Analysis of an SEIR epidemic model with the effects of awareness programs

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Abstract

In this paper, a new SEIR model incorporating the effects of awareness programs on the epidemic spreading is analyzed. Two types of equilibria and the basic reproduction number of the model are given, and an algebraic approach is used to prove the global stability of the equilibria. Then the sensitivity analysis of the basic reproduction number and endemic equilibrium is performed. Moreover, the effects of awareness and media parameters on system dynamics are analyzed. Finally, we conduct numerical simulation to verify the analytical results.

Keywords: Awareness programs; Epidemic; Global stability; Sensitivity index

1 Introduction

In recent decades, the infectious diseases have become a great threat to the global personal and property security. Public health services always search for new ways to reduce the spread of infectious diseases, such as vaccination, isolation, treatment, and so on. Among these new ways, vaccination is one of the most effective public policies to prevent the transmission of epidemic [1]. But some vaccines are not completely effective. Even, there are some diseases that do not have vaccines [2]. For example, Corona virus disease 2019 is an infectious disease associated with the fever and pneumonia, it is considered as the biggest global threat worldwide because of tens of millions of confirmed infections, accompanied by hundreds of thousands of deaths over the world. [3]. The limitation of the medical resources, the absence of specific therapeutic treatment and effective vaccine, all make it more difficult to curb the transmission of the COVID-19 [4]. In this case, it is very important to educate people about the disease prevention via various media (e.g., TV, newspaper, social networking sites), this usually results in people raising the awareness of the epidemic and take a series of protective measures to lower their probability of becoming infected.

Many mathematical models are proposed to study the effects of awareness on epidemics. These models can be divided into two major classes: network-based models and mean-field models. There are two ways to explain the effects of awareness: (i) By changing the rate of diseases transmission and taking preventive measures [5-9]. Kiss et al. [5] extended a simple SIRS model to account for the treatment class. They proved that the awareness of the whole population could reduce the prevalence of infection. An SIRS model that considering the effects

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of private and public awareness on epidemic was studied [6,7]. (ii) By introducing a mass media compartment to represent the public interaction with mass media $[10-16]$. Greenhalgh et al. [11] presented a brief and nice commentary on the literature related to awareness and their effects on the dynamics of infectious diseases. Xiao et al. [12] quantified and evaluated the media impacts on the control of emerging infectious diseases, they modeled such media impacts using a piecewise smooth function depending on both the case number and its rate of change. Zhou et al. [15] considered that optimal media reporting intensity on mitigating spread of an emerging infectious disease. They formulate the novel media function $f(I, M, \alpha_1, \alpha_2)$ with $\alpha_i (i = 1, 2)$ denoting the weight of infected humans and media reports.

In this paper, on the basis of [7] and [16], we develop a new SEIR model to analysis the effects of awareness programs on epidemic. In this paper, the unaware humans are the humans without disease awareness or the humans who have disease awareness but do not take effective protective measures. Since people will get the progress of the epidemic through the media, when the new cases reported by the media decrease, some aware individuals may no longer take protective measures, thus becoming unaware individuals. Therefore, the loss of awareness is related to media reports. Further, we assume that the increase of infected and exposed individuals will have an impact on media reports [16]. In addition, we assume that the medical personals and hospitalized humans have taken necessary protective items. That is, only exposed and infected humans can spread diseases, the hospitalized humans can not spread the virus. The birth and death of humans are considered in our model, so the total population is not constant.

The reminder of the paper is organized as follows. The SEIR epidemic model is established in Section 2. The basic reproduction number and the stability analysis of equilibria are given in Section 3. The sensitivity analysis of basic reproduction number and endemic equilibrium are discussed, effects of parameters on system dynamics and the numerical simulations are performed in Section 4. We end with Section 5 of discussion.

2 The model description

In this section, we formulate an SEIR model based on some realistic assumptions. The total population $N(t)$ is sub-divided into six compartments: unaware susceptible (S), aware susceptible (S_a), exposed (E), infected (I), hospitalized (H) and recovered (R). M represents the cumulative density of awareness programs driven by the media reports. The transition relations of all compartments are summarised in Fig. 1. Dotted lines represent the effects of the exposed and infected humans on the awareness programs.

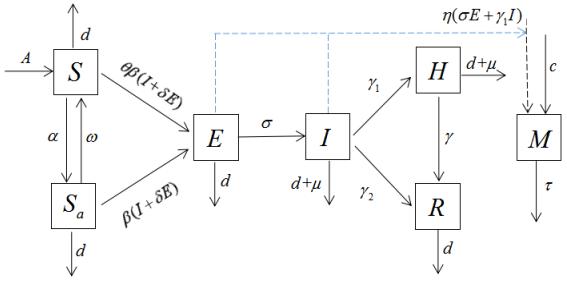


Fig. 1. Classes and transitions in the model.

Under the above assumptions, the relevant differential equations are

$$\begin{cases} \frac{dS}{dt} = A - \theta\beta S(I+\delta E) - \alpha SM + \omega S_a M - dS, \\ \frac{dS_a}{dt} = -\beta S_a(I+\delta E) + \alpha SM - \omega S_a M - dS_a, \\ \frac{dE}{dt} = \theta\beta S(I+\delta E) + \beta S_a(I+\delta E) - (d+\sigma)E, \\ \frac{dI}{dt} = \sigma E - (d+\mu+\gamma_1+\gamma_2)\, I, \\ \frac{dH}{dt} = \gamma_1 I - (\mu+d+\gamma)H, \\ \frac{dR}{dt} = \gamma_2 I + \gamma H - dR, \\ \frac{dM}{dt} = c + \eta(\sigma E + \gamma_1 I) - \tau M. \end{cases} \tag{2.1}$$

With initial conditions $S(0) > 0, S_a(0) > 0, E(0) > 0, I(0) > 0, H(0) > 0, R(0) > 0, M(0) > 0$. Note that

$$\frac{dN}{dt} = A - dN - \mu(I+H) \geq 0.$$

So we consider only solutions with initial conditions inside the biologically feasible region

$$\Gamma = \{(S, S_a, E, I, H, R, M) \in R_+^7 | S + S_a + E + I + H + R \leq \tfrac{A}{d}, 0 < M \leq \tfrac{c}{\tau}\}.$$

Here, A is the constant recruitment rate to the unaware susceptible humans, β is infection rate from aware susceptible to exposed humans, which is increased by the factors $\theta > 1$ for unaware susceptible humans, and the increase occurs due to their failure to take enough protective measures. δ is the relative infection rate of exposed to infected humans. Disease awareness is assumed to spread from the aware susceptible to the unaware susceptible humans at a rate α and to be lost at a rate ω. d is the natural death rate. $1/\sigma$ is the incubation period. μ is the disease induced death rate. γ_1 is the hospitalized rate for infected humans. γ_2 and γ are the cured rates for infected and hospitalized humans, respectively. c is the constant recruitment rate of awareness programs. τ represents the spontaneous disappearance rate of awareness programs.

Since the variables R and H do not appear in the other five equations in system (2.1), for convenience of calculation, the system (2.1) is simplified as follows

$$\begin{cases} \frac{dS}{dt} = A - \theta\beta S(I+\delta E) - \alpha SM + \omega S_a M - dS, \\ \frac{dS_a}{dt} = -\beta S_a(I+\delta E) + \alpha SM - \omega S_a M - dS_a, \\ \frac{dE}{dt} = \theta\beta S(I+\delta E) + \beta S_a(I+\delta E) - (d+\sigma)E, \\ \frac{dI}{dt} = \sigma E - (d+\mu+\gamma_1+\gamma_2)\, I, \\ \frac{dM}{dt} = c + \eta(\sigma E + \gamma_1 I) - \tau M. \end{cases} \tag{2.2}$$

3 Analysis of the model

3.1 Disease-free equilibrium and basic reproduction number

It is easy to obtain the system (2.2) has a disease-free equilibrium

$$P^0 = \left(S^0, S_a^0, E^0, I^0, M^0\right) = \left(\tfrac{(\omega c+d\tau)A}{d(\alpha c+\omega c+d\tau)}, \tfrac{\alpha cA}{d(\alpha c+\omega c+d\tau)}, 0, 0, \tfrac{c}{\tau}\right).$$

As we all know, the transmissibility of a virus at the initial stage of an epidemic is measured by the basic reproduction number R_0, which measures the average number of new infections generated by one infected individual in the population during the average infection period. According to the concepts of next generation matrix and the basic reproduction number presented in [17], we define

$$F = \begin{bmatrix} \theta\beta\delta S + \beta\delta S_a & \theta\beta S + \beta S_a \\ 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} d+\sigma & 0 \\ -\sigma & d+\mu+\gamma_1+\gamma_2 \end{bmatrix},$$

$$FV^{-1}\left(P^0\right) = \begin{bmatrix} \dfrac{\theta\beta\delta S^0 + \beta\delta S_a^0}{d+\sigma} + \dfrac{(\theta\beta S^0 + \beta S_a^0)\sigma}{(d+\sigma)(d+\mu+\gamma_1+\gamma_2)} & \dfrac{\beta S^0\theta + \beta S_a^0}{d+\mu+\gamma_1+\gamma_2} \\ 0 & 0 \end{bmatrix}.$$

So a straightforward calculation of $\rho(FV^{-1})$ gives that the basic reproduction number of disease is R_0 and

$$R_0 = \frac{\beta A((d+\mu+\gamma_1+\gamma_2)\delta+\sigma)(d\tau\theta + c\omega\theta + c\alpha)}{d(d\tau + c\alpha + c\omega)(d+\sigma)\,(d+\mu+\gamma_1+\gamma_2)}.$$

3.2 Existence of endemic equilibrium

Theorem 3.1 *If $R_0 > 1$, there exists a unique endemic equilibrium for system (2.2), which is recorded as $P^* = (S^*, S_a^*, E^*, I^*, M^*)$.*

Proof. It is easy to obtain the endemic equilibrium P^* satisfies

$$\begin{cases} A - \theta\beta S(I+\delta E) - \alpha SM + \omega S_a M - dS = 0, \\ -\beta S_a(I+\delta E) + \alpha SM - \omega S_a M - dS_a = 0, \\ \theta\beta S(I+\delta E) + \beta S_a(I+\delta E) - (d+\sigma)E = 0, \\ \sigma E - (d+\mu+\gamma_1+\gamma_2)\,I = 0, \\ c + \eta(\sigma E + \gamma_1 I) - \tau M = 0. \end{cases} \tag{3.1}$$

From the 4th and 5th equations of system (3.1), obtain

$$E^* = \frac{(d+\mu+\gamma_1+\gamma_2)\,I^*}{\sigma}, \quad M^* = \frac{(d+\mu+\gamma_1+\gamma_2)\,I^*\eta + \eta\gamma_1 I^* + c}{\tau}. \tag{3.2}$$

Then substituting (3.2) into the third equation of system (3.1) and obtain

$$S_a^* = \frac{((\beta\tau\theta(\delta T+\sigma) + \alpha\eta\sigma(T+\gamma_1))I^* + \sigma(\alpha c + d\tau))S^* - A\sigma\tau}{\sigma\omega\eta I^*(T+\gamma_1) + \sigma\omega c}, \tag{3.3}$$

substituting (3.2) and (3.3) into the second equation of system (3.1) and obtain

$$S^* = \frac{\left(\left(\omega\left(T+\gamma_1\right)\eta\sigma + \tau\beta\sigma + \tau\beta\delta T\right)I^* + \sigma(\omega c + d\tau)\right)(d+\sigma)T}{\left(\left(\left(T+\gamma_1\right)(\omega\theta+\alpha)\eta\sigma + \theta\tau\beta\sigma + \theta\tau\beta\delta T\right)I^* + \sigma(\theta d\tau + c\omega\theta + c\alpha)\right)\beta(\delta T+\sigma)}. \tag{3.4}$$

Then

$$S_a^* + S^* = \frac{b_1 I^* + b_2}{c_1 I^* + c_2},$$

where

$$b_1 = ((T+\gamma_1)(\omega+\alpha)\eta\sigma + \tau\beta\sigma + \tau\beta\delta T)(d+\sigma)T, \quad b_2 = (d\tau + c\omega + c\alpha)\sigma(d+\sigma)T,$$

$$c_1 = \beta\left(\left(T+\gamma_1\right)(\omega\theta+\alpha)\eta\sigma + \theta\tau\beta\sigma + \theta\tau\beta\delta T\right)(\delta T+\sigma), \quad c_2 = \beta\sigma(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma),$$

$$T = d + \gamma_1 + \gamma_2 + \mu.$$

From system (2.1), we have $S_a^* + S^* = \frac{A-(d+\sigma)E^*}{d}$, so $S_a^* + S^* = \frac{A\sigma-(d+\sigma)TI^*}{d\sigma}$. We denote

$$\begin{aligned} f(I) =& b_1 I + b_2 = b_1 I + (d\tau + c\omega + c\alpha)\sigma(d+\sigma)T, \\ g(I) =& \frac{A\sigma - (d+\sigma)TI}{d\sigma}(c_1 I + c_2) \\ =& -\frac{c_1(d+\sigma)T}{d\sigma}I^2 + \frac{Ac_1 - \beta(d+\sigma)(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma)T}{d}I + \frac{\beta A\sigma(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma)}{d}. \end{aligned}$$

Then at the endemic equilibrium we have

$$f(I) = g(I), \quad I \in \left(0, \frac{A\sigma}{T(d+\sigma)}\right). \tag{3.5}$$

We define

$$\begin{aligned} \varphi(I) =& f(I) - g(I) \\ =& \frac{c_1(d+\sigma)T}{d\sigma}I^2 + \left(b_1 - \frac{Ac_1 - \beta(d+\sigma)(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma)T}{d}\right)I \\ &+ (d\tau + c\omega + c\alpha)\sigma(d+\sigma)T - \frac{\beta A\sigma(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma)}{d}. \end{aligned}$$

Assuming that I_1^* and I_2^* are the roots of $\varphi(\lambda) = 0$ and obtain

$$\begin{aligned} I_1^* =& \frac{\sqrt{(Tc_2(d+\sigma) + (b_1 d + Ac_1)\sigma)^2 - 4c_1 d\sigma(Tb_2(d+\sigma) + Ab_1\sigma)} - Tc_2(d+\sigma) - b_1 d\sigma + Ac_1\sigma}{2c_1 T(d+\sigma)} \\ <& \frac{\sqrt{(Tc_2(d+\sigma) + (b_1 d + Ac_1)\sigma)^2} - Tc_2(d+\sigma) - b_1 d\sigma + Ac_1\sigma}{2c_1 T(d+\sigma)} < \frac{A\sigma}{T(d+\sigma)}, \\ I_2^* =& \frac{-\sqrt{(Tc_2(d+\sigma) + (b_1 d + Ac_1)\sigma)^2 - 4c_1 d\sigma(Tb_2(d+\sigma) + Ab_1\sigma)} - Tc_2(d+\sigma) - b_1 d\sigma + Ac_1\sigma}{2c_1 T(d+\sigma)}. \end{aligned}$$

According to the *Vieta* theorem,

$$I_1^* I_2^* = \frac{d\sigma^2 T(d+\sigma)(d\tau + c\omega + c\alpha) - \beta A\sigma^2(\theta d\tau + c\omega\theta + c\alpha)(\delta T+\sigma)}{c_1(d+\sigma)T} = \frac{d\sigma^2 T(d+\sigma)(d\tau + c\omega + c\alpha)}{c_1(d+\sigma)T}(1 - R_0).$$

So if $R_0 > 1$, then $I_1^* I_2^* < 0$, and there exists a unique positive root of equation $\varphi(I) = 0$, which is I_1^*. So there is only an endemic equilibrium P^* for system (2.2).

3.3 The stability of equilibria

Theorem 3.2 *The disease-free equilibrium P^0 is locally asymptotically stable if $R_0 < 1$ and it is unstable if $R_0 > 1$.*

Proof. The Jacobian matrix of system (2.2) at P^0 is

$$J\left(P^0\right) = \begin{pmatrix} -\frac{\alpha c}{\tau} - d & \frac{\omega c}{\tau} & -\theta\beta\delta S^0 & -\theta\beta S^0 & -\alpha S^0 + \omega S_a^0 \\ \frac{\alpha c}{\tau} & -\frac{\omega c}{\tau} - d & -\beta\delta S_a^0 & -\beta S_a^0 & \alpha S^0 - \omega S_a^0 \\ 0 & 0 & \beta\delta(\theta S^0 + S_a^0) - d - \sigma & \theta\beta S^0 + \beta S_a^0 & 0 \\ 0 & 0 & \sigma & -d - \mu - \gamma_1 - \gamma_2 & 0 \\ 0 & 0 & \eta\sigma & \eta\gamma_1 & -\tau \end{pmatrix}.$$

The characteristic equation of $J\left(P^0\right)$ is

$$\Phi(\lambda) = (\lambda + d)\left(\lambda + \tfrac{\omega c}{\tau} + \tfrac{\alpha c}{\tau} + d\right)(\lambda + \tau)(\lambda^2 + k_1\lambda + k_2) = 0,$$

where

$$k_1 = (d + \mu + \gamma_1 + \gamma_2) + (d + \sigma)\left(1 - \tfrac{\beta\delta(\theta S^0 + S_a^0)}{(d+\sigma)}\right), \quad k_2 = (d + \sigma)(d + \mu + \gamma_1 + \gamma_2)(1 - R_0).$$

where λ denotes the eigenvalue and

$$\lambda_1 = -d,\ \lambda_2 = -\left(\tfrac{\omega c}{\tau} + \tfrac{\alpha c}{\tau} + d\right),\ \lambda_3 = -\tau.$$

We define $g(\lambda) = \lambda^2 + k_1\lambda + k_2$, then λ_4 and λ_5 are the roots of equation $g(\lambda) = 0$. If $R_0 < 1$, then $\frac{\beta\delta(\theta S^0 + S_a^0)}{(d+\sigma)} < 1$, so $\lambda_4 + \lambda_5 = -k_1 < 0$, $\lambda_4\lambda_5 = k_2 > 0$. Then all roots of $\Phi(\lambda)$ have negative real parts. Hence, the disease-free equilibrium P^0 is locally asymptotically stable if $R_0 < 1$ and it is unstable if $R_0 > 1$.

Theorem 3.3 *If $R_0 < 1$, then the disease-free equilibrium P^0 is globally asymptotically stable.*

Proof. Motivated by the method in [18], consider a Lyapunov function defined by

$$V_1 = S - S^0 - S^0 \ln\frac{S}{S^0} + S_a - S_a^0 - S_a^0 \ln\frac{S_a}{S_a^0} + E + \frac{\beta(\theta S^0 + S_a^0)}{d + \mu + \gamma_1 + \gamma_2} I.$$

For simplicity, denote $X = \frac{S}{S^0}$, $Y = \frac{S_a}{S_a^0}$, $Z_1 = E$, $Z_2 = I$, $Z_3 = \frac{M}{M^0}$. So the derivative of function V_1 along solutions of system (2.2) is given by

$$\begin{aligned} \dot{V}_1(P^0) =& \dot{S}\left(1 - \frac{1}{X}\right) + \dot{S}_q\left(1 - \frac{1}{Y}\right) + \dot{E} + \frac{\beta(\theta S^0 + S_a^0)}{d + \mu + \gamma_1 + \gamma_2}\dot{I} \\ =& X\left(-dS^0\right) + \frac{1}{X}(-A) + Y\left(-dS_a^0\right) + \frac{YZ_3}{X}\left(-\omega S_a^0 M^0\right) + \frac{XZ_3}{Y}\left(-\alpha S^0 M^0\right) \\ &+ Z_1\left(\frac{\beta\sigma(\theta S^0 + S_a^0)}{d + \mu + \gamma_1 + \gamma_2} - (d + \sigma) + \beta\delta\left(\theta S^0 + S_a^0\right)\right) + Z_3\left(\alpha M^0 S^0 + \omega S_a^0 M^0\right) + A + dS^0 + dS_a^0. \end{aligned}$$

Denote

$$\begin{aligned}\dot{G}_1 =&B_1\left(X+\frac{1}{X}-2\right)+B_2\left(Z_3+\frac{1}{Z_3}-2\right)+B_3\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right)\\&+B_4\left(\frac{XZ_3}{Y}+\frac{YZ_3}{X}+\frac{1}{Z_3}+\frac{1}{Z_3}-4\right)+Z_1(d+\sigma)\left(R_0-1\right).\end{aligned}$$

Let $\dot{V}_1(P^0)=\dot{G}_1$, since their coefficients correspond to equal, we can obtain

$$\begin{cases}B_1=-dS^0=-\frac{(\omega c+d\tau)A}{(\alpha c+\omega c+d\tau)}, & B_2=\alpha M^0S^0+\omega S_a^0M^0=\frac{\alpha cA(d\tau+2\omega c)}{d\tau(\alpha c+\omega c+d\tau)},\\ B_3=-dS_a^0M^0=-\frac{\alpha c^2A}{\tau(\alpha c+\omega c+d\tau)}, & B_4=-\omega S_a^0M^0=-\frac{\alpha\omega c^2A}{\tau d(\alpha c+\omega c+d\tau)}.\end{cases}$$

Denote

$$\begin{cases}D_1=Z_3+\frac{1}{Z_3}-2, & D_2=\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4,\\ D_3=\frac{XZ_3}{Y}+\frac{YZ_3}{X}+\frac{1}{Z_3}+\frac{1}{Z_3}-4.\end{cases}$$

According to the properties of Mean Inequality, we can obtain $D_3\geq 2(Z_3+\frac{1}{Z_3}-2)$, so

$$\begin{aligned}\dot{V}_1(P^0)=&\dot{G}_1\leq B_1\left(X+\frac{1}{X}-2\right)+B_2\left(Z_3+\frac{1}{Z_3}-2\right)+B_3\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right)\\&+2B_4\left(Z_3+\frac{1}{Z_3}-2\right)+Z_1(d+\sigma)\left(R_0-1\right)\\=&-\frac{(\omega c+d\tau)A}{(\alpha c+\omega c+d\tau)}\left(X+\frac{1}{X}-2\right)+\frac{\alpha cA}{(\alpha c+\omega c+d\tau)}\left(Z_3+\frac{1}{Z_3}-2\right)\\&+B_3\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right)+Z_1(d+\sigma)\left(R_0-1\right)\\=&-\frac{((\omega-\alpha)c+d\tau)A}{(\alpha c+\omega c+d\tau)}\left(X+\frac{1}{X}-2\right)+B_3\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right)+Z_1(d+\sigma)\left(R_0-1\right)\end{aligned}$$

Since when $R_0<1$, the disease does not spread, so human's protection measures are reduced, that is the awareness spread rate α is less than the awareness loss rate ω [7], which is $\omega-\alpha>0$. So $\dot{V}_1\leq 0$. $\dot{V}_1=0$ if and only if $X=\frac{1}{X}=\frac{1}{Z_3}=Y=1$, $Z_1=0$. That is $S=S^0$, $E=0$, $S_a=S_a^0$, $M=M^0$. That is $\{(S,S_a,E,I,M))\,|\dot{V}_1=0\}=\left\{P^0\right\}$. By LaSalle's Invariance Principle[19], the disease-free equilibrium P^0 is globally asymptotically stable.

Theorem 3.4 *If $R_0>1$, then the endemic equilibrium P^* is globally asymptotically stable.*

Proof. [See Appendix A] Consider a Lyapunov function defined by

$$\begin{aligned}V_2=&S-S^*-S^*\ln\frac{S}{S^*}+S_a-S_a^*-S_a^*\ln\frac{S_a}{S_a^*}+E-E^*-E^*\ln\frac{E}{E^*}\\&+\frac{\beta(\theta S^*+S_a^*)}{d+\mu+\gamma_1+\gamma_2}(I-I^*-I^*\ln\frac{I}{I^*}).\end{aligned}$$

For simplicity, denote $X = \frac{S}{S^*}$, $Y = \frac{S_a}{S_a^*}$, $Z_1 = \frac{E}{E^*}$, $Z_2 = \frac{I}{I^*}$, $Z_3 = \frac{M}{M^*}$. So the derivative of function V_2 along solutions of system (2.2) is given by

$$\begin{aligned}
\dot{V}_2(P^*) =& \dot{S}\left(1-\frac{1}{X}\right)+\dot{S}_a\left(1-\frac{1}{Y}\right)+\dot{E}\left(1-\frac{1}{Z_1}\right)+\frac{\beta(\theta S^*+S_a^*)}{d+\mu+\gamma_1+\gamma_2}\dot{I}\left(1-\frac{1}{Z_2}\right)\\
=& X(-dS^*-\beta\delta\theta E^*S^*)+\frac{1}{X}(-A)+Y\left(-\beta\delta E^*S_a^*-dS_a^*\right)+\frac{YZ_3}{X}\left(-\omega S_a^*M^*\right)\\
&+\frac{XZ_3}{Y}\left(-\alpha S^*M^*\right)+Z_1\left(-(d+\sigma)E^*+\beta\delta E^*\left(\theta S^*+S_a^*\right)+\beta I^*\left(\theta S^*+S_a^*\right)\right)\\
&+Z_3\left(\alpha M^*S^*+\omega S_a^*M^*\right)+\frac{YZ_2}{Z_1}\left(-\beta I^*S_a^*\right)+\frac{Z_1}{Z_2}\left(-\beta I^*\left(\theta S^*+S_a^*\right)\right)\\
&+\frac{XZ_2}{Z_1}(-\beta\theta I^*S^*)+A+dS^*+dS_a^*+(d+\sigma)E^*+\beta I^*(\theta S^*+S_a^*).
\end{aligned}$$

Denote

$$\begin{aligned}
\dot{G}_2 =& B_1\left(X+\frac{1}{X}-2\right)+B_2\left(Z_3+\frac{1}{Z_3}-2\right)+B_3\left(\frac{Z_1}{Z_2}+\frac{XZ_2}{Z_1}+\frac{1}{X}-3\right)+B_4\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right)\\
&+B_5\left(\frac{XZ_3}{Y}+\frac{YZ_3}{X}+\frac{1}{Z_3}+\frac{1}{Z_3}-4\right)+B_6\left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+\frac{Z_1}{Z_2}+\frac{YZ_2}{Z_1}-5\right).
\end{aligned}$$

Let $\dot{V}_2(P^*) = \dot{G}_2$, since their coefficients correspond to equal, we can obtain

$$\begin{cases}
B_1 = -dS^*-\beta\delta\theta E^*S^*, & B_2 = \alpha M^*S^*+\omega S_a^*M^*,\\
B_3 = -\beta\theta I^*S^*, & B_4 = -(\beta\delta E^*+d)S_a^*,\\
B_5 = -\omega S_a^*M^*, & B_6 = -\beta I^*S_a^*.
\end{cases}$$

Denote

$$\begin{cases}
D_1 = Z_3+\frac{1}{Z_3}-2, & D_2 = \left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+Y-4\right),\\
D_3 = \left(\frac{XZ_3}{Y}+\frac{YZ_3}{X}+\frac{1}{Z_3}+\frac{1}{Z_3}-4\right), & D_4 = \left(\frac{XZ_3}{Y}+\frac{1}{X}+\frac{1}{Z_3}+\frac{Z_1}{Z_2}+\frac{YZ_2}{Z_1}-5\right),\\
D_{min} = min\{D_i\}, i = 1,2,4.
\end{cases}$$

According to the properties of Mean Inequality, we can obtain $D_3 \geq 2(Z_3+\frac{1}{Z_3}-2)$, so

$$\begin{aligned}
\dot{V}_2(P^*) = \dot{G}_2 \leq& B_1\left(X+\frac{1}{X}-2\right)+B_3\left(\frac{Z_1}{Z_2}+\frac{XZ_2}{Z_1}+\frac{1}{X}-3\right)+(B_2+B_4+2B_5+B_6)D_{min}\\
=& -\left(dS^*+\beta\delta\theta E^*S^*\right)\left(X+\frac{1}{X}-2\right)-\beta\theta I^*S^*\left(\frac{Z_1}{Z_2}+\frac{XZ_2}{Z_1}+\frac{1}{X}-3\right)\\
&+(\alpha M^*S^*+\omega S_a^*M^*-(\beta\delta E^*+d)S_a^*-2\omega S_a^*M^*-\beta I^*S_a^*)D_{min}\\
=& -\left(dS^*+\beta\delta\theta E^*S^*\right)\left(X+\frac{1}{X}-2\right)-\beta\theta I^*S^*\left(\frac{Z_1}{Z_2}+\frac{XZ_2}{Z_1}+\frac{1}{X}-3\right).
\end{aligned}$$

Since P^* exists under the condition $R_0 > 1$, so if $R_0 > 1$, then $\dot{V}_2 \leq 0$. $\dot{V}_2 = 0$ if and only if $X = \frac{1}{X}$, $Z_1 = \frac{1}{Z_1}$, $Z_2 = \frac{1}{Z_2}$. That is $S = S^*$, $E = E^*$, $I = I^*$. That is $\{(S, S_a, E, I, M))\,|\dot{V}_2 = 0\} = \{P^*\}$. By LaSalle's Invariance Principle[19], the endemic equilibrium P^* is globally asymptotically stable.

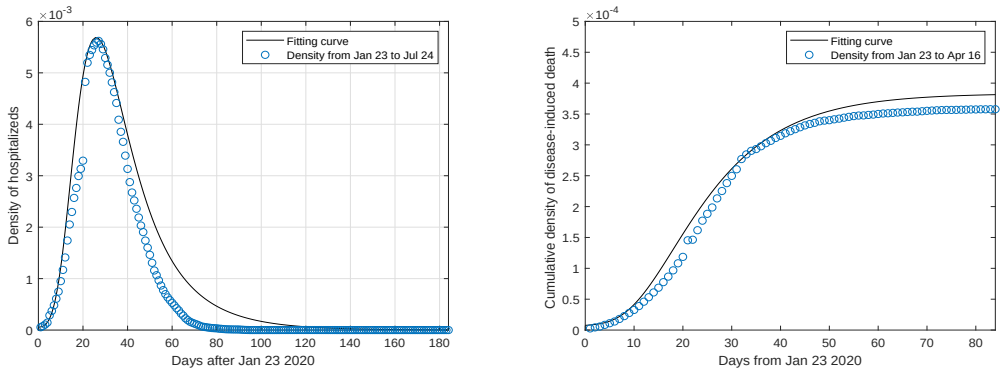
4 Numerical simulation

Since the data of the recent outbreak of COVID-19 is available, so the parameter selection in this paper is based on the parameter of disease COVID-19[20]. Since Wuhan implemented the strategy of closing the city on January 23, 2020, the data we fitted start from January 23, 2020. Data information includes the cumulative number of confirmed cases and the number of death cases, shown in Fig. 2.

Table 1. Estimated initial values of variables and parameters for system (2.2).

Parameters	Description	Value	Source
A	The constant recruitment rate to S	0.017	data
θ	Disease intensity increasing factor	75	[4]
β	Infection rate from S_a to E	0.027	[21,4]
$\theta\beta$	Infection rate from S to E	0.027	[21,4]
δ	Relative infection probability of E compared with I	0.23	[4]
α	Awareness spreading rate from S to S_a	0.152	Fitted
ω	Awareness losing rate from S_a to S	3×10^{-6}	Fitted
γ_1	Hospitalized rate of I to H	0.11	[16]
γ_2	Recovery rate of I	0.330	[16]
γ	Recovery rate of H	0.13	[16]
σ	Progression rate of E to I	0.2	[16]
d	Natural death rate	0.01	data
μ	Disease-induced death rate	0.003	[16]
c	Natural recruitment rate of media reports	0.8	[4]
η	Implementation rate affected by epidemic	2.951	[16]
τ	Depletion rate of media reports	0.735	[16]
Variables	Description	Initial quantity	Source
S	Unaware susceptible humans	9.00×10^6	[22]
S_a	Aware humans	2132	[22]
E	Exposed humans	4000	[22]
I	Infected humans	935	[22]
H	Hospitalized humans	494	[22]
R	Recovered humans	34	[22]
M	Media items	163	[16]

Accord to the data in Table 1, we calculate and assume that initial values of population density are $S(0) = 0.999156823$, $S_a(0)$=2.366894E-04, $E(0)$=4.4407E-04, $I(0)$=1.03801E-04, $H(0)$=5.48426E-05, $R(0)$=3.77459E-06, $M(0)$=1.80958E-05, respectively, that is, the initial values of total population is $N(0) = 1$. Then we fit the model to the daily data of confirmed cases and deaths as shown in Fig. 2. It is easy to see that some values on the fitting curve are larger than the actual values, the reason is that after a period of disease outbreak: (i)The hospitals had some treatment experiences, so the recovery rate of patients increased, (ii) The government actively adopts protective measures such as home isolation and disinfection, which reduces the infection rate, thus reducing the number of infected and hospitalized humans.



(a) (b)

Fig. 2. The black curves are the fitting curves, the blue circles represent (a) the density of confirmed cases per day from Jan 23 to Jul 24, (b) the cumulative density of disease-induced death from Jan 23 to Apr 16.

4.1 Sensitivity analysis based on perturbation of fixed point estimations

Understanding the relative importance of parameters is helpful to guide in developing of efficient intervention strategies in infectious disease areas where resources are scarce. Sensitivity analysis is commonly used to determine the robustness of system predictions to parameter values. In this section, sensitivity analysis using the fixed point estimation has been applied to determine the relative importance of different parameters responsible[23].

Definition. The normalized forward sensitivity index of a variable, h, that depends differentially on a parameter, m, is defined as:

$$\Upsilon_m = \Upsilon_m^h := \frac{\partial h}{\partial m} \times \frac{m}{h}. \tag{4.1}$$

4.1.1 Sensitivity analysis of R_0

Since the initial disease transmission is directly related to basic reproduction number R_0, thus, we calculate the sensitivity indexes of R_0 are given in Table 2.

Table 2. Sensitivity indexes of R_0

Parameters	Sensitivity indexes of R_0	Corresponding % changes[a]
A	$\Upsilon_A = +1.0000000000$	-1.00000000
θ	$\Upsilon_\theta = +0.8193245475$	-1.22051752
β	$\Upsilon_\beta = +1.0000000000$	-1.00000000
δ	$\Upsilon_\delta = +0.3425161904$	-2.91957000
α	$\Upsilon_\alpha = -0.7623081596$	$+1.32823580$
ω	$\Upsilon_\omega = +0.00000000001$	-4015.71245
γ_1	$\Upsilon_{\gamma_1} = -0.159653905$	$+6.36028540$
γ_2	$\Upsilon_{\gamma_2} = -0.478961716$	$+2.12009510$
σ	$\Upsilon_\sigma = -0.2948971420$	$+3.50418150$
μ	$\Upsilon_\mu = -0.0043541974$	$+233.210464$
c	$\Upsilon_c = -0.7620593242$	$+1.32867530$
τ	$\Upsilon_\tau = +0.7620593248$	-1.60513058

[a] The corresponding % changes needed to affect a 1% decrease in the value of R_0.

From Table 2, we note that the sensitivity index may depend on several parameters of the system, also can

be a constant. For example, $\Upsilon_A = \Upsilon_\beta = +1$ indicates that increasing (decreasing) A or β by a given percentage increases (decreases) always R_0 by that same percentage. Except Υ_A and Υ_β, other indexes are the functions of other parameters, the sensitivity indexes of R_0 will change with the changes in parameter values. The sensitivity indexes $\Upsilon_A, \Upsilon_\theta, \Upsilon_\beta, \Upsilon_\delta, \Upsilon_\omega$ and Υ_τ are positive impacts on R_0 and the rest of the parameters have negative impacts. That is, 1% increase in δ, resulting in 0.3425161904% increase in R_0, on the other hand 1% increase in α, will decrease R_0 by 0.7623081596%. It's easy to see parameters A, β, θ, α, τ and c are most sensitive to R_0, a small variation in these parameter will lead to relatively large quantitative change in R_0.

4.1.2 Sensitivity analysis of the P^*

Sensitivity analysis of the endemic equilibrium has been used to determine the relative importance of different parameters responsible for equilibrium disease prevalence[23,24]. So by carrying out similar argument to the R_0, obtain

Table 3. The sensitivity indexes of P^*

Parm	Sensitivity indexes				
	S	S_a	E	I	M
A	−0.013436645	−1.007742642	+0.523538652	+0.231142893	+0.522481371
θ	−0.9874059e-4	−0.001838875	+0.922673e-4	+0.407361e-4	+0.920810e-4
β	−0.243485264	−4.582431196	+0.229806228	+0.101459704	+0.229342137
δ	−0.013416143	−0.252494927	+0.012662422	+0.005590473	+0.012636851
α	−0.012016129	+0.901210758	−0.042342549	−0.018694282	−0.042257039
ω	+0.197431496	−14.90138000	+0.695710988	+0.307157169	+0.694306005
γ_1	+0.013066122	+0.246794533	−0.012374363	−0.005874713	−0.011782796
γ_2	+0.013078626	+0.245856439	−0.012330301	−0.005855260	−0.012487102
σ	+0.013281899	+0.250144895	−0.014553368	−0.005493466	−0.012417572
μ	+0.013078626	+0.245856439	−0.012330301	−0.005855260	−0.012487102
c	−0.002273524	+0.170513988	−0.008011467	−0.003537072	+0.128059134
τ	+0.002484162	−0.186311053	+0.008753715	+0.003864775	−0.139923580

The sensitivity indexes of endemic equilibrium $P^* = (0.09244, 1.5189, 0.004221, 0.001864, 0.001434)$, with respect to the twelve input parameters, are given in Table 1, in particular, here $\beta = 0.037$. From Table 3, the most sensitive parameter for S^* is β followed by $\omega, A, \delta, \sigma, \mu, \gamma_2, \gamma_1, \alpha, \tau, c, \theta$. The most sensitive parameter for S_a^* is ω followed by $\beta, A, \alpha, \delta, \sigma, \gamma_1, \gamma_2, \mu, \tau, c, \theta$. The most sensitive parameter for E^* is ω followed by $A, \beta, \alpha, \sigma, \delta, \gamma_1, \gamma_2, \mu, \tau, c, \theta$. The most sensitive parameter for I^* is ω followed by $A, \beta, \alpha, \gamma_1, \gamma_2, \mu, \delta, \sigma, \tau, c, \theta$. The most sensitive parameter for M^* is ω followed by $A, \beta, \tau, c, \alpha, \delta, \mu, \gamma_2, \sigma, \gamma_1, \theta$.

4.2 Effects of awareness programs on system dynamics

Since we are focusing on the awareness impacts on the transmission of epidemic, accord to the data in Table 1, we plotted the density of exposed($E(t)$), infected and hospitalized humans $(I(t) + H(t))$ with different values of the awareness parameters α, ω and media parameters c, η, τ, as shown in Fig. 3.

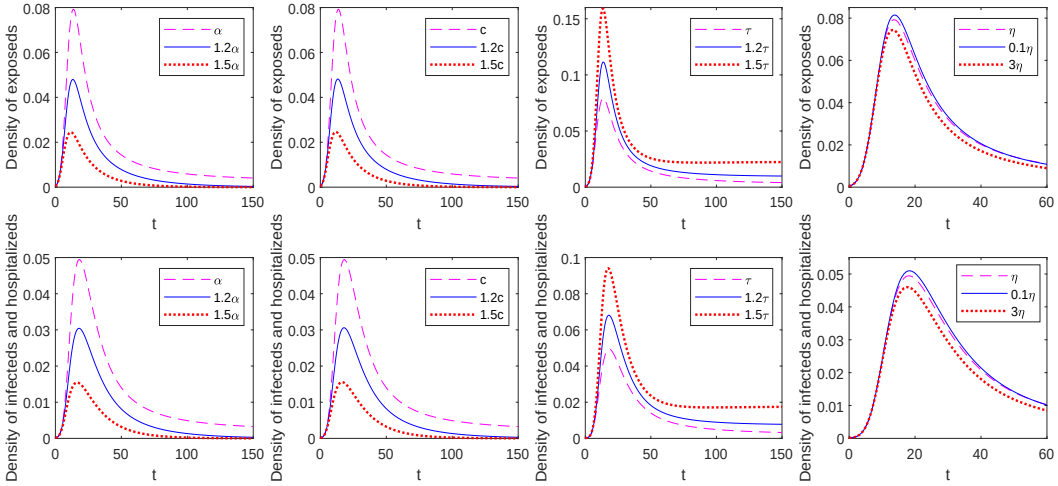


Fig. 3. The effects of α, c, τ and η on the density of exposed, infected and hospitalized humans.

Fig. 3 illustrates that increasing α, c and η by three times from its baseline value would lead to the peak time of humans advanced and decrease the peak size of humans. Otherwise, decreasing τ could lead to the peak time of humans advanced and decrease the peak size of humans.

4.3 Numerical verification of global stability of equilibria

In this section, we give simulation result for the exposed, infected and hospitalized humans by using the data in the Table 1. In particular, we take $\beta = 0.037$ in studying endemic equilibrium, and take $\beta = 0.027$ at the disease-free equilibrium.

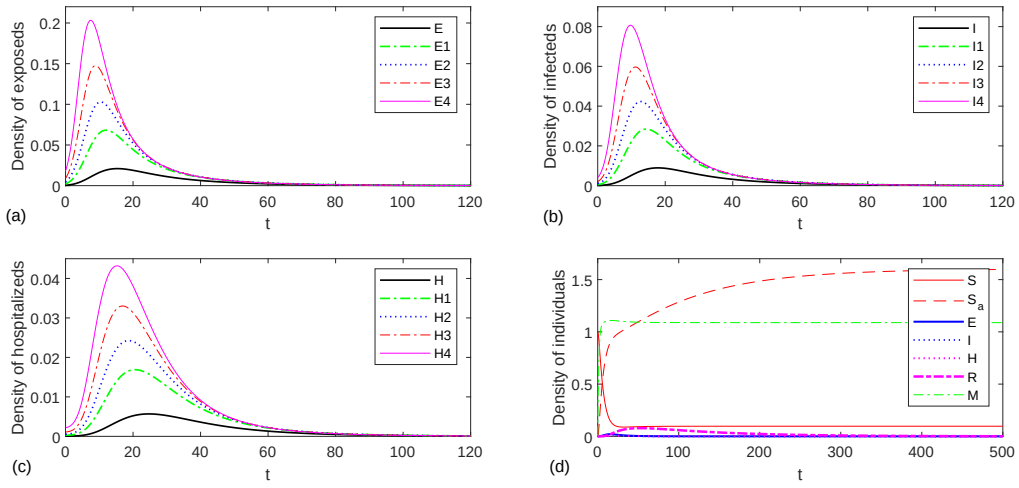


Fig. 4. Dynamical behavior around disease-free equilibrium P^0.

Fig. 4 depicts variations of the density of humans as a function of time. Fig. 4(a,b,c) show the trajectories

of E, I and H with different initial conditions $x = x(0)$, $x1 = 5x(0)$, $x2 = 10x(0)$, $x3 = 20x(0)$ and $x4 = 40x(0)$,($x = E$ or I or H). Fig. 4(d) depicts the global stability of disease-free equilibrium, and it is clear that all solutions converge to the DFE (P^0). In this case, $\beta = 0.027$, $R_0 = 0.7663$ and disease-free equilibrium $P^0 = (0.09692, 1.60307, 0, 0, 0, 1.08844)$. Similarly, for $\beta = 0.037$, $R_0 = 1.0497$ and endemic equilibrium $P^* = (0.09244, 1.5189, 0.004221, 0.001864, 0.001434, 1.0926)$, and all solutions converge to the EE (P^*) as shown in Fig. 5.

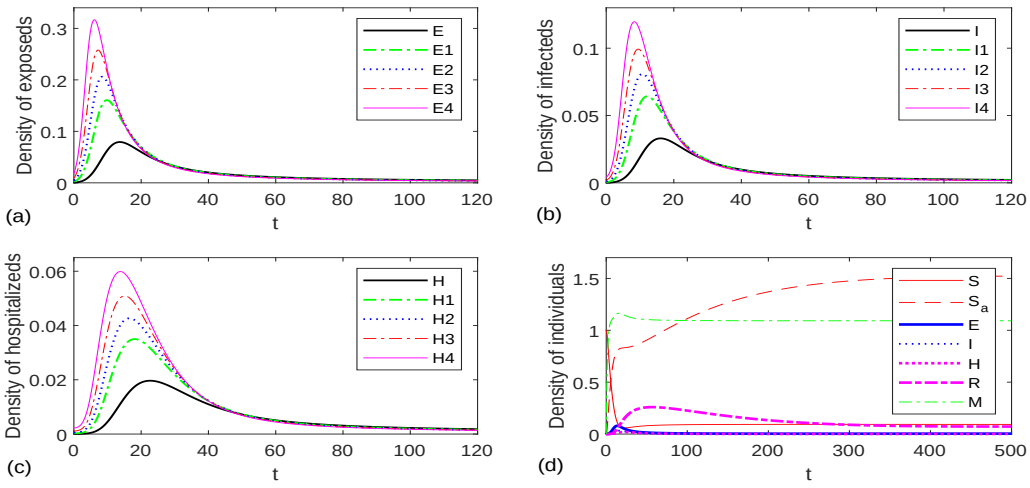


Fig. 5. Dynamical behavior around endemic equilibrium P^*.

5 Discussion

In order to study the effects of disease awareness on the spread of infectious diseases, we have proposed and analyzed an SEIR mathematical model. A detailed analysis shows that the proposed system posses two equilibria, namely one disease free and one endemic whose existence and asymptotic stability criteria depend on the numerical value of basic reproduction number R_0. That is if $R_0 < 1$, the disease-free equilibrium P^0 is globally asymptotically stable, if $R_0 > 1$, the endemic equilibrium P^* is globally asymptotically stable, but the disease-free equilibrium P^0 is unstable. To control the spread of disease better, we analyzed the sensitivity of R_0 and P^* with respect to different parameters, respectively, we also discussed the effects of awareness and media parameters on system dynamics. The results show that increasing the awareness transmission can not only directly reduce the incidence of diseases, but also reduce the incidence of diseases by reducing the disease infection rate and increasing the recovery rate, and so on.

Hence, when an epidemic occurs in a region, we should (1) reduce the humans input as much as possible so as to reduce the susceptible humans(reduce A), (2) go to densely populated places as little as possible(reduce β and θ), (3) actively take protective measures, wash hands frequently and wear masks, etc(increase α), (4) intensify epidemic propaganda effects and actively report the progress of epidemic(increase c and reduce τ).

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Appendix A.The global stability of endemic equilibrium

Consider the following positive definite function

$$\begin{aligned}V_2 =&S - S^* - S^* \ln \frac{S}{S^*} + a_1(S_a - S_a^* - S_a^* \ln \frac{S_a}{S_a^*}) + a_2(E - E^* - E^* \ln \frac{E}{E^*}) + \\ &+ a_3(I - I^* - I^* \ln \frac{I}{I^*}) + a_4(M - M^* - M^* \ln \frac{M}{M^*}).\end{aligned}$$

For simplicity, denote $X = \frac{S}{S^*}$, $Y = \frac{S_a}{S_a^*}$, $Z_1 = \frac{E}{E^*}$, $Z_2 = \frac{I}{I^*}$, $Z_3 = \frac{M}{M^*}$. Then the derivative of function V_2 along solutions of system (2.2) is given by

$$\begin{aligned}\dot{V}_2(P^*) =&\dot{S}\left(1 - \frac{1}{X}\right) + \dot{S}_a\left(1 - \frac{1}{Y}\right) + \dot{E}\left(1 - \frac{1}{Z_1}\right) + \dot{I}\left(1 - \frac{1}{Z_2}\right) + \dot{M}\left(1 - \frac{1}{Z_3}\right)\\ =&X(-dS^* - a_2\beta\delta\theta E^* S^*) + \frac{1}{X}(-A) + Y\left(-a_2\beta\delta E^* S_a^* - a_1 d S_a^*\right) + XZ_1\left(a_2\beta\delta\theta E^* S^* - \beta\delta\theta E^* S^*\right)\\ &+ XZ_2\left(a_2\beta\theta I^* S^* - \beta\theta I^* S^*\right) + Z_1\left(-a_2(d+\sigma)E^* + \beta\delta E^*\left(\theta S^* + a_1 S_a^*\right) + a_3\sigma E^* + a_4\eta\sigma E^*\right)\\ &+ Z_2\left(-a_3\left(d + \mu + \gamma_1 + \gamma_2\right) + a_1\beta S_a^* + a_4\eta\gamma_1 + \beta\theta S^*\right) I^* + XZ_3\left(-\alpha S^* M^* + a_1\alpha S^* M^*\right)\\ &+ Z_3\left(-a_4\tau M^* + \alpha M^* S^* + a_1\omega S_a^* M^*\right) + YZ_1\left(-a_1\beta\delta E^* S_a^* + a_2\beta\delta E^* S_a^*\right)\\ &+ YZ_2\left(-a_1\beta I^* S_a^* + a_2\beta I^* S_a^*\right) + YZ_3\left(\omega S_a^* M^* - a_1\omega S_a^* M^*\right) + \frac{YZ_3}{X}\left(-\omega S_a^* M^*\right)\\ &+ \frac{XZ_3}{Y}\left(-a_1\alpha S^* M^*\right) + \frac{XZ_2}{Z_1}(-a_2\beta\theta I^* S^*) + \frac{YZ_2}{Z_1}\left(-a_2\beta I^* S_a^*\right)\\ &+ \frac{Z_1}{Z_2}\left(-a_3\sigma E^*\right) + \frac{Z_1}{Z_3}\left(-a_4\sigma\eta E^*\right) + \frac{Z_2}{Z_3}\left(-a_4\eta\gamma_1 I^*\right) + \frac{1}{Z_3}\left(-a_4 c\right)\\ &+ A + a_4\left(c + \tau M^*\right) + dS^* + a_1 d S_a^* + a_2 E^*(d+\sigma) + a_3 I^*\left(d + \mu + \gamma_1 + \gamma_2\right).\end{aligned}$$

There are following seven cases

$$\begin{gathered}\left\{X, \tfrac{1}{X}\right\}, \left\{Z_3, \tfrac{1}{Z_3}\right\}, \left\{\tfrac{Z_1}{Z_2}, \tfrac{XZ_2}{Z_1}, \tfrac{1}{X}\right\}, \left\{XZ_3, \tfrac{1}{X}, \tfrac{1}{Z_3}\right\},\\ \left\{\tfrac{XZ_3}{Y}, \tfrac{1}{X}, \tfrac{1}{Z_3}, Y\right\}, \left\{\tfrac{XZ_3}{Y}, \tfrac{YZ_3}{X}, \tfrac{1}{Z_3}, \tfrac{1}{Z_3}\right\}, \left\{\tfrac{Z_2 Y}{Z_1}, \tfrac{XZ_3}{Y}, \tfrac{1}{X}, \tfrac{Z_1}{Z_2}, \tfrac{1}{Z_3}\right\}.\end{gathered}$$

Denote

$$\begin{aligned}\dot{G}_2 =&B_1\left(X + \frac{1}{X} - 2\right) + B_2\left(Z_3 + \frac{1}{Z_3} - 2\right) + B_3\left(\frac{Z_1}{Z_2} + \frac{XZ_2}{Z_1} + \frac{1}{X} - 3\right)\\ &+ B_4\left(XZ_3 + \frac{1}{X} + \frac{1}{Z_3} - 3\right) + B_5\left(\frac{XZ_3}{Y} + \frac{1}{X} + \frac{1}{Z_3} + Y - 4\right)\\ &+ B_6\left(\frac{XZ_3}{Y} + \frac{YZ_3}{X} + \frac{1}{Z_3} + \frac{1}{Z_3} - 4\right) + B_7\left(\frac{Z_2 Y}{Z_1} + \frac{XZ_3}{Y} + \frac{1}{X} + \frac{Z_1}{Z_2} + \frac{1}{Z_3} - 5\right).\end{aligned}$$

In the following we let $\dot{V}_2(P^*) = \dot{G}_2$ to determine the coefficients $a_i(i = 1, 2, 3, 4)$ and $B_i(i = 1, 2, ..., 7)$. Since the terms $Z_1, Z_2, Z_1X, Z_2X, Z_1Y, Z_2Y, Z_3Y, \frac{Z_1}{Z_3}$ and $\frac{Z_2}{Z_3}$ of function $\dot{V}_2(P^*)$ do not appear in function $\dot{G}_2$, their coefficients should be equal to zero, so $a_i(i = 1, 2, 3, 4)$ can be uniquely determined as

$$a_1 = a_2 = 1, a_3 = \frac{\beta(\theta S^* + S_a^*)}{d+\mu+\gamma_1+\gamma_2}, a_4 = 0.$$

Then by $\dot{V}_2(P^*) = \dot{G}_2$, comparing the coefficients of the like terms between them yields, which gives

$$\begin{cases} B_1 = -dS^* - \beta\delta\theta E^* S^*, \quad B_2 = \alpha M^* S^* + \omega S_a^* M^*, \\ B_3 = -\beta\theta I^* S^*, \quad B_4 = 0, \quad B_5 = -(\beta\delta E^* + d)S_a^*, \\ B_6 = -\omega S_a^* M^*, \qquad\qquad B_7 = -\beta I^* S_a^*. \end{cases}$$

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