THE TEMPERATURE STATE OF A PLANE DIELECTRIC LAYER AT CONSTANT VOLTAGE AND FIXED TEMPERATURE OF ONE OF THE SURFACES OF THIS LAYER

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Abstract. The paper formulates the nonlinear problem of steady-state heat conduction at the constant electric potential difference on the surfaces of a plane dielectric layer with the temperature-dependent heat conduction coefficient and electrical resistivity. A fixed temperature value is set on one of the layer surfaces, and the convective heat exchange with the ambient medium occurs on the opposite surface. The formulation of the problem is transformed to integral ratios, which allows the calculation of the temperature distribution over the layer thickness, governed both by the monotonic and nonmonotonic function. The quantitative assay of the temperature state of a layer of a polymer dielectric made of amorphous polycarbonate is given as an example, as well as the analysis of nonuniformity of the absolute value of electric field intensity over the thickness of this layer.

Keywords: nonlinear mathematical model, polymer dielectric, one-dimensional temperature distribution.

INTRODUCTION

Materials, including polymers, used in various electrical and radio engineering devices as dielectrics have very high electrical resistivity, which at the temperature of about 300 K has the values of $10^{14}...10^{18}$ Ohm [1-5]. With a large electric potential difference on the surfaces of the dielectric layer, even at such values of electrical resistivity, the current passing through the layer causes the release of Joule heat and the dielectric temperature increase, which, as a consequence, reduces the level of electrical resistivity. This, in turn, leads to a further increase in electric current intensity and volumetric energy release. The comparatively low heat conduction coefficient of the materials used and the insufficient intensity of heat removal of the released energy into the ambient medium predetermine a positive feedback, due to which there is a rapid temperature increase, which results in thermal destruction of the dielectric material, i.e. melting, carburization. Such a process has been called the thermal breakdown of a dielectric, in contrast to the electric breakdown [6, 7].

The reliable functioning of a dielectric with a high electrical potential difference is facilitated by the intensificating of the removal of the Joule heat released in it and the choice of the material with the highest possible value of heat conduction. It is desirable to increase the value with the increasing temperature, which is characteristic for some polymer materials. One of the areas of application of polymer dielectrics is associated with making high-voltage cables, in which such a dielectric is the electrical insulation of the conducting cores [8, 9]. The available polymeric materials allow for the increase in the cable operating voltage, including the cables used in direct current lines, which have a number of advantages in comparison with AC voltage cable lines [10].

The quantitative assay of the dielectric temperature state necessary for the evaluation of its performance requires modern methods of mathematical modeling [11, 12] and is related to the solution of a rather sophisticated nonlinear problem that takes into account the interrelationship of the temperature dependences of the dielectric electrical resistivity and its heat conduction coefficient. This paper states a nonlinear problem of steady-state heat conduction for the constant electric potential difference on the surfaces of a plane dielectric layer with a fixed temperature on one of these surfaces and given conditions of the convective heat transfer on the opposite surface. The problem forms the basis of the differential form of the mathematical model. This model describes the temperature distribution over the thickness of the layer, which also determines the degree of uneven distribution of the absolute value of the electric field intensity in the dielectric affecting the dielectric strength [6, 13, 14].

For the quantitative assay of the mathematical model, the resulting differential form is transformed to integral relations, which, using experimental data on the temperature dependence of the electrical resistivity and the heat conduction coefficient of amorphous polycarbonate [1], as a possible dielectric material, allowed us to calculate the temperature distributions and the absolute value of electric field intensity in the dielectric layer. Such information can form the basis for the rational choice of a dielectric material in relation to operating conditions of high-voltage equipment.

1. RESEARCH OBJECTIVE

One of the surfaces of a plane dielectric layer of the fixed thickness h has a fixed temperature T_0, and on the opposite surface there occurs the convective heat exchange with the ambient medium having a temperature T^*, whose intensity determines the heat transfer coefficient α. On the surfaces of the layer a constant electric potential difference $U_* > 0$ is defined. The heat conduction coefficient $\lambda(T)$ and the electrical resistivity $\rho(T)$ of the dielectric depend on temperature T. The computing origin of the dimensionless coordinate $\xi \in [0;1]$ is chosen on the surface of the layer with a given temperature T_0.

The one-dimensional steady-state temperature $T(\xi)$ distribution over the thickness of the dielectric layer is governed by a nonlinear differential equation [15]

$$\frac{d}{d\xi}\left(\lambda(T)\frac{dT(\xi)}{d\xi}\right)+q_V h^2=0,\quad \xi\in(0;1). \tag{1}$$

Here q_V is the the volume power of energy release in the layer, caused by the conversion of some electric energy into Joule heat when passing through the electric current layer. The solution of the equation (1) must meet the boundary conditions

$$T(0)=T_0,\quad \lambda(T)\frac{dT(\xi)}{d\xi}\bigg|_{\xi=1}=\alpha(T^*-T_1)h, \tag{2}$$

where $T_1=T(1)$.

The volume power of energy release in the dielectric layer can be represented in the form

$$q_V=j^2\rho=jE, \tag{3}$$

where j and $E=|d\varphi/d\xi|/h$ are the modules of the electric current density and electric field intensity vectors directed perpendicular to the layer surfaces, and φ is the potential of this field (the choice of reference zero φ does not affect the value E). From the condition of mobile electric charges conservation in the dielectric layer follows the equality $j=E/\rho=\text{const}$, or, according to the equality (3), it follows that

$$\frac{d}{d\xi}\left(\frac{1}{\rho}\left|\frac{d\varphi}{d\xi}\right|\right)=0. \tag{4}$$

2. THE FIRST STAGE OF THE PROBLEM SOLUTION

Taking into account the equality (3), we represent the equation (1) in the form

$$\frac{d}{d\xi}\left(\lambda(T)\frac{dT(\xi)}{d\xi}\right)+(jh)^2\rho(T)=0,\quad \xi\in(0;1). \tag{5}$$

Using the well-known procedure for reducing the derivative order in (5), similar applied at integration of the movement equation in analytic mechanics [16], we write

$$\lambda(T)\frac{dT}{d\xi}=\pm h\left(\left(\alpha(T^{*}-T_1)\right)^2+2j^2\int_{T}^{T_1}F(T')dT'\right)^{1/2}. \tag{6}$$

The relation (6) is a differential equation with separable variables, but the choice of the sign on its right side is not unique and depends on heat exchange conditions on the layer surface at $\xi=1$. Moreover, the derivative $dT/d\xi$ under certain conditions can change the sign within the interval $(0;1)$. It follows from the equation (5) that the left side of the equality (6) decreases monotonically in this interval, but the function $T(\xi)$ can not only decrease or increase monotonically, but also reach maximum within this interval at the point $\xi=\xi_*$ when a positive value of the derivative $dT/d\xi$ in the neighborhood of this point changesinto the negative one.

It is possible to clarify the further solution of the problem by substituting the second boundary condition (2) for the condition of an ideal thermal insulation of the layer surface at $\xi=1$. In this case, the derivative $dT/d\xi$ in the interval $[0;1]$ will decrease from a certain positive value at $\xi=0$ to zero at $\xi=1$, and the constant in the first integral of the equation (5) will be

$$C_1'=(jh)^2\int_{T_0}^{T_1}F(T)dT.$$

Then, instead of the equation (6), we obtain

$$\lambda(T)\frac{dT}{d\xi}=jh\left(2\int_{T}^{T_1^{\circ}}F(T')dT'\right)^{1/2}, \tag{7}$$

where T_1° is the temperature on the ideally insulated surface. From the equality of the heat flow density

$$q_0'=\frac{\lambda(T)}{h}\frac{dT(\xi)}{d\xi}\bigg|_{\xi=0}=j\left(2\int_{T_0}^{T_1^{\circ}}F(T')dT'\right)^{1/2},$$

removed through the surface of the layer at $\xi=0$ and the total energy release capacity jU_*, necessary per unit area of this surface, we find the relation

$$U_*=\left(2\int_{T_0}^{T_1^{\circ}}F(T)dT\right)^{1/2}, \tag{8}$$

characteristic for a dielectric plane layer with one ideally insulated surface [13] and in this case the determining value T_1° is one-valued.

If $T^* > T_1^\circ$, the heat flow from the ambient medium will enter the dielectric layer, i.e. $T^* > T_1$ and $dT/d\xi\big|_{\xi=1} > 0$, which under the condition $U_* = \text{const}$ will lead to a certain increase in the positive derivative $dT/d\xi\big|_{\xi=0}$. Conversely, if $T^* < T_1^\circ$, the ambient medium will cool the dielectric layer, i.e. $T^* < T_1$ and $dT/d\xi\big|_{\xi=1} < 0$. Hence, the conventional value $T_p^* = T_1^\circ$ of the ambient medium temperature determines the level, the excess of which at $T^* > T_p^*$ ensures that the left side of the equation (6) is positive in the whole interval of the coordinate variation ξ.

The value of the ambient medium temperature T_m^*, which, when the condition $T^* < T_m^*$ is satisfied provides a monotone decrease of the function $T(\xi)$ in the interval $(0;1)$, can be found, by setting $T^* = T_m^*$, $T = T_0$ and $dT/d\xi\big|_{\xi=0} = 0$ in the eqaution (6). The latter equality in the monotone decrease of the derivative $dT/d\xi$ in this interval ensures its negative value within the whole interval. Then from the equation (6) we obtain

$$\alpha\left(T_1^* - T_m^*\right) = j\left(2\int_{T_1^*}^{T_0} F(T)dT\right)^{1/2},$$

where T_1^* is the temperature of the layer surface at $\xi = 1$ and the temperature of the ambient medium T_m^*.

Since the surface of the layer with the given temperature T_0 is simultaneously and ideally thermally insulated, the following equality is correct

$$U_* = \left(2\int_{T_1^*}^{T_0} F(T)dT\right)^{1/2}. \tag{9}$$

It is analogous to the equality (8) and uniquely determines the temperature T_1^*. From the two latter formulae we find $j = \alpha(T_1^* - T_m^*)/U_*$. To calculate the value T_m^*, we need one more condition, which follows from the solution of the equation (6), applied to the considered situation, and takes the following form

$$\lambda(T)\frac{dT}{d\xi} = -\alpha\left(T_1^* - T_m^*\right)h\left(1 - \frac{2}{U_*^2}\int_{T_1^*}^{T} F(T')dT'\right)^{1/2}.$$

After integrating and determining the constant from the condition $T(0)=T_0$ we obtain

$$\alpha\left(T_1^*-T_m^*\right)r_1\ln\frac{r}{r_0}=\int_{T(r)}^{T_0}\left(1-\frac{2}{U_*^2}\int_{T_1^*}^{T}F(T')dT'\right)^{-1/2}\lambda(T)dT. \tag{10}$$

Hence, after setting $\xi=1$ and $T(1)=T_1^*$, for the known value T_1^* we find the temperature T_m^*.

At $T^*\le T_m^*$ the left side of the equation (6) will be negative in the whole interval $(0;1)$, and when $T_m^*<T^*<T_p^*$ the sign of the derivative $dT/d\xi$ changes, i.e. $dT/d\xi\big|_{\xi=\xi_*}=0$ at some point $\xi_*\in(0;1)$, in which the function $T(\xi)$ takes up the maximum value $T_*(\xi_*)$. This property to a large extend complicates the subsequent solution of the problem.

3. THE SECOND STAGE OF THE SOLUTION OF THE PROBLEM

The subsequent procedure for solving the problem depends on the result of comparing the fixed value T^* of the temperature of the ambient medium with the previously calculated values $T_p^*=T_1^\circ$ and T_m^*. If $T^*\ge T_p^*$, on the right side of the relation (6) we should choose the plus sign and after integrating and determining the constant from the condition $T(0)=T_0$ write

$$h\xi=\int_{T_0}^{T(\xi)}\left(\left(\alpha(T^*-T_1)\right)^2+2j_0^2\int_{T}^{T_1}F(T')dT'\right)^{-1/2}\lambda(T)dT, \tag{11}$$

and in the particular case $T^*=T_p^*$ in this formula it is necessary to set $T_1\equiv T_p^*$ and

$$j=\frac{1}{h}\int_{T_0}^{T_p^*}\left(2\int_{T}^{T_p^*}F(T')dT'\right)^{-1/2}\lambda(T)dT. \tag{12}$$

With the strict inequality $T^*>T_p^*$, equating the product jU_* of the difference of heat flows removed through the surface at $\xi=0$ and supplied through the surface at $\xi=1$, we obtain

$$jU_*=\left(\left(\alpha(T^*-T_1)\right)^2+2j^2\int_{T_0}^{T_1}F(T)dT\right)^{1/2}-\alpha(T^*-T_1).$$

This implies

$$j=2U_*\alpha(T^*-T_1)\left(2\int_{T_0}^{T_1}F(T)dT-U_*^2\right)^{-1}, \tag{13}$$

which allows us to exclude j from the formula (11) and write

$$\alpha(T^* - T_1)h\xi = \int_{T_0}^{T(\xi)} \left(1 + 8U_*^2 \left(2\int_{T_0}^{T_1} F(T)dT - U_*^2\right)^{-2} \int_{T}^{T_1} F(T')dT'\right)^{-1/2} \lambda(T)dT. \quad (14)$$

This relation makes it possible to first calculate the value T_1, if we set $\xi = 1$ and $T(\xi) = T_1$, and then calculate the temperature distribution $T(\xi)$ over the thickness of the dielectric layer.

If $T^* \le T_m^*$, the function $T(\xi)$ monotonically decreases in the interval $(0;1)$. In the particular case $T^* = T_m^*$ after calculating the value T_1 this function from the equlity (9) is uniquely determined by the relation (10). With the strict inequality $T^* > T_m^*$ after choosing the minus sign in the equation, integrating and determining the constant from the condition $T(0) = T_0$ nstead of the formula (10), we obtain

$$h\xi = \int_{T(\xi)}^{T_0} \left(\left(\alpha(T_1 - T^*)\right)^2 - 2j^2 \int_{T_1}^{T} F(T')dT'\right)^{-1/2} \lambda(T)dT. \quad (15)$$

To find the temperature $T(\xi)$ distribution in the dielectric layer, it is necessary to apply the relation (15), which requires preliminary calculation of the values T_1 and j from two independent equations. The first such equality follows from this relation when $T(\xi) = T_1$ and $\xi = 1$, and the second one — from the equality of the product jU_* of the difference of heat flows, removed through the surface when $\xi = 1$ and supplied through the surface when $\xi = 0$:

$$jU_* = \alpha(T_1 - T^*) - \left(\left(\alpha(T_1 - T^*)\right)^2 - 2j^2 \int_{T_1}^{T_0} F(T)dT\right)^{1/2}.$$

From here it follows that

$$j = 2U_*\alpha(T_1 - T^*)\left(U_*^2 + 2\int_{T_1}^{T_0} F(T)dT\right)^{-1}, \quad (16)$$

and after plugging j in the relation (15) we obtain

$$\alpha(T_1 - T^*)h\xi = \int_{T(\xi)}^{T_0} \left(1 - 8U_*^2 \left(U_*^2 + 2\int_{T_1}^{T_0} F(T)dT\right)^{-2} \int_{T_1}^{T} F(T')dT'\right)^{-1/2} \lambda(T)dT. \quad (17)$$

The relation (17) allows us first to determine the value T_1, if we set $T(\xi) = T_1$ and $\xi = 1$, and after that find the temperature $T(\xi)$ distribution over the thickness of the dielectric layer.

When the inequality $T_m^* < T^* < T_p^*$ is satisfied, the calculation of the nonmonotone temperature $T(\xi)$ distribution in the dielectric layer is the most complicated. First, owing to the condition $dT/d\xi\big|_{\xi=\xi_*} = 0$ at $\xi \in (0;\xi_*)$ by integrating the equation analogous to the equation (7)

$$\lambda(T)\frac{dT}{d\xi} = jh\left(2\int_T^{T_*} F(T')dT'\right)^{1/2}, \qquad (18)$$

and defining the constant from the condition $T(0) = T_0$, we obtain

$$h\xi = \int_{T_0}^{T(\xi)} \left(2j^2 \int_T^{T_*} F(T')dT'\right)^{-1/2} \lambda(T)dT, \quad \xi \in [0;\xi_*]. \qquad (19)$$

When $\xi > \xi_*$ it is necessary to choose the minus sign in the equation (6) and after integrating and finding the constant from the condition $T(1) = T_1$ to write

$$h(1-\xi) = \int_{T_1}^{T(\xi)} \left(\left(\alpha(T_1 - T^*)\right)^2 - 2j^2 \int_{T_1}^{T} F(T')dT'\right)^{-1/2} \lambda(T)dT, \quad \xi \in [\xi_*;1]. \qquad (20)$$

After setting in the formulas (19) and (20) $T(\xi) = T_*$ and $\xi = \xi_*$, and termwise adding these formulas, we obtain

$$h = \int_{T_0}^{T_*} \left(2j^2 \int_T^{T_*} F(T')dT'\right)^{-1/2} \lambda(T)dT + \int_{T_1}^{T_*} \left(\left(\alpha(T_1 - T^*)\right)^2 - 2j^2 \int_{T_1}^{T} F(T')dT'\right)^{-1/2} \lambda(T)dT. \qquad (21)$$

By analogy with the formulas (8) and (16), we can write

$$\left(2\int_{T_0}^{T_*} F(T)dT\right)^{1/2} + \left(2\int_{T_1}^{T_*} F(T)dT\right)^{1/2} - U_* = f_1(T_*, T_1) = 0. \qquad (22)$$

Moreover, from the equality of the product jU_* to the sum of the heat flows removed through both surfaces of the dielectric layer, follows the relation

$$jU_* = \left(2j^2 \int_{T_0}^{T_*} F(T)dT\right)^{1/2} + \alpha(T_1 - T^*).$$

From this and from the formula (22) we find

$$j = \alpha(T_1 - T^*)\left(2\int_{T_1}^{T_*} F(T)dT\right)^{-1/2} \qquad (23)$$

and plugging it the relation (21), we write

$$\left(\int_{T_1}^{T_*} F(T)dT\right)^{1/2} \left(\int_{T_0}^{T_*}\left(\int_{T}^{T_*} F(T')dT'\right)^{-1/2} \lambda(T)dT + \int_{T_1}^{T_*}\left(\int_{T}^{T_*} F(T')dT'\right)^{-1/2} \lambda(T)dT\right) - \\ -\alpha(T_1 - T^*)h = f_2(T_*, T_1) = 0. \qquad (24)$$

The relations (22) and (24) make it possible to calculate the values of T_1 and T_* and then, using formulae (19) and (20), as well as the equation (23), to find the temperature $T(\xi)$ distribution in the dielectric layer.

4. THE CALCULATION EXAMPLE

Let us do the quantitative assay of the obtained options for solving the problem through the example of a plane dielectric layer made of polymeric material – amorphous polycarbonate, for which the experimentally obtained temperature dependences of the heat conduction coefficient and electric conductivity are known [1]. Fig. 1 shows the function graph $\lambda(T)$, and fig. 2 in semilogarithmic coordinates gives the function graph $\rho(T)$, where $\rho_0 = 3{,}16 \cdot 10^{14}$ Ohm. As initial data, we take $h = 0{,}01$ m, $T_0 = 300$ K, $\alpha = 10$ W/(m^2·K) and $U_* = 40$ mV.

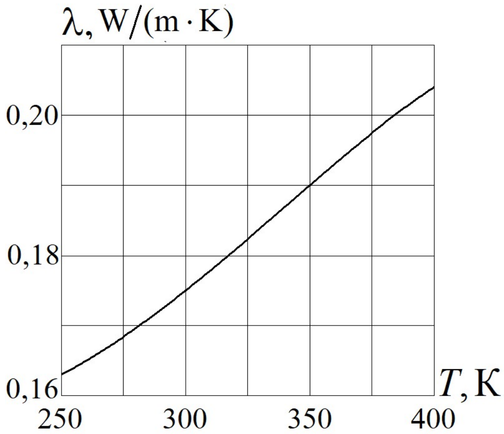

Fig. 1. Temperature dependence of polycarbonate heat conduction coefficient

According to the equations (8) and (9), temperatures $T_p^* = T_1^\circ \approx 315{,}7$ K and $T_1^* \approx 286{,}5$ K correspond to the given data. The first value of the temperature corresponds to the condition of an ideal thermal insulation on the surface of the layer at $\xi = 1$, and the second one corresponds to the absence of heat flow removal from the surface of the layer at $\xi = 0$, when the temperature of the ambient medium has the following value $T_m^* \approx 238{,}4$, according to the equation (10). It should be noted that using this relation to determine the value T_m^* is formally connected with the calculation of the improper integral and requires the use of special algorithms, since the replacement of the lower limit of the outer integral in accordance with the equation $T(1)=T_1^*$ leads to an unlimited increase in the subintegral function, as the upper limit of the inner integral tends to the value T_0.

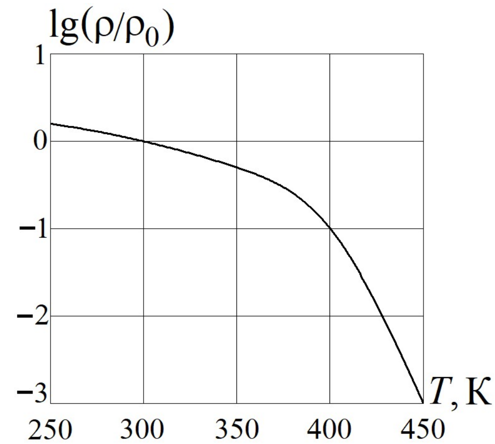

Fig. 2. Temperature dependence of polycarbonate electrical resistivity

In addition, it is well to bear in mind that the temperature T_1^* can be found only approximately by using the equation (9), and then it is possible to use its rounded value. Not to obtain the result in the form of a complex number and the unlimited subintegral function increase when calculating the outer integral in the relation (10), the rounding of the value T_1^* in this case should be ample.

The calculated values of the temperatures T_1° and T_1^* correspond to monotone temperature distributions $T(\xi)$ over the thickness of the dielectric layer. The graph of such distribution at $T_1=T_1^\circ$ is plotted in Fig. 3 by the relation (11) with the value of the electric current density $j\approx 14{,}34\cdot 10^{-6}$ A/m^2, calculated from the formula (12). In the case when $T_1=T_1^*$ we used the relation (10) when plotting the function graph in this figure. The electric current density was approximately 12 microamperes per square meter.

For the value $T^*=320$ K, exceeding T_p^*, after setting in the relation (14) $T(\xi)=T_1$ and $\xi=1$, we obtain $T_1\approx 318{,}4$ K. In this case from the formula (13) it follows that $j\approx 14{,}59\cdot 10^{-6}$ A/м^2. Fig. 3 shows the monotone temperature $T(\xi)$ distribution curve, which is built according to the relation (14) and located above the graph, corresponding to the ideal thermal insulation of the layer surface at $\xi=1$. When $T^*=230$ K $<T_m^*$ the calculation of the temperature $T(\xi)$ distribution, whose graph is shown in Fig. 3 as well, is done according to the formula (17), and the value $j\approx 11{,}82\cdot 10^{-6}$ A/m^2 is obtained from the eqality (16).

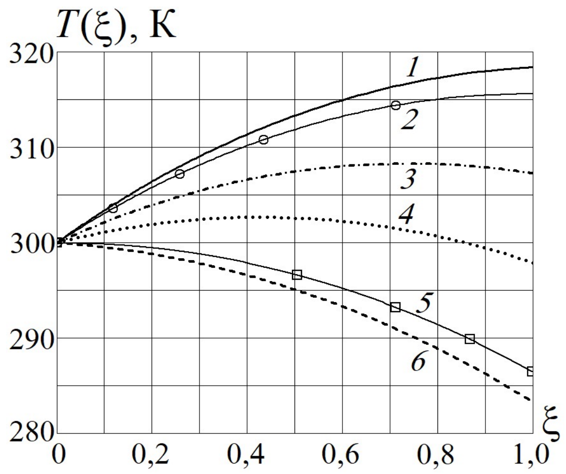

Fig. 3. Temperature $T(\xi)$ distributions over the thickness of the dielectric layer at different values of the ambient temperature: 1 – $T^* = 323$ K; 2 – $T_p^* = T_1^\circ \approx 315,7$ K; 3 – $T^* = 293,2$ K; 4 – $T^* = 268,4$ K; 5 – $T_m^* \approx 238,4$ K; 6 – $T^* = 230$ K

Since the relations (22) or (24) written in the form of homogeneous equations do not let express explicitly any of the required temperatures, to solve these simultaneous equations, it is expedient to apply one of the methods of unconstrained minimization [17, 18] of the function of two variables

$$f(T_*, T_1) = \left(f_1(T_*, T_1)\right)^2 + \left(f_2(T_*, T_1)\right)^2.$$

Then the desired values T_* and T_1 will correspond to the zero value of the nonnegative function. $f(T_*, T_1)$. By minimizing this function at $T^* = 293,2$ K we found the values $T_* \approx 308,3$ K and $T_1 \approx 307,3$ K, which, according to the equation (23), correspond to the value $j \approx 13,6 \cdot 10^{-6}$ A/m^2, and at $T^* = 268,4$ K we obtained $T_* \approx 302,7$ K, $T_1 \approx 297,9$ K and $j \approx 12,85 \cdot 10^{-6}$ A/m^2 respectively. The temperature $T(\xi)$ distribution graphs plotted according to the formulae (19) and (20) and with the use of the specified parameter values, are shown in Fig. 3.

Using the equations (3) and (4), we can write the relation

$$E_*(\zeta) = \frac{E(\zeta)h}{U_*} = \frac{jh}{U_h}\rho\left(T(\zeta)\right).$$

It allows us to find the distribution $E_*(\zeta)$ of the dimensionless absolute value of the electric field intensity over the thickness of the dielectric layer. Fig. 4 shows such distributions for all calculated and given in Fig. 3 options for temperature $T(\xi)$ distribution. The most nonuniform distribution $E_*(\xi)$ corresponds to the value of the temperature T^*, which is the largest among those taken in the calculations. Moreover, the greatest deviation from the uniform distribution corresponding in Fig. 4 to a horizontal straight line with an ordinate equal to one, arises on the surface of the layer with a given temperature $T_0 = 300$ K.

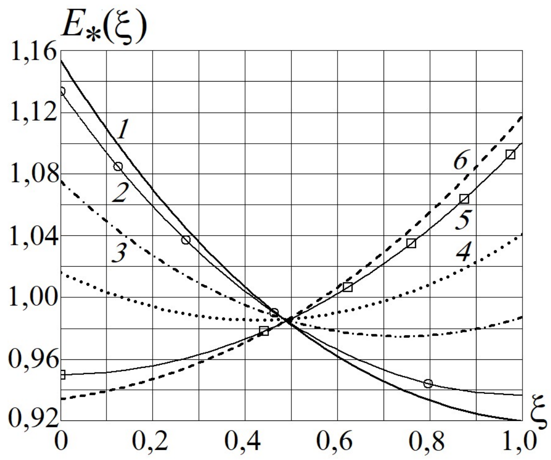

Fig. 4. Distributions over the thickness of the dielectric layer of the relative magnitude of the electric field intensity (the designations of the curves are identical to those in Fig. 3)

With a decrease in the value, first there occurs the tendency for the flattening of distribution $E_*(\xi)$. When $T^* \approx 268,4$ K and the temperature distribution is also the closest to the uniform distribution (see Figure 3), the values E_* deviate to a lesser extent from the uniform distribution, and the function $E_*(\xi)$ acquires an explicitly nonmonotone character. However, a further decrease in the value T^* leads to an increase in the nonuniformity of the temperature distribution over the thickness of the dielectric layer and, as a consequence, to an increase in the nonuniformity of the distribution $E_*(\xi)$, but now the largest unit deviations of the values E_* occur on the cooled surface of the layer at $\xi = 1$.

CONCLUSION

Based on the formulated nonlinear mathematical model of steady-state heat conduction in a plane dielectric layer at the constant electric potential difference on the surfaces of this layer, integral ratios are obtained that allowed us according to the temperature dependences of the thermal conduction coefficient and electrical resistivity of the dielectric material to estimate the temperature distribution and the absolute value of the electric field intensity. The obtained results make it possible to evaluate the feasibility of using a specific material, including polymer, as a dielectric in the high-voltage electrical devices being designed. A calculation example is given for the dielectric layer made of amorphous polycarbonate.

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