
Modelling sediment transport capacity of loessial slopes based on effective stream power

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Abstract: The sediment transport capacity must be considered because it provides a theoretical basis for accurate prediction of soil erosion. Existing studies tended to study sediment transport capacity using a particular soil, but the models derived from one kind of soil cannot be applicable to other soil types. To obtain a prediction model for a variety of soils and evaluate its applicability, sandy loess and loess soil ($d_{50}=0.095$ mm and $d_{50}=0.04$ mm) were chosen in the indoor artificial simulated sediment transport experiments. The experimental slopes ranged from 7% to 38.4% and the unit discharges were adjusted from 0.00014 to 0.00526m²/s. Moreover, this study combined the experimental data with cohesive soil and cohesionless sand from four scholars so as to analyze the response relationship between sediment transport capacity and each flow intensity parameter through dimensionless processing. Results showed that the dimensionless sediment transport capacity varied with its power function relationship with the flow intensity parameters. Through analysis, the effective stream power could be seen as an optimum indicator ($R^2=0.9692$). After considering the effective stream power and volume sediment concentration, this study derived a formula for calculating the sediment transport capacity. It was better than the ANSWERS (Areal Nonpoint Source Watershed Environment Response Simulation) model, improved WEPP (Water Erosion Prediction Project) model, Zhang's formula and Ali's model due to its superior applicability to cohesive soil and cohesionless sand. These findings lay a basis for establishing prediction models of soil erosion.

Key words: overland flow; sediment transport capacity; flow intensity parameters; effective stream power; volume sediment concentration

0 Introduction

Soil erosion, as a major environmental problem, has attracted public concern from all over the world (Ali et al., 2012). In China, the Loess Plateau is located in the upper and middle reaches of the Yellow River, covering an area of 630,000 square kilometers. This region plays important roles in energy sources and food production, as well as economic development. But, severe soil erosion has posed threats to the environmental sustainability of the Loess Plateau (Liu et al., 2012; Zhao et al., 2013a). At present, over 70% of the area becomes a gully-hill-dominated region due to massive

soil erosion over the past thousands of years (Tian et al., 2016). Although in recent decades, this phenomenon has been mitigated significantly because of numerous soil and water conservation practices (Zhao et al., 2013b), analysing the sediment transport capacity helps to provide a reference for establishing prediction models for soil erosion.

Water erosion is the most prominent way, which refers to the destruction of soil framework and particles transport due to fluid movement. It includes three dynamic processes: soil detachment, sediment transport and sediment deposition. The sediment transport capacity of overland flow mainly means the maximum flux of sediment transport under specific slopes and discharges (Li and Abrahams, 1999; Zhang et al., 2009). It plays a critical role in defining areas of net erosion where the actual sediment concentration falls below the transport capacity, and it also determines net deposition where the transport capacity is exceeded (Yu et al., 2015). This is because of the coordinative role of sediment concentration and sediment transport capacity (Merten et al., 2001; Nearing et al., 1989). When sediment transport rate is less than sediment transport capacity, the soil detachment dominates; When sediment transport rate is equal to sediment transport capacity, the detachment process and sediment deposition process could reach a dynamic equilibrium. Adversely, the sediment deposition would occur due to excessive soil particles transported by currents (Ahmadi et al., 2006). Therefore, accurate prediction of the sediment transport capacity is a key issue when investigating soil erosion.

There are numerous factors affecting the sediment transport capacity of overland flow including energy slopes, slope length, flow discharge and flow velocity (Hao et al., 2019), and they also impact the flow intensity parameters (Liu et al., 2014; Wu et al., 2017). These influence factors can be classified into three categories: slope form like slope length, hydraulic parameters like flow velocity, energy gradient and flow discharges, as well as soil properties like particle size and viscosity (Prosser, and Rustomji, 2000). The actual slope length in the Loess Plateau area is generally more than tens of meters. However, the test length could not reach that value due to limited experimental conditions (Zhang, 2000). Zhang et al. (2017) studied relationships between hydraulic parameters and sediment yield under different slope lengths, and concluded that the step length of the fluctuation process in overland flow was three meters. Most noteworthy, relevant studies focused more on the relationship between sediment transport capacity and mean flow velocity, energy gradient, flow discharge, as well as mean flow velocity (Abrahams et al. 1996; Govers et al. 2010; Nearing et al. 1997). For example, Guy et al (2009) pointed that the flow discharge and energy slopes are two primary factors affecting the sediment transport capacity. Zhang et al. (2009) concluded that there was a linear relationship between mean flow velocity and sediment transport capacity. And the flow velocity is usually affected by hydraulic and surface conditions (Zhang, et al., 2010). But under the same condition, Wang et al. (2015) believed that the relationship between mean flow velocity and sediment transport capacity was exponential. Moreover, the sediment transport capacity is closely related to the sediment properties, such as median particle size, viscosity and shapes of sediment particles (Pieri et al., 2009). With

an increasing particle diameter, the incipient velocity and the settling velocity of soils increased exponentially (Zhang et al., 2011), while the sediment transport capacity showed an exponential decline. So, different hydraulic parameters should be considered when calculating the sediment transport capacity.

The sediment transport capacity can also be calculated by compound hydraulic parameters, such as shear stress, stream power, and unit stream power. Finkner et al. (1989) chose the flow shear stress τ in WEPP model, while Beasley et al. (1980) and De Roo (1996) selected the unit stream power P in EUROSEM model and LISEM model. However, Mahmoodabadi et al. (2014a, 2014b) proposed that sediment transport capacity obtained by the GUEST model had a higher accuracy than that modeled using the WEPP. There are many existing formulas for calculating the sediment transport capacity based on sediment transport theories in open-channels and natural rivers.

There are many Beasley et al. (1980) proposed the formula for sediment transport capacity on the basis of a single influence factor, as shown in formula (1). It has been applied to the ANSWERS model.

$$T_c = 146Sq^{0.5} \quad q \leq 0.046 \quad (1a)$$

$$T_c = 14600Sq^2 \quad q \geq 0.046 \quad (1b)$$

Where T_c is the flow sediment transport capacity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ and S is the energy slope, %. And q is the unit discharge, m^3/min .

Finkner et al. (1989) proposed a compound factor and then established the WEPP model after simplifying the Yalin formula (Yalin, 1963):

$$T_c = K\tau^{3/2} \quad (2)$$

Where T_c is the flow sediment transport capacity, $\text{kg}/(\text{m} \cdot \text{s})$ and τ is the shear stress, Pa . Sediment transport coefficient K is a comprehensive coefficient representing the general state of water flows and soil sediments.

Then, Zhang et al. (2008) carried out steep slope tests and found that the sediment transport coefficient K increased in terms of its power function with an increasing shear stress τ . It is also found that the WEPP model underestimated the influence of flow shear force. So, the WEPP model was revised by considering the exponent as two:

$$T_c = K\tau^2 \quad (3)$$

Ali et al. (2012) obtained the relations between the total flow discharges, median particle size and energy slope through sediment transport experiments with four different median particle sizes as follows:

$$T_c = 0.17 \times 10^6 \frac{Q^{1.46}}{d_{50}^{0.5}} S^{2.89} \quad (4)$$

84 Where T_c is the flow sediment transport capacity, kg/ (m s) and Q is the total flow discharge, m³/s. d_{50} means the
85 median particle size, mm and S is the energy slope, %.

86 Zhang et al. (2011) established a formula for sediment transport capacity which is applicable to sediments with
87 diverse soil characteristics:

$$T_c = 0.024 \sum_{i=1}^n d_{50i}^{-0.313} P_i \tau^2 \quad (5)$$

88 Where T_c is the flow sediment transport capacity, kg/ (m s) and n is the number of different sediment particle sizes.
89 And d_{50i} represents a particular sediment median particle size, mm. P_i indicates the proportion of a certain median particle
90 size in total sediments.

91 Furthermore, Luan et al. (2016) simulated the transport process of non-cohesive sand on steep slopes and evaluated
92 the influence of stream power on sediment transport capacity. Then, a formula was obtained by analyzing its relation with
93 d_{50i} , P_i and W :

$$T_c = 0.142 \sum_{i=1}^n d_{50i}^{-0.322} P_i W^{1.25} \quad (6)$$

94 Where W is the stream power, w/ m².

95 Apparently, there were no consistent conclusions regarding the selection of flow intensity parameters, and existing
96 prediction models were aimed at a certain kind of soils like cohesive soil and sand, so the prediction model cannot be
97 applied to each other. The experiment here is similar to that of Zhang et al. (2011), Wang et al. (2016), Ail et al. (2012)
98 and Aziz and Scott (1989), which all could be called overland flow sediment transport experiment. There are only
99 differences between slope gradients, flow discharge and the median diameter. Therefore, this study synthesized the data
100 of above four scholars. The objectives of this study were:

- 101 (1) to evaluate the relationship between dimensionless sediment transport capacity and dimensionless flow intensity
102 parameters.
103 (2) to establish the sediment transport capacity for rill flow considering the volume sediment concentration and an
104 optimum flow intensity parameter.
105 (3) to evaluate the applicability by comparing the prediction model in this study with four existing models.

106 1 Materials and Methods

107 1.1 Soil samples

108 Test soil materials were Shenmu sand loess from Shenmu county (110° 30' E, 38° 49' 48" N) and Huangmian

soil collected from Ansai County (109° 19' 23" E, 36° 51' 30" N), as shown in Fig.1. Then, these two soil materials were air dried and first passed through a 2 mm sieve to remove gravel and residues. Table 1 showed the particle size distribution of Shenmu sand loess ($d_{50}=0.095\text{mm}$) and Huangmian soil ($d_{50}=0.04\text{mm}$).

1.2 Experimental set-up

These experiments were carried out in July 2017 at the State Key Laboratory of Soil Erosion and Dryland Farming on the Loess Plateau, Yangling, China. The experimental set-up consisted of a test flume, a constant water tank, a sediment hopper and sediment collection devices (as shown in Fig. 2). A $4.5 \times 0.3 \times 0.1$ m (L \times W \times H) rill flume with plexiglass sides was selected in our experiment. The slope of the flume could be adjusted to a desired slope using lifting bolts. In order to make the roughness of the bed surface uniform, an artificial sand cloth (particle sizes = 0.38mm) was pasted onto the bed surface. The flow discharge was controlled by seven drain valves at the outlet of the constant water tank. A sediment hopper which was 0.5m distant from the upper side of this flume was used to provide the water flow with sediments. The rate of adding sediments was controlled by guide plates and rollers, as shown in Fig.2. Three kinds of slopes (10.5%, 15.6% and 20.8%) and seven unit discharges (0.14, 0.28, 0.42, 0.56, 0.69, 0.83 and $1.11 \times 10^{-3} \text{m}^2 \text{s}^{-1}$) were systematically investigated using Shenmu sand loessial samples. As for Huangmian soil materials, five types of slopes (7.0%、10.5%、14.1%、17.6% and 21.3%) and six unit discharges (0.19, 0.39, 0.58, 0.75, 0.95 and $1.12 \times 10^{-3} \text{m}^2 \text{s}^{-1}$) were set up in this test. Also, these designed unit discharges ensured that the water depth was tested within a wide range.

1.3 Measurement

Before adding sediments into the water flow, slope gradients and flow discharges were adjusted to designed values. And flow discharges were measured by the weighing method with an accuracy of 0.001L/s. When it reached a constant value, hydraulic parameters related to this study were measured. There were three cross sections for observation in this flume, positioned at 1.0, 2.0, and 3.0 m along the slope from top to bottom. For each cross section, three measurement points were set transversely to observe the surface velocity using the KMnO₄ dye tracer method. Surface flow velocity was measured 20 times for every point, then we took the average value of the 20 measurements to increase reliability of data. So at each cross section, three relatively accurate surface flow velocities were obtained from the left and right of this flume. The time during which the tracer was required to traverse a marked distance (1m) was determined using a stop watch (Wu et al., 2017), as shown in Fig.3 (b). It's remarkable that the start and stop of the timing depends on the front of the dyeing currents. An SX401 digital probe tester (Chongqing Hydrological Instrument Factory, Chongqing, China) with 0.01- mm precision was used to measure water depth, as shown in Fig.3 (a). The velocity and water depth for every measurement point were obtained. Their average values were mean water depth and surface flow velocity, respectively.

In addition, as shown in Table 2 and Table 3, the water depth is shallow and it increases as the flow discharges increases. The sediment transport capacity of loess sand ($d_{50}=0.095\text{mm}$) increases comparatively faster as flow discharges and slope gradient increases. And when the energy slope and unit discharges are small, the sediment transport capacity of loess sand ($d_{50}=0.095\text{mm}$) and Huangmian soil ($d_{50}=0.04\text{mm}$) has some fluctuations. This may be because sediment-laden flow destabilizes and forms a series of waves having great influence on the observational result when the the energy slope and unit discharges are relatively small, as shown in Figure 4. On the other hand the dynamic conditions of sediment incipience and sediment subsidence may be affected by rolling waves. And as the sediment concentration at wave crests and troughs was over-saturated or nearly saturated respectively, the number of wave crests and troughs would influence the measurement of sediment concentration in flow-water.

1.4 Experiment Procedures

This study chose Formula (7) (Zhang et al., 2011) to predict a rough T_c value in advance. It is because these experimental conditions are similar, and they are all indoor simulated tests. Then, numerous pre-experiments with initial sediment discharges approximating this value were carried out. When the sediments in flows just began to settle on the surface of this flume, we considered that the T_c value at this time was close to sediment transport capacity, and then our experiments began.

$$T_c = 2382.32q^{1.269}S^{1.637}d_{50}^{-0.345} \quad (7)$$

Where T_c is the sediment transportation capacity in overland flow, $\text{kg}/(\text{s m})$ and q is the unit discharge, m^2/s . S means the energy slope, % and d_{50} is the median particle size, mm .

In formal experiments, once the flow velocity and depth were measured, the sediment delivery machine operated until not all of the sediment could be carried and deposition was observed to occur at the bed surface, at which point the transport capacity was assumed to have been reached (Mu et al., 2019). When measuring the sediment transport capacity, the rate of adding sand could be controlled by the electric motor and sediment hopper until it kept stable within five minutes. Also, the sediment and water were fully mixed with iron bars. As the discharge of sediment-laden water was constant, sampling buckets with different numbers were used at the outlet of the flume. The sampling time determined by flow discharges and the size of containers could be recorded by a stopwatch. After around 30s, the sediment samples were sent to be deposited and dried, and the M value in Formula (8) was the average the mass of these sand samples. The sediment transport capacity could be calculated by Formula (8):

$$T_c = \frac{M}{Tb} \quad (8)$$

Where T_c is the sediment transportation capacity in overland flow, $\text{kg}/(\text{s m})$ and M is the mass of dried sand samples,

kg. And the parameter b is the width of this test flume while T is the sampling time, s.

1.5 Calculation of hydraulic parameters

In sediment transport mechanics, flow intensity parameters are classified into three categories: flow velocity parameters (surface velocity, bottom velocity and mean velocity), dynamic parameters (flow shear stress) and power parameters (unit stream power, stream power and effective stream power).

However, due to disputes about the definition of surface velocity and bottom velocity, these two parameters have not been widely used. This study observed the surface velocity of flow using the KMnO_4 dye tracer method. Although previous studies have proposed that mean velocity can be estimated by the surface velocity multiplied by a correction coefficient, but the correction factor a (the ratio of the average to maximum velocity) has been shown to vary significantly with a number of parameters (Myers, 2002). For example, Luk and Merz (1992), Dunkerley (2001) and Ali et al (2012) all gave a reasonable correction coefficient. Therefore, in order to avoid the interference of these unsuitable correction coefficients, the mean velocity V which has been extensively used could be expressed as follows:

$$V = \frac{Q}{bh} \quad (9)$$

Where V is the mean velocity, m/s and Q is the flow discharge, m^3/s . h is the water depth, m and b is the width of the flume, m.

The flow shear stress refers to the force acting along slopes during the movement of fluids. It can be expressed as:

$$\tau = \rho ghS \quad (10)$$

Where τ is the flow shear stress, Pa and ρ is the density of water flow, kg^3/m . g is the acceleration due to gravity (9.81 m/s^2) and the h is water depth, m. S is the energy slope, %.

Bagnold (1966) proposed the concept of stream power, i.e. the change rate of the water potential energy per unit time, which can be expressed as:

$$W = \tau V = \rho g S q \quad (11)$$

Where W is the stream power, w/m^2 . Where q means the unit discharge, m^2/s . Compared with stream power, the unit stream power considers the mass of water flow. It represents the energy loss of sediment transport per unit mass of water flow.

The relationship between unit stream power and mean velocity, as well as slopes is:

$$P = VS \quad (12)$$

Where P is the unit stream power, m/s.

[Govers \(1992\)](#) proposed the effective stream power on the basis of flow shear stress, which represents the actual output power of water flows. The formula is:

$$W_{eff} = \frac{(\tau V)^{1.5}}{h^{0.67}} \quad (13)$$

Where W_{eff} is the effective stream power, $N^{1.5}/(s^{1.5}m^{2.17})$.

1.6 Theoretical analysis

The dimensionless formulas are shown in Table 4.

There are numerous non-dimensional formulas of sediment transport capacity for the reason that it could be dimensionless using parameters related to sediment properties and flow conditions. This study combined experimental data with [Zhang et al. \(2011\)](#), [Wang et al. \(2016\)](#), [Ali et al. \(2012\)](#) and [Aziz and Scott \(1989\)](#), 348 groups in total. All data were randomly divided into two groups without human interference. The first group contain 181 sets of data, which could be used to analyze the relationship between flow intensity parameters and sediment transport capacity, and to derive formulas for calculating the sediment transport capacity of overland flow. Another 166 sets of data were chosen to evaluate the applicability of these formulas. When evaluating its applicability, the correlation coefficient R^2 and Nash coefficient N_{SE} were selected to verify the simulation of these formulas.

$$R^2 = \frac{\left[\sum_{i=1}^n (O_i - \bar{O})(P_i - \bar{P}) \right]^2}{\sum_{i=1}^n (O_i - \bar{O}) \sum_{i=1}^n (P_i - \bar{P})^2} \quad (14)$$

$$N_{SE} = 1 - \frac{\sum (O_i - P_i)^2}{\sum (O_i - \bar{O})^2} \quad (15)$$

Where P_i is the simulated value and \bar{P} is the average of simulated values. O_i is the measured value while \bar{O} is the average of measured values. The parameter n is the sample number. The Nash coefficient N_{SE} is close to 1, indicating that the formula is applicable to simulate actual situations. When Nash coefficient N_{SE} approaches zero, it shows that these simulation results are close to the average of observed values. That is, the overall results are credible although there are still large errors in the simulation process. If Nash coefficient N_{SE} is far less than zero, the simulation results of these formulas are questionable.

2 Results and analysis

2.1 The dimensionless process

This study chose experimental data with Zhang et al. (2011), Wang et al. (2016), Ali et al. (2012) and Aziz and Scott (1989) totaled 348 groups due to data availability, as shown in Table 5. This is because these studies are all flume experiments and there are only differences between the energy slope, discharge and median particle size of soils. In this study, there were 5.8% cohesive soil particles with a median particle size of 0.095 mm. And the cohesive particles with 0.04mm covered 6.85 percent, which is the same as that in Wang et al. (2016). However, the cohesionless fine sand was chosen in Zhang et al. (2011), Ali et al. (2012) and Aziz and Scott (1989).

It can be seen that different scholars chose diverse median particle sizes, so it is necessary to make parameters dimensionless, as shown in Table 4. This process considered the settling velocity because in quiescent water, the median particle size has little influence on the sediment settling velocity. When the related flow intensity parameters and the sediment transport capacity are all dimensionless, the influence of median particle size on flow intensity parameters would be weakened.

2.2 Selection of parameters

The first group was used to analyze the relationship between flow intensity parameters and sediment transport capacity while another 166 sets of data were chosen to evaluate the applicability of these formulas.

(1) The response relationship between dimensionless mean velocity V^* and dimensionless sediment transport capacity Φ was shown in Fig.5 (a). It showed that the sediment transport capacity Φ increased in terms of its power function with an increasing mean velocity ($R^2=0.664$). The relation can be expressed by formula (16):

$$\Phi = 0.0214V^{*3.009} \quad R^2 = 0.664 \quad (16)$$

Where Φ is the dimensionless sediment transport capacity in overland flow while V^* is the dimensionless mean velocity.

Then, Formula (16) was validated by the second set of data, as shown in Fig. 5 (b). The fitting precision R^2 was 0.664 and the N_{SE} value was 0.664. This formula could better predict the sediment transport capacity of silty sand containing cohesive particles in this test and Wang et al. (2016). But it is not suitable for Zhang et al. (2011), Ali et al. (2012) and Aziz and Scott's (1989) cohesionless sand tests. Overall, the prediction results using mean velocity are not that ideal.

(2) The response relationship between dimensionless flow shear stress θ and dimensionless sediment transport capacity Φ was shown in Fig.6 (a). It showed that the sediment transport capacity Φ increased in terms of its power function with an increasing flow shear stress ($R^2=0.9498$). The relation can be expressed by formula (17):

$$\Phi = 0.0369\theta^{2.0307} \quad R^2 = 0.9498 \quad (17)$$

Where θ is the dimensionless flow shear stress. Then, Formula (17) was validated by the second set of data, as

shown in Fig.6 (b). The fitting precision R^2 was 0.920 and the N_{SE} value was 0.806.

(3) The response relationship between dimensionless stream power W^* and dimensionless sediment transport capacity Φ was shown in Fig.7 (a). It showed that the sediment transport capacity Φ increased in terms of its power function with an increasing stream power ($R^2=0.9458$). The relation can be expressed by formula (18):

$$\Phi = 0.0209W^{*1.3067} \quad R^2 = 0.9458 \quad (18)$$

Where W^* is the dimensionless stream power. Then, Formula (18) was validated by the second set of data, as shown in Fig. 7 (b). The fitting precision R^2 was 0.970 and the N_{SE} value was 0.721. It can be seen from Fig.7 (a) that Aziz and Scott (1989) obtained a smaller sediment transport capacity under the same stream power, meaning that this formula is not well applicable to Aziz and Scott (1989). The reason is that Aziz and Scott (1989) carried out their experiments using a soil-covered flume under gentle slopes (3~10%) and large flow discharges. So, the measured sediment transport capacity is the unsaturated value.

(4) The response relationship between dimensionless effective stream power W_{eff}^* and dimensionless sediment transport capacity Φ was shown in Fig.8 (a). It showed that the sediment transport capacity Φ increased in terms of its power function with an increasing effective stream power ($R^2=0.9692$). The relation can be expressed by formula (19):

$$\Phi = 0.0036W_{eff}^{*1.0927} \quad R^2 = 0.969 \quad (19)$$

Where W_{eff}^* is the dimensionless effective stream power. Then, Formula (19) was validated by the second set of data, as shown in Fig.8 (b). The fitting precision R^2 was 0.975 and the N_{SE} value was 0.797. The simulated and measured values are around the 1:1 line, indicating that the effective stream power can better predict the sediment transport capacity.

2.3 Empirical formula for sediment transport capacity

The relationship between the dimensionless sediment transport capacity and dimensionless effective stream power. K can be expressed by formula (20):

$$\Phi = KW_{eff}^{*b} \quad (20)$$

Where Φ is the dimensionless sediment transport capacity in overland flow and W_{eff}^* is the dimensionless effective stream power. K is the sediment transport coefficient and b is the power exponent.

Sediment transport coefficient K is a comprehensive coefficient representing the overall state of water flows and soils. It is sensitive to multiple factors such as the equilibrium condition, saturation and sediment characteristics. In this study, the dimensionless theory has eliminated the influence of median particle size on sediment transport capacity. Therefore, the sediment transport coefficient K would be mainly affected by the equilibrium condition and the saturation.

As the volume sediment concentration S_v indicates the percentage of sediments in water flows per unit volume. It could depict the saturated state of sediments. The relationship between volume sediment concentration S_v and sediment transport coefficient K is analyzed as shown in Fig.9.

It showed that sediment transport coefficient K underwent an upward trend as the volume sediment concentration S_v increased. According to SPSS correlation analysis, the Pearson index between sediment transport coefficient K and volume sediment concentration is 0.393. These two parameters are negatively correlated and their relationship is as follows:

$$K = 0.0576S_v^{0.1551} \quad R^2 = 0.393 \quad (21)$$

Then, the dimensionless effective flow power W_{eff}^* and the volume sediment concentration S_v are taken as independent variables, and the dimensionless sediment transport capacity Φ is taken as a dependent variable. It would be:

$$\Phi = 0.1742S_v^{0.322}W_{eff}^{*0.949} \quad R^2 = 0.989 \quad (22)$$

Where S_v is the volume sediment concentration.

This formula considered the volume sediment concentration, indicating whether the sediment is saturated or not. And the dimensionless effective stream power is correlated with mean velocity, water depth, slope gradients, sediment particle density and gravitational constant. So, this formula has high fitting accuracy and can better predict the sediment transport capacity. However, in formula (22), both volume sediment concentration S_v and dimensionless sediment transport capacity are unknown parameters, so it needs to be further deduced by the implicit function method.

As sediment transport capacity can also be expressed by:

$$T_c = S_v \gamma_s q \quad (23)$$

Where T_c is the sediment transport capacity and S_v is the volume sediment concentration. Where q is the unit discharge.

By introducing formula (22) into the formula for calculating dimensionless sediment transport capacity in Table 1, it can be obtained that:

$$\Phi = \frac{S_v \gamma_s q}{\gamma_s \sqrt{\left(\frac{\gamma_s}{\gamma} - 1\right)} g d_{50}^3} \quad (24)$$

When considering the formula (24) is equal to formula (22), it can be concluded that:

$$\frac{S_v \gamma_s q}{\gamma_s \sqrt{\left(\frac{\gamma_s}{\gamma} - 1\right)} g d_{50}^3} = 0.1742 S_v^{0.322} W_{eff}^{*0.949} \quad (25)$$

The volume soil concentration can be calculated as:

$$S_v^{0.678} = \frac{0.1742 W_{eff}^{*0.949} \sqrt{\left(\frac{\gamma_s}{\gamma} - 1\right)} g d_{50}^3}{q} \quad (26)$$

If the parameter related to soil types (d_{50} , γ_s) and runoff conditions (W_{eff}^* , q) can be obtained, the volume soil concentration S_v can be determined. Then a formula for dimensionless sediment transport capacity is derived by integrating formula (23) and formula (26):

$$T_c = S_v \gamma_s q = \frac{0.076 W_{eff}^{*1.4} \gamma_s g^{0.7374} d_{50}^{2.2125} (\gamma_s - \gamma)^{0.7374}}{q^{0.475} \gamma^{0.7374}} \quad (27)$$

By substituting $\gamma=1$, $g=9.8$ in formula (27), it would be:

$$T_c = 0.41 \left[\gamma_s (\gamma_s - 1)^{0.74} d_{50}^{2.21} \right] \frac{W_{eff}^{*1.4}}{q^{0.48}} \quad R^2 = 0.989 \quad (28)$$

$$X = \gamma_s (\gamma_s - 1)^{0.74} d_{50}^{2.21} \quad (29)$$

The formula (28) can be used to calculate the sediment transport capacity. When the soil types and flow conditions are determined, the X is a constant and sediment transport capacity would be mainly affected by the effective stream power and unit discharges. This formula is simple in form and has a high fitting degree. In addition, this formula considers the relationship between volume sediment concentration and sediment transport capacity. In the case of both unknown, an implicit function mathematical method was used for getting a simplified calculation formula. And as this formula took the effective stream power as an important parameter, it can be applied to both cohesive and cohesionless soils, which will provide better results for predicting the sediment transport capacity.

To evaluate the applicability of formula (28), the Figure 10 shows the relationship between the measured value and calculated value of sediment transport capacity.

It can be seen in Fig.10 that the calculated and measured value are around the 1:1 line ($R^2=0.985$, $N_{SE}=0.884$), although this study included a series of data from this experiment, Wang's silty test, Zhang's, Ali's and Aziz's non-cohesive soil tests (Wang et al. 2016; Zhang et al. 2011; Ali et al. 2012; Aziz and Scott 1989). It means that this formula is applicable to calculate the sediment transport capacity of cohesive and cohesionless soil particles.

3 Discussion

3.1 Parameter selection

The dimensionless effective stream power W_{eff}^* appeared later than other parameters, which is proposed by [Govers \(1992\)](#) based on the concept of flow shear stress. It mainly refers to the residual net output power of water flow without losses due to the shear stress. The value of the effective flow power can also reflect sediment transport capacity because it represents the actual output power of water flow. The larger the effective flow power is, the greater the output energy of the water flow is. And the flow with large energy can transport more sediments, this is why the effective flow power is most highly correlated with the sediment transport capacity.

Actually in the real world, both flow velocity and shear stress are easier to obtain as they all can be calculated by the measured water depth. But under natural conditions, both cohesive and cohesionless soils can be transported by water flow. And their transport processes are different. That is, because of mutual adhesion between the particles, the sediment incipience of cohesive soil particles is in blocks when they are eroded by currents while the cohesionless soil particles are transported from one to the next. In order to avoid the impact of transient transport characteristics of different soil types on sediment transport, the energy parameter would be more appropriate compared with flow velocity or shear stress. And energy parameters can also reflect the energy consumption in the whole process of sediment transport. So this study chose the effective stream power to derive a formula for sediment transport capacity due to the higher R^2 (0.969) and N_{SE} (0.975).

3.2 Comparison of prediction models

The formulas of sediment transport capacity obtained in this study are compared with Equation (1), (3), (4), and (5) as follows. The equation (4) depicts the relationship between sediment transport capacity T_c , median particle size d_{50} , slope gradients S and flow discharges Q . As the flow discharge is sensitive to the width of test flumes, it should be converted into the unit discharge q . In this way, the equation (4) can be expressed as:

$$T_c = 0.17 \times 10^6 \frac{(0.2q)^{1.46}}{d_{50}^{0.5}} S^{2.89} \quad (30)$$

It also can be concluded that:

$$T_c = 16216 \frac{q^{1.46}}{d_{50}^{0.5}} S^{2.89} \quad (31)$$

Where T_c is the sediment transport capacity, kg/ (m s) and q is the unit discharge, m²/s. Where d_{50} is the median particle size, mm and S is the energy slope, %.

According to the Equation (1), (3), (30), and (5), the relationship between measured and simulated values of sediment transport capacity is drawn as follows:

It can be seen that in the left half of the Fig.11 (a), the simulated values obtained by the ANSWERS model (Beasley et al. 1980) and the measured data showed zonal distribution. In the rear segment, the distribution of points was relatively scattered. It maybe because this model only considered the unit discharge as a single factor affecting sediment transport capacity. The fitting degree R^2 equals to 0.854 and N_{SE} equals to 0.796, indicating the ANSWERS model can predict a relatively accurate T_c . It is only possible that the prediction of sediment transport capacity on different soil types is not accurate enough. The Fig.11 (b) showed that when the improved WEPP model (Zhang et al. 2008) is used in this study, the simulated values are mostly larger than the measured data. The fitting accuracy R^2 equals to 0.67 and N_{SE} equals to 0.188. This maybe because the improved WEPP model was established when choosing only one sediment particle size ($d_{50} = 0.28\text{mm}$), which cannot reflect the influence of the varied median sediment particle size on its transport capacity. Therefore, there is a certain deviation when predicting the sediment transport capacity of flows with other median particle sizes. And the Fig.11 (c) illustrates that the fitting degree R^2 of Zhang's formula (Zhang et al. 2011) equals to 0.464 and N_{SE} equals to 0.017. Its simulated values are larger than measured ones, and their deviations maybe due to the median particle size in soils. This study, Wang et al. (2016) and Aziz and Scott (1989) all chose loess containing cohesive particles, and they had the most prominent deviations between measured and simulated values. The Zhang's, Aziz's and Ali's test chose cohesionless sand, and the deviation between measured data and simulated values was not that obvious (Zhang et al. 2011; Aziz and Scott 1989; Ail et al. 2012). It indicates that Zhang's formula (Zhang et al. 2011) is more applicable for predicting the sediment transport capacity of cohesionless soils. It can be seen from Fig.11 (d) that the fitting degree R^2 of the Ali's model (Ali et al. 2012) equals to 0.797 and N_{SE} equals to 0.425. When the sediment transport capacity is at a low or a high value, the deviation between measured and simulated values occurs. It indicates that the Ali's formula is more applicable to soil-laden flows with a transport capacity of 0.2~2kg/ (m s).

From the above comparison, it is concluded that each model has its advantages and limitations. The median particle size, hydraulic parameters, experimental condition and soil characteristics all influence the prediction of sediment transport capacity. The dimensionless method can be seen as an effective method and considering compound parameters can better model transport capacity. But to model sediment transport capacity in natural conditions, it is necessary to explore more direct factors affecting the sediment transport capacity in the process of water erosion in future research.

4 Conclusions

This study chose sandy loess and loess soil with different median particle sizes when carrying out a sediment transport experiment. After synthesizing the data from four scholars, an appropriate formula for the sediment transport capacity was derived based on the effective stream power and volume sediment concentration. Following are the main conclusions:

(1) The selection of flow parameters is of vital importance when calculating the sediment transport capacity. It can be seen from the existing research that the flow velocity parameter was not a reliable indicator in cohesionless soils. The shear force could not be used to calculate the sediment transport capacity of vicious soils, but it served as an effective parameter when it comes to the cohesionless soils. Compared with the above parameters, the compound factor stream power parameters especially the effective stream power was the best predictor.

(2) The influence of median particle sizes on the sediment transport capacity could be eliminated after the dimensionless treatment. Based on this, the relationship between the sediment transport coefficient K and volume sediment concentration was analyzed theoretically. And it was concluded that the volume sediment concentration is a necessary factor when calculating the sediment transport capacity.

(3) Then, this study chose the dimensional effective stream power and the volume sediment concentration as two influence factors. And the power function model was used to calibrate the sediment transport capacity. It is apparent that the sediment transport mechanism in overland flow is different from that in rivers. But the predictional model obtained here had a wide applicability and could calculate the sediment transport capacity accurately.

Acknowledgements

This research was supported financially by the National Natural Science Foundation of China [Grant No.41877076].

Conflict of Interest

I confirm that all authors have no conflicts of interest.

Data Availability Statement

The data used to support the findings of this study are available from the corresponding author upon request.