ENERGY HARVESTING FROM A ROTATING PARAMETRIC PENDULUM: SINGULAR OPTIMAL CONTROL

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ABSTRACT

Sea waves represents a very promising energy source. Pioneered by Wiercigroch a pendulum system would be feasible for such an energy harvesting purpose (see for instance[1] and [2]).

These devices consist of a pendulum with a vertical motion induced by the sea waves. As it is well known, pendulum's stable rotations generate enough energy able to be extracted by an electrical generator attached to its axis [2].

In this paper, using a brushless dc motor as an input torque $u(t)$ to maintain stable pendulum rotations, following the ideas implemented in [3], a singular optimal control formalism provides a very simple control law using the mechanical model given by equation (1):

$$\ddot{\theta}(t) + \beta \cdot \dot{\theta}(t) + \left(R \cdot cos(\omega \cdot t) + \lambda \cdot R \cdot \frac{\Lambda_3}{\Lambda_1^3} + \lambda \cdot \frac{\Lambda_2}{\Lambda_1} + 1 \right) + u(t) = 0 \tag{1}$$

Besides the main objective in this paper: stable controlled rotations, in order to control every motion possibility: rotation, stability or even chaos, a singular optimal control policy is defined given in equation (2) rewriting in state-space form equation (1):

$$min_{u \in U} \quad \frac{1}{2} \left(\dot{\theta}(t) - \phi(t) \right)^2 \tag{2}$$

such that:

$$\dot{X}(t) = \begin{bmatrix} x_2(t) \\ h(x_1, x_2, t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot u(t)$$

where $x_1 = \theta(t), x_2(t) = \dot{\theta}(t)$ and $\phi(t)$ acts as a set-point (desired pendulum's trajectory). Notice that tuning the function $\phi(t)$, different controlled behaviours for the pendulum's orbits can be achieved.

Then, Pontryagin's principle solve this singular optimal control problem, providing a very simple controller in equation (3):

$$u(t) = -K \cdot sign \left(\dot{\theta}(t) - \phi(t) \right) \tag{3}$$

with $K \in \Re^+$ and arbitrary constant and $sign(.)$ the classic sign function. A salient property of the the resulting control law lies on its very simple hardware implementation bang-bang control: only sign is needed using Arduino and operational amplifiers (see Figure 1).

While Matlab/Simulink simulations will be presented, the possibility of being tuned for stable/unstable rotations or even asymptotic stability is also an interesting analysis. Conclusions and future work are also depicted.

Keywords: Parametric pendulum, Singular optimal control, Energy harvesting

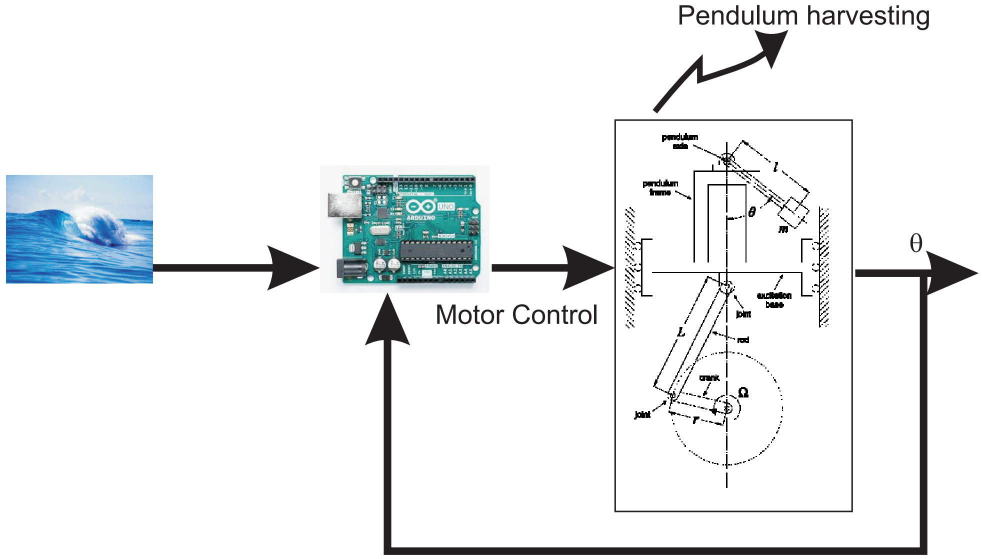

Figure 1: Matlab-Simulink model with singular optimal control

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