\bar{b}_{3}\left(0.5\left(y_{21} \bar{y}_{22}+\bar{y}_{21} y_{22}\right)-3.7 \bar{y}_{23}\right)+\bar{c}_{3} \dot{\bar{x}}_{13}\left(\bar{p}_{2} \bar{x}_{22}+\bar{q}_{1} \bar{x}_{31}\right)+\bar{c}_{3} \bar{x}_{13}\left(\bar{p}_{2} \dot{\bar{x}}_{22}+\bar{q}_{1} \dot{\bar{x}}_{31}\right)+\bar{\sigma}_{2}\right) e_{23321}\right] \\
& +\frac{1}{2}\left[\left(\bar{a}_{3}\left(0.5\left(y_{11} \bar{y}_{12}+\bar{y}_{11} y_{12}\right)-5 \bar{y}_{13}\right)+\bar{b}_{1}\left(14\left(\bar{y}_{22}-\bar{y}_{21}\right)\right)+\bar{c}_{2} \dot{\bar{x}}_{12}\left(\bar{p}_{3} \bar{x}_{23}+\bar{q}_{2} \bar{x}_{32}\right)+\bar{c}_{2} \bar{x}_{12}\left(\bar{p}_{3} \dot{\bar{x}}_{23}+\bar{q}_{2} \dot{\bar{x}}_{32}\right)+\bar{\sigma}_{3}\right) e_{31232}\right] \\
& +\frac{1}{2}\left[\bar{e}_{12312}\left(a_{1}\left(40\left(y_{12}-y_{11}\right)\right)+b_{2}\left(35 y_{21}-y_{21} y_{23}-y_{22}\right)+c_{3} \dot{x}_{13}\left(p_{1} x_{21}+q_{2} x_{32}\right)+c_{3} x_{13}\left(p_{1} \dot{x}_{21}+q_{2} \dot{x}_{32}\right)+\sigma_{1}\right)\right] \\
& +\frac{1}{2}\left[\bar{e}_{23321}\left(a_{2}\left(22 y_{12}-y_{11} y_{13}\right)+b_{3}\left(0.5\left(\bar{y}_{21} y_{22}+y_{21} \bar{y}_{22}\right)-3.7 y_{23}\right)+c_{3} \dot{x}_{13}\left(p_{2} x_{22}+q_{1} x_{31}\right)+c_{3} x_{13}\left(p_{2} \dot{x}_{22}+q_{1} \dot{x}_{31}\right)+\sigma_{2}\right)\right] \\
& +\frac{1}{2}\left[\bar{e}_{31232}\left(a_{3}\left(0.5\left(\bar{y}_{11} y_{12}+y_{11} \bar{y}_{12}\right)-5 y_{13}\right)+b_{1}\left(14\left(y_{22}-y_{21}\right)\right)+c_{2} \dot{x}_{12}\left(p_{3} x_{23}+q_{2} x_{32}\right)+c_{2} x_{12}\left(p_{3} \dot{x}_{23}+q_{2} \dot{x}_{32}\right)+\sigma_{3}\right)\right]
\end{aligned} \tag{33}$$

Substituting the numerical value equations of $\sigma_{1}, \sigma_{2}, \sigma_{3}$ in (33), we obtain

$$\dot{V}=-\bar{e}_{12312} e_{12312}-\bar{e}_{23321} e_{23321}-\bar{e}_{31232} e_{31232}<0 \tag{34}$$

Hence V is positive definite, the derivative of V is negative definite, indicating that the CCCMSAS error system has reached asymptotically stable state.

In the numerical modeling, the initial states of the leader systems (21)-(23) and the follower systems (24), (25) are randomly given by $\left(x_{11}, x_{12}, x_{13}\right)^{T}=(2+3 j, 5+6 j, 9)^{T}$, $\left(x_{21}, x_{22}, x_{23}\right)^{T}=(6+9 j, 5+7 j, 12)^{T}$, $\left(x_{31}, x_{32}, x_{33}\right)^{T}=(8+7 j, 6+8 j, 7)^{T}$, $\left(y_{11}, y_{12}, y_{13}\right)^{T}=(1+2 j, 3+4 j, 5)^{T}$, $\left(y_{21}, y_{22}, y_{23}\right)^{T}=(3+6 j,-3+j, 3)^{T}$. We assume $a_{1}=1+j$, $a_{2}=1-j$, $a_{3}=1$, $b_{1}=1+j$, $b_{2}=1-2 j$, $b_{3}=1$, $c_{1}=1-2 j$, $c_{2}=1+j$, $c_{3}=1$

$p_{1}=1+j$, $p_{2}=1-j$, $p_{3}=1$, $q_{1}=1+j$, $q_{2}=1-j$, $q_{3}=1$. The initial values of complex compound-combination multi switching anti-synchronization (CCCMSAS) error will be $\left(e_{12312}, x_{23321}, x_{31232}\right)^T = \left(97+163j, 127+154j, -36+303j\right)^T$. Figure 6 shows the time response of CCCMSAS errors e_{12312}, e_{23321} and e_{31232} between the leader systems (21)-(23) and follower systems(24)-(25), where the curves of blue, red and black represent error states e_{12312}, e_{23321}, e_{31232} and solid and dotted lines present the real-parts and imaginary parts of error states $e_{12312}, e_{23321}, e_{31232}$, respectively. Figures 7-12 show the time response of the real parts and imaginary parts of the complex compound-combination multi switching anti-synchronization synchronized states $a_1 y_{11} + b_2 y_{22}$ and $c_3 x_{13}\left(p_1 x_{21} + q_2 x_{32}\right)$, $a_2 y_{12} + b_3 y_{23}$ and $c_3 x_{13}\left(p_2 x_{22} + q_1 x_{31}\right)$, $a_3 y_{13} + b_1 y_{21}$ and $c_2 x_{12}\left(p_3 x_{23} + q_2 x_{32}\right)$ of the leader systems (21)-(23) and follower systems (24)-(25) respectively. It should be noted that the controller involves the integer-order derivative of the leader variables, but the known leader systems are fractional-orders. Therefore, in this paper, the numerical simulation of state variable response is carried out by using the engineering time-domain and frequency-domain conversion method to convert the given fractional order leader system into integer order operation. As expected, numerical simulation results show that CCCMSAS has been implemented between five different complex chaotic systems (21)-(23) and (24)-(25), which is consistent with the theoretical analysis and further demonstrates the effectiveness of the controller.

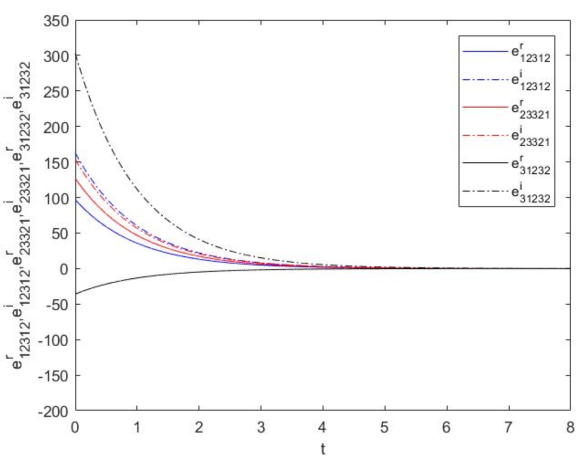

FIGURE 6 The CCCMSAS errors e_{12312}, e_{23321} and e_{31232} between the leader systems (21)-(23) and follower systems (24)-(25), where the full lines are depicted as real parts of $e_{12312}, e_{23321}, e_{31232}$

and the dash lines represent imaginary parts of errors $e_{12312}, e_{23321}, e_{31232}$.

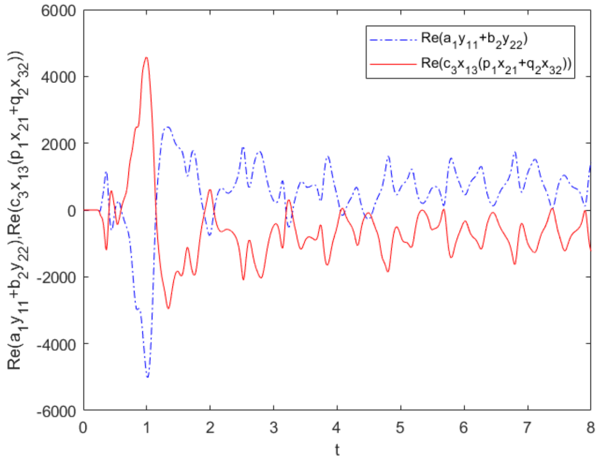

FIGURE 7 Response for the real parts of the states $a_1 y_{11} + b_2 y_{22}$ and $c_3 x_{13}\left(p_1 x_{21} + q_2 x_{32}\right)$, where

$$\mathrm{Re}\left(a_1 y_{11} + b_2 y_{22}\right) = y_{11}^r - y_{11}^i + y_{22}^r + 2y_{22}^i, \mathrm{Re}\left(c_3 x_{13}\left(p_1 x_{21} + q_2 x_{32}\right)\right) = x_{13}\left(x_{21}^r - x_{21}^i + x_{32}^r + x_{32}^i\right).$$

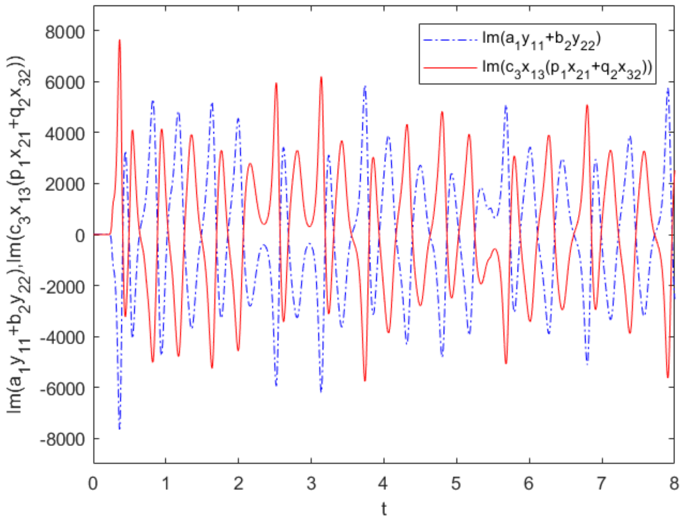

FIGURE 8 Response for the imaginary parts of the states $a_1 y_{11} + b_2 y_{22}$ and $c_3 x_{13}\left(p_1 x_{21} + q_2 x_{32}\right)$, where $\mathrm{Im}\left(a_1 y_{11} + b_2 y_{22}\right) = y_{11}^r + y_{11}^i + y_{22}^i - 2y_{22}^r$, $\mathrm{Im}\left(c_3 x_{13}\left(p_1 x_{21} + q_2 x_{32}\right)\right) = x_{13}\left(x_{21}^i + x_{21}^r + x_{32}^i - x_{32}^r\right)$.

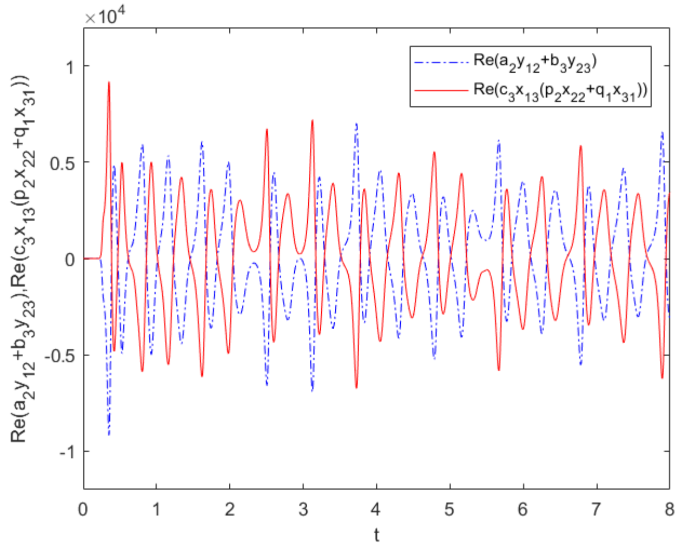

FIGURE 9 Response for the real parts of the states $a_2 y_{12} + b_3 y_{23}$ and $c_3 x_{13}\left(p_2 x_{22} + q_1 x_{31}\right)$, where

$$\mathrm{Re}\left(a_2 y_{12} + b_3 y_{23}\right) = y_{12}^r + y_{12}^i + y_{23},\ \mathrm{Re}\left(c_3 x_{13}\left(p_2 x_{22} + q_1 x_{31}\right)\right) = x_{13}\left(x_{22}^r + x_{22}^i + x_{32}^r - x_{32}^i\right).$$

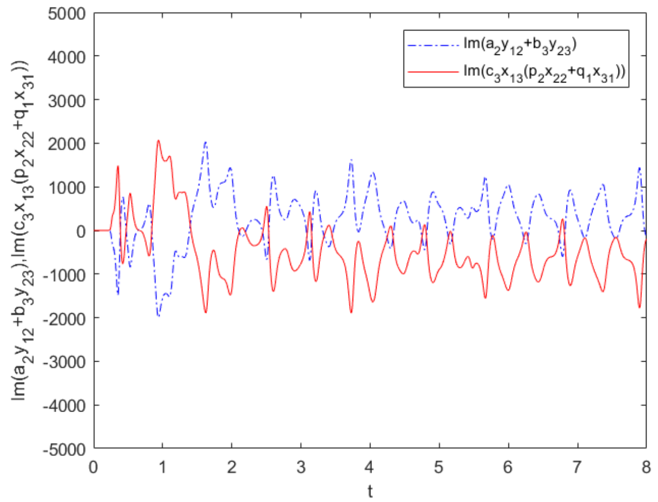

FIGURE 10 Response for the imaginary parts of the states $a_2 y_{12} + b_3 y_{23}$ and $c_3 x_{13}\left(p_2 x_{22} + q_1 x_{31}\right)$, where $\mathrm{Im}\left(a_2 y_{12} + b_3 y_{23}\right) = y_{12}^i - y_{12}^r$, $\mathrm{Im}\left(c_3 x_{13}\left(p_2 x_{22} + q_1 x_{31}\right)\right) = x_{13}\left(x_{22}^i - x_{22}^r + x_{31}^i + x_{31}^r\right)$.

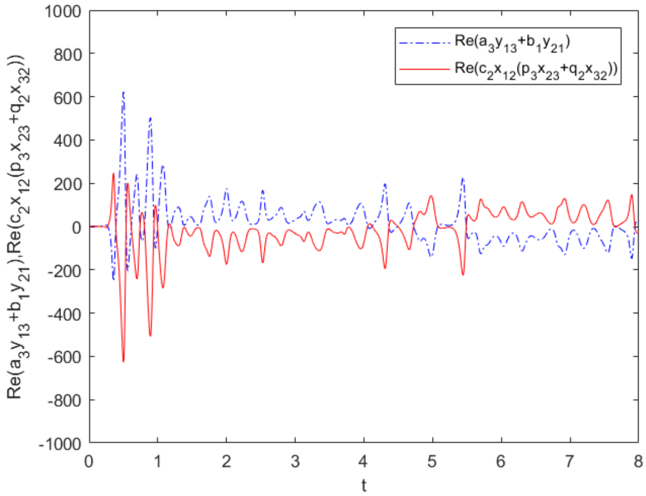

FIGURE 11 Response for the real parts of the states $a_3y_{13}+b_1y_{21}$ and $c_2x_{12}\left(p_3x_{23}+q_2x_{32}\right)$, where $\mathrm{Re}\left(a_3y_{13}+b_1y_{21}\right)=y_{21}^r-y_{21}^i+y_{13}$, $\mathrm{Re}\left(c_2x_{12}\left(p_3x_{23}+q_2x_{32}\right)\right)=x_{12}^r x_{23}+2x_{12}^r x_{32}^r-x_{12}^i x_{23}-2x_{12}^i x_{32}^i$.

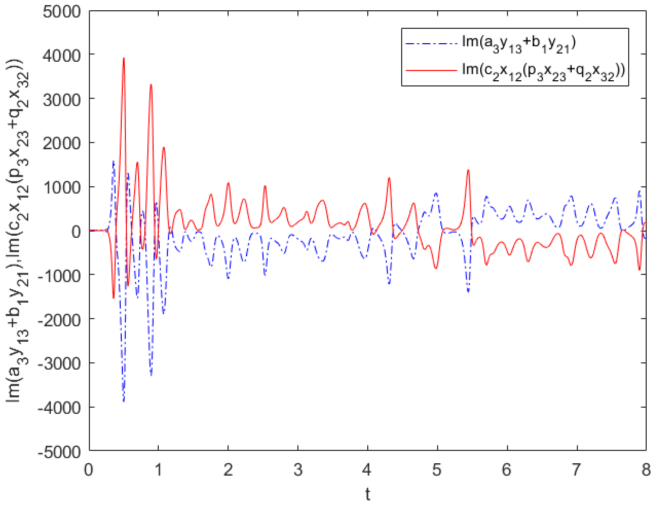

FIGURE 12 Response for the imaginary parts of the states $a_3y_{13}+b_1y_{21}$ and $c_2x_{12}\left(p_3x_{23}+q_2x_{32}\right)$, where $\mathrm{Im}\left(a_3y_{13}+b_1y_{21}\right)=y_{21}^i+y_{21}^r$, $\mathrm{Im}\left(c_2x_{12}\left(p_3x_{23}+q_2x_{32}\right)\right)=x_{12}^i x_{23}+2x_{12}^i x_{32}^r+x_{12}^r x_{23}+2x_{12}^r x_{32}^i$.

4 | CCCMSAS AMONG FIVE HYPERCHAOTIC SYSTEMS

In this section, we will implement the CCCMSAS among five hyperchaotic complex systems.

Suppose three fractional-order complex hyperchaotic leader systems are

$$
\begin{cases}
\dfrac{d^{0.95}x_{11}}{dt^{0.95}} = 15\left(x_{12}-x_{11}\right) + jx_{14} \\
\dfrac{d^{0.95}x_{12}}{dt^{0.95}} = 36x_{11} - x_{12} - x_{11}x_{13} + jx_{14} \\
\dfrac{d^{0.95}x_{13}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{11}x_{12} + x_{11}\overline{x}_{12}\right) - 4.5x_{13} \\
\dfrac{d^{0.95}x_{14}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{11}x_{12} + x_{11}\overline{x}_{12}\right) - 12x_{14}
\end{cases} \tag{35}
$$

$$
\begin{cases}
\dfrac{d^{0.95}x_{21}}{dt^{0.95}} = 15\left(x_{22}-x_{21}\right) + jx_{24} \\
\dfrac{d^{0.95}x_{22}}{dt^{0.95}} = 36x_{21} - x_{22} - x_{21}x_{23} + jx_{24} \\
\dfrac{d^{0.95}x_{23}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{21}x_{22} + x_{21}\overline{x}_{22}\right) - 4.5x_{23} \\
\dfrac{d^{0.95}x_{24}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{21}x_{22} + x_{21}\overline{x}_{22}\right) - 12x_{24}
\end{cases} \tag{36}
$$

$$
\begin{cases}
\dfrac{d^{0.95}x_{31}}{dt^{0.95}} = 15\left(x_{32}-x_{31}\right) + jx_{34} \\
\dfrac{d^{0.95}x_{32}}{dt^{0.95}} = 36x_{31} - x_{32} - x_{31}x_{33} + jx_{34} \\
\dfrac{d^{0.95}x_{33}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{31}x_{32} + x_{31}\overline{x}_{32}\right) - 4.5x_{33} \\
\dfrac{d^{0.95}x_{34}}{dt^{0.95}} = \dfrac{1}{2}\left(\overline{x}_{31}x_{32} + x_{31}\overline{x}_{32}\right) - 12x_{34}
\end{cases} \tag{37}
$$

where $x_{11} = x_{11}^{r} + jx_{11}^{i}$, $x_{12} = x_{12}^{r} + jx_{12}^{i}$, $x_{21} = x_{21}^{r} + jx_{21}^{i}$, $x_{22} = x_{22}^{r} + jx_{22}^{i}$, $x_{31} = x_{31}^{r} + jx_{31}^{i}$ $x_{32} = x_{32}^{r} + jx_{32}^{i}$ are complex variables, x_{13}, x_{14}, x_{23} , x_{24}, x_{33}, x_{34} are real variables.

Suppose the two integer order complex hyperchaotic follower systems with nonlinear controllers are

$$
\begin{cases}
\dot{y}_{11} = 15\left(y_{12}-y_{11}\right) + jy_{14} + u_{11} \\
\dot{y}_{12} = 36y_{11} - y_{12} - y_{11}y_{13} + jy_{14} + u_{12} \\
\dot{y}_{13} = \dfrac{1}{2}\left(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}\right) - 4.5y_{13} + u_{13} \\
\dot{y}_{14} = \dfrac{1}{2}\left(\overline{y}_{11}y_{12} + y_{11}\overline{y}_{12}\right) - 12y_{14} + u_{14}
\end{cases} \tag{38}
$$

$$\begin{cases}\dot{y}_{21}=15\left(y_{22}-y_{21}\right)+jy_{24}+u_{21}\\ \dot{y}_{22}=36y_{21}-y_{22}-y_{21}y_{23}+jy_{24}+u_{22}\\ \dot{y}_{23}=\dfrac{1}{2}\left(\overline{y}_{21}y_{22}+y_{21}\overline{y}_{22}\right)-4.5y_{23}+u_{23}\\ \dot{y}_{24}=\dfrac{1}{2}\left(\overline{y}_{21}y_{22}+y_{21}\overline{y}_{22}\right)-12y_{24}+u_{24}\end{cases} \tag{39}$$

, where $y_{11}=y_{11}^{r}+jy_{11}^{i}$, $y_{12}=y_{12}^{r}+jy_{12}^{i}$, $y_{21}=y_{21}^{r}+jy_{21}^{i}$, $y_{22}=y_{22}^{r}+jy_{22}^{i}$ are complex variables, $y_{13},y_{14},y_{23},y_{24}$ are real variables.

Arbitrary switched error states are selected as follows:

$$\begin{cases}e_{14324}=a_1y_{11}+b_4y_{24}+c_3x_{13}\left(p_2x_{22}+q_4x_{34}\right)\\ e_{23113}=a_2y_{12}+b_3y_{23}+c_1x_{11}\left(p_1x_{21}+q_3x_{33}\right)\\ e_{32421}=a_3y_{13}+b_2y_{22}+c_4x_{14}\left(p_2x_{22}+q_1x_{31}\right)\\ e_{41123}=a_4y_{14}+b_1y_{21}+c_1x_{11}\left(p_2x_{22}+q_3x_{33}\right)\end{cases} \tag{40}$$

Theorem 3. *If the controller according to Theorem 1 is designed to Eq. (41), the leader systems (35)-(37) and follower systems (38)-(39) can achieve CCCMSAS:*

$$\begin{cases}\sigma_1=-a_1\left(15\left(y_{12}-y_{11}\right)+jy_{14}+y_{11}\right)-b_4\left(0.5\left(\overline{y}_{21}y_{22}+y_{21}\overline{y}_{22}\right)-12y_{24}+y_{24}\right)\\ \qquad -c_3\dot{x}_{13}\left(p_2x_{22}+q_4x_{34}\right)-c_3x_{13}\left(p_2\dot{x}_{22}+q_4\dot{x}_{34}+p_2x_{22}+q_4x_{34}\right)\\ \sigma_2=-a_2\left(36y_{11}-y_{12}-y_{11}y_{13}+jy_{14}+y_{12}\right)-b_3\left(0.5\left(\overline{y}_{21}y_{22}+y_{21}\overline{y}_{22}\right)-4.5y_{23}+y_{23}\right)\\ \qquad -c_1\dot{x}_{11}\left(p_1x_{21}+q_3x_{33}\right)-c_1x_{11}\left(p_1\dot{x}_{21}+q_3\dot{x}_{33}+p_1x_{21}+q_3x_{33}\right)\\ \sigma_3=-a_3\left(0.5\left(\overline{y}_{11}y_{12}+y_{11}\overline{y}_{12}\right)-4.5y_{13}+y_{13}\right)-b_2\left(36y_{21}-y_{22}-y_{21}y_{23}+jy_{24}+y_{22}\right)\\ \qquad -c_4\dot{x}_{14}\left(p_2x_{22}+q_1x_{31}\right)-c_4x_{14}\left(p_2\dot{x}_{22}+q_1\dot{x}_{31}+p_2x_{22}+q_1x_{31}\right)\\ \sigma_4=-a_4\left(0.5\left(\overline{y}_{11}y_{12}+y_{11}\overline{y}_{12}\right)-12y_{14}+y_{14}\right)-b_1\left(15\left(y_{22}-y_{21}\right)+jy_{24}+y_{21}\right)\\ \qquad -c_1\dot{x}_{11}\left(p_2x_{22}+q_3x_{33}\right)-c_1x_{11}\left(p_2\dot{x}_{22}+q_3\dot{x}_{33}+p_2x_{22}+q_3x_{33}\right)\end{cases}$$

(41)

, where $\sigma_1=a_1u_{11}+b_4u_{24},\sigma_2=a_2u_{12}+b_3u_{23},\sigma_3=a_3u_{13}+b_2u_{22},\sigma_4=a_4u_{14}+b_1u_{21}$.

The initial states of the leader systems (35)-(37) and the follower systems (38), (39) are randomly taken as $\left(x_{11},x_{12},x_{13},x_{14}\right)^T=\left(12+10j,2+7j,9,10\right)^T$, $\left(x_{21},x_{22},x_{23},x_{24}\right)^T=\left(10+12j,5+6j,4,8\right)^T$, $\left(x_{31},x_{32},x_{33},x_{34}\right)^T=\left(8+7j,6+8j,7,8\right)^T$, $\left(y_{11},y_{12},y_{13},y_{14}\right)^T=\left(12+10j,2+7j,9,10\right)^T$, $\left(y_{21},y_{22},y_{23},y_{24}\right)^T=\left(8+7j,6+8j,7,8\right)^T$.

The matrices elements of A,B,C,P,Q are selected as $a_1 = 1 + j$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $b_1 = 1 + j$, $b_2 = 1$, $b_3 = 1$, $b_4 = 1$, $c_1 = 1 + j$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1$, $p_1 = 1 + j$, $p_2 = 1$, $p_3 = 1$, $p_4 = 1$, $q_1 = 1 + j$, $q_2 = 1$, $q_3 = 1$, $q_4 = 1$. The initial values for the CCCMSAS errors $\left(e_{14324}, e_{23113}, e_{32421}, e_{41123}\right)^T$ are $\left(127 + 76j, -467 + 161j, 45 + 228j, -97 + 511j\right)^T$. Figure 13 shows the time response of complex compound-combination multi switching anti-synchronization errors $e_{14324}, e_{23113}, e_{32421}$ and e_{41123} between the leader systems (35)-(37) and follower systems(38)-(39), where the curves of blue, black , red and green represent error states $e_{14324}, e_{23113}, e_{32421}$, e_{41123} and solid and dotted lines are depicted as the real parts and imaginary parts of $e_{14324}, e_{23113}, e_{32421}$, e_{41123} respectively. We can see that CCCMSAS errors $e_{14324}, e_{23113}, e_{32421}$ and e_{41123} progressively converge to 0. Figs.14-21 indicate the time response of the real parts and imaginary parts of the complex compound-combination multi switching anti-synchronization synchronized states $a_1 y_{11} + b_4 y_{24}$ and $c_3 x_{13}\left(p_2 x_{22} + q_4 x_{34}\right)$, $a_2 y_{12} + b_3 y_{23}$ and $c_1 x_{11}\left(p_1 x_{21} + q_3 x_{33}\right)$, $a_3 y_{13} + b_2 y_{22}$ and $c_4 x_{14}\left(p_2 x_{22} + q_1 x_{31}\right)$, $a_4 y_{14} + b_1 y_{21}$ and $c_1 x_{11}\left(p_2 x_{22} + q_3 x_{33}\right)$ of the leader systems (35)-(37) and follower systems (38)-(39) respectively. Sure enough, the simulation results indicate that CCCMSAS has been accomplished well among five complex hyperchaotic systems (35)-(37) and (38)-(39), which accord with the theoretical analysis and further illustrates the feasibility of proposed nonlinear controller.

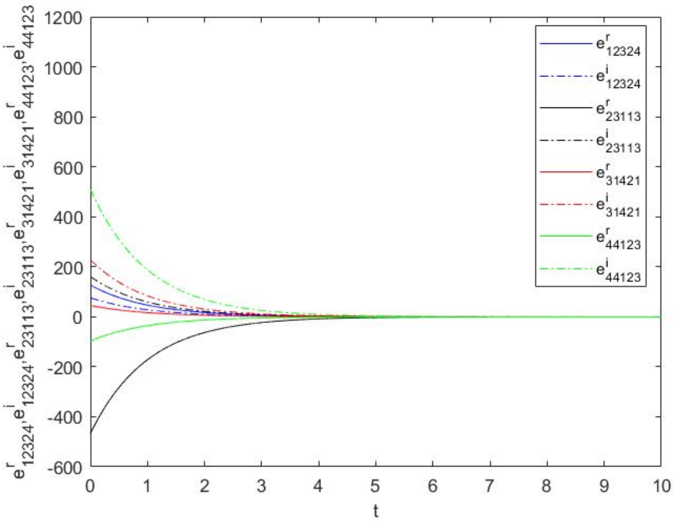

FIGURE 13 CCCMSAS errors $e_{14324}, e_{23113}, e_{32421}, e_{41123}$ between the leader systems (35)-(37) and follower systems (38)-(39), where the full lines represent real parts of $e_{14324}, e_{23113}, e_{32421}, e_{41123}$ and the dash lines are depicted as imaginary parts of errors $e_{14324}, e_{23113}, e_{32421}, e_{41123}$.

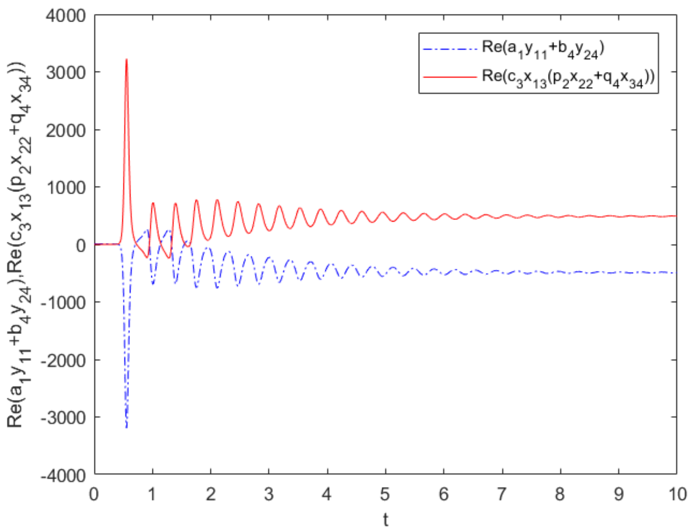

FIGURE 14 Response for the real parts of the states $a_1 y_{11} + b_4 y_{24}$ and $c_3 x_{13}\left(p_2 x_{22} + q_4 x_{34}\right)$, where

$$\mathrm{Re}\left(a_1 y_{11} + b_4 y_{24}\right) = y_{11}^r - y_{11}^i + y_{24}, \mathrm{Re}\left(c_3 x_{13}\left(p_2 x_{22} + q_4 x_{34}\right)\right) = x_{13}\left(x_{22}^r + x_{34}\right).$$

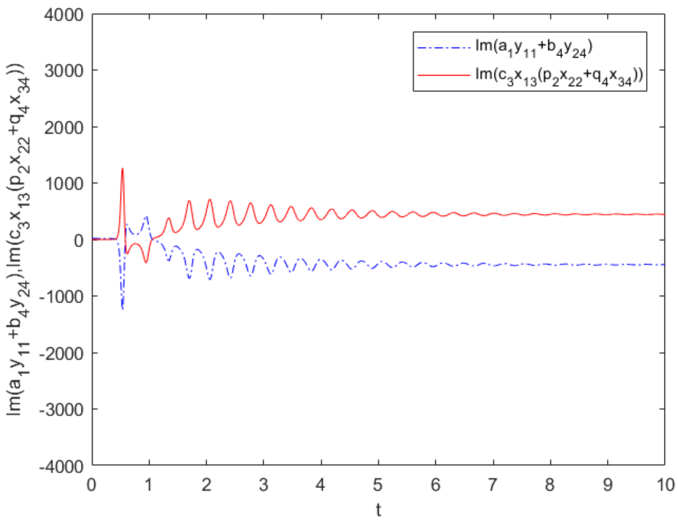

FIGURE 15 Response for the imaginary parts of the states $a_1y_{11}+b_4y_{24}$ and $c_3x_{13}\left(p_2x_{22}+q_4x_{34}\right)$, where $\operatorname{Im}\left(a_1y_{11}+b_4y_{24}\right)=y_{11}^r+y_{11}^i$, $\operatorname{Im}\left(c_3x_{13}\left(p_2x_{22}+q_4x_{34}\right)\right)=x_{13}x_{22}^i$.

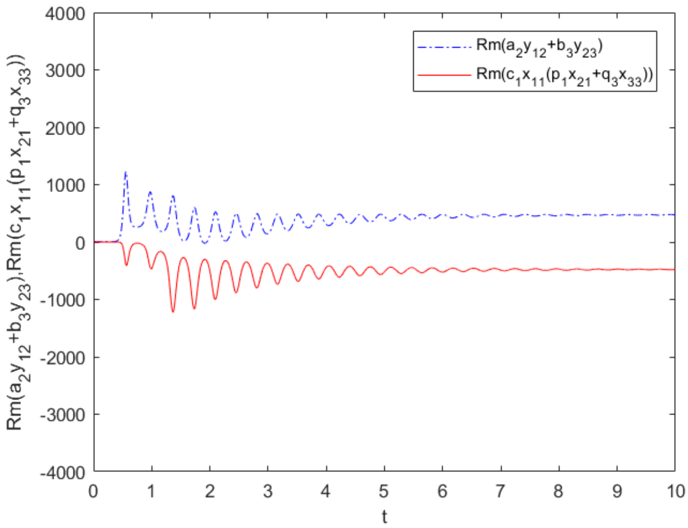

FIGURE 16 Response for the real parts of the states $a_2y_{12}+b_3y_{23}$ and $c_1x_{11}\left(p_1x_{21}+q_3x_{33}\right)$, where $\operatorname{Re}\left(a_2y_{12}+b_3y_{23}\right)=y_{12}^r+y_{23}$, $\operatorname{Re}\left(c_1x_{11}\left(p_1x_{21}+q_3x_{33}\right)\right)=x_{11}^r\left(x_{33}-2x_{21}^i\right)-x_{11}^i\left(x_{33}+2x_{21}^r\right)$.

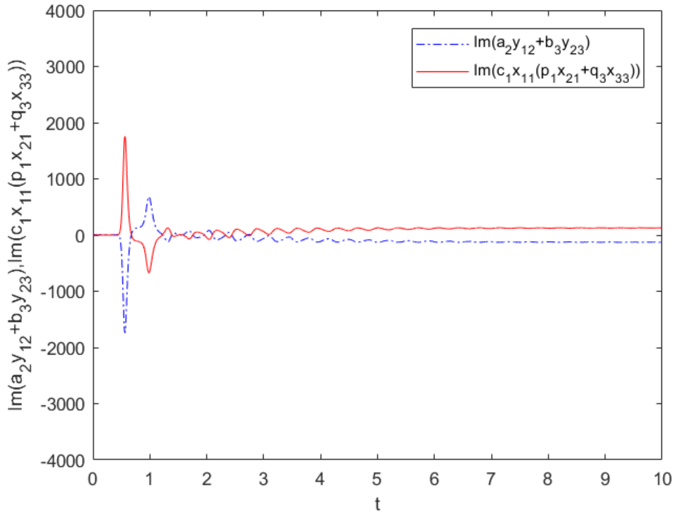

FIGURE 17 Response for the imaginary parts of the states $a_2y_{12}+b_3y_{23}$ and $c_1x_{11}\left(p_1x_{21}+q_3x_{33}\right)$, where $\operatorname{Im}\left(a_2y_{12}+b_3y_{23}\right)=y_{12}^i$, $\operatorname{Im}\left(c_1x_{11}\left(p_1x_{21}+q_3x_{33}\right)\right)=x_{11}^r\left(x_{33}+2x_{21}^r\right)+x_{11}^i\left(x_{33}-2x_{21}^i\right)$.

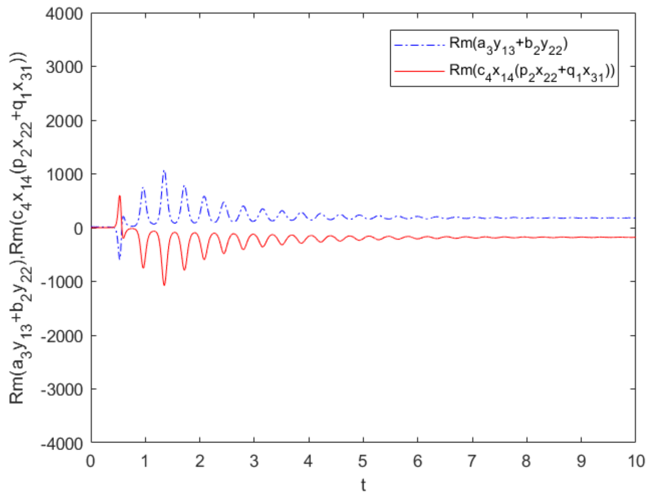

FIGURE 18 Response for the real parts of the states $a_3y_{13}+b_2y_{22}$ and $c_4x_{14}\left(p_2x_{22}+q_1x_{31}\right)$, where

$$\operatorname{Re}\left(a_3y_{13}+b_2y_{22}\right)=y_{13}+y_{22}^r,\ \operatorname{Re}\left(c_4x_{14}\left(p_2x_{22}+q_1x_{31}\right)\right)=x_{14}\left(x_{22}^r+x_{31}^r-x_{31}^i\right).$$

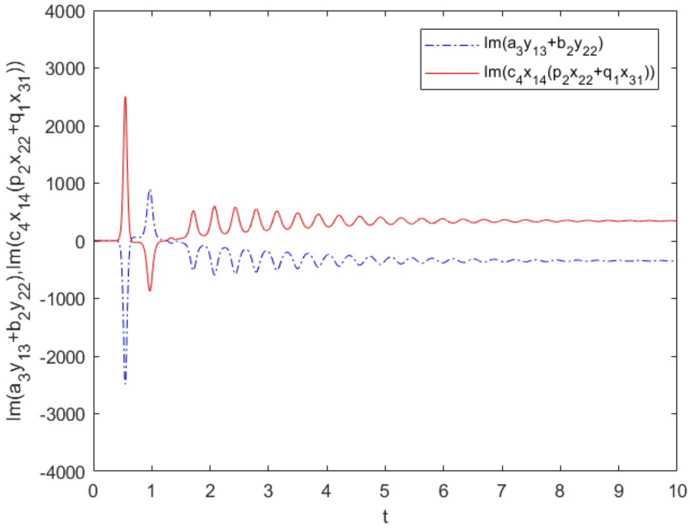

FIGURE 19 Response for the imaginary parts of the states $a_3y_{13}+b_2y_{22}$ and $c_4x_{14}\left(p_2x_{22}+q_1x_{31}\right)$, where $\mathrm{Im}\left(a_3y_{13}+b_2y_{22}\right)=y^{i}_{22}$, $\mathrm{Im}\left(c_4x_{14}\left(p_2x_{22}+q_1x_{31}\right)\right)=x_{14}\left(x^{i}_{22}+x^{r}_{31}+x^{i}_{31}\right)$.

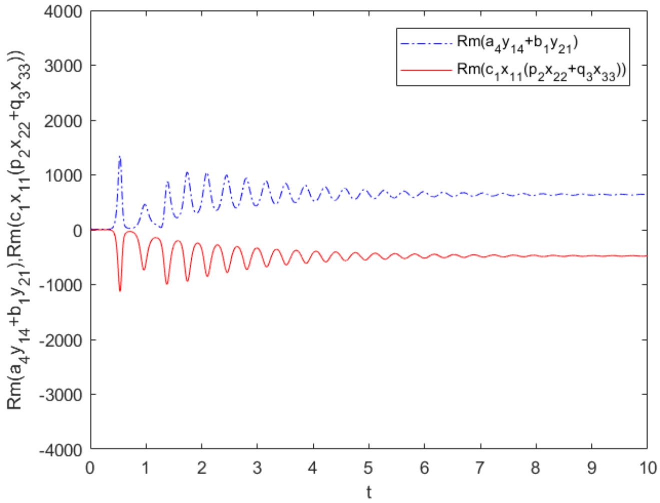

FIGURE 20 Response for the real parts of the states $a_4y_{14}+b_1y_{21}$ and $c_1x_{11}\left(p_2x_{22}+q_3x_{33}\right)$, where $\mathrm{Re}\left(a_4y_{14}+b_1y_{21}\right)=y_{14}+y^{r}_{21}-y^{i}_{21}$, $\mathrm{Re}\left(c_1x_{11}\left(p_2x_{22}+q_3x_{33}\right)\right)=\left(x^{r}_{11}-x^{i}_{11}\right)\left(x^{r}_{22}+x_{33}\right)-x^{i}_{22}\left(x^{r}_{11}+x^{i}_{11}\right)$.

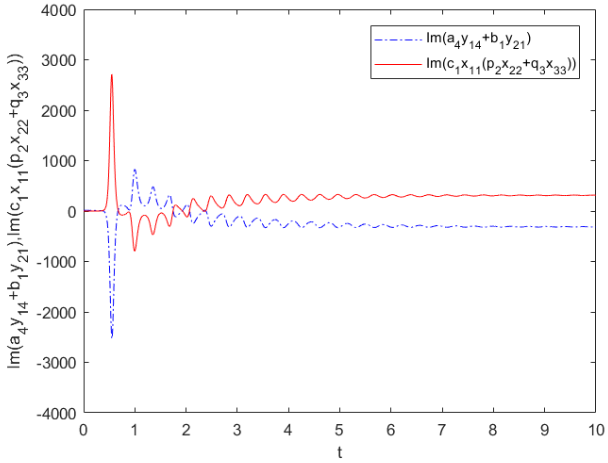

FIGURE 21 Response for the imaginary parts of the states $a_4y_{14}+b_1y_{21}$ and $c_1x_{11}\left(p_2x_{22}+q_3x_{33}\right)$,

where $\operatorname{Im}\left(a_4y_{14}+b_1y_{21}\right)=y_{22}^r+y_{22}^i$, $\operatorname{Im}\left(c_1x_{11}\left(p_2x_{22}+q_3x_{33}\right)\right)=\left(x_{11}^r+x_{11}^i\right)\left(x_{22}^r+x_{33}\right)+x_{22}^i\left(x_{11}^r-x_{11}^i\right)$.

5 | CONCLUSIONS

In this article, we provide CCCMSAS among three fractional-order complex chaotic systems and two integer order complex chaotic systems. The proposed scheme extends the scale factors of multi-switch synchronization from the real space to the complex space, and is distinct in research objects from the reported multi-switch papers is that the leader and follower systems of this article are three fractional-order chaotic systems and two integer-order complex chaotic systems, respectively. This scheme may supply better security and greater versatility in secure communication. Complex universal controllers are designed to lead to compound-combination complex anti-synchronization in the form of multiswitching. With the view of to perform the correctness and validity of the proposed complex compound-combination multi switching anti-synchronization scheme in Sect.2, two examples are accomplished well between different 3D fractional-order complex chaotic leader systems and 3D integer-order complex chaotic follower systems as well as hyperchaotic complex fractional-order chaotic systems and hyperchaotic integer-order complex chaotic systems, respectively. The facts of the availability and reliability of the method have been triumphantly confirmed by the graphical presentations. This scheme not only greatly improving the security of communication process information transmission, but also has potential advantages in realizing intelligent synchronization.

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