Circuit theory approach to stability and passivity analysis of nonlinear dynamical systems

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In this paper, we address the problem of global asymptotic stability and strong passivity analysis of nonlinear and nonautonomous systems controlled by second-order vector differential equations. First, we construct this system or the differential equation from a nonlinear time varying network of the circuit theory. Our system and with its real energy function generalize and improve upon some well-known studies in the literature. This system and its special forms have ample applications in many scientific investigations. We realized that most of the first- and second-order ordinary differential equations can be represented by LRC circuits. So, the energy (Lyapunov) functions of the systems can be constructed directly without much trial and error. By this way, the application of Lyapunov's direct method may become a standard technique for physical systems. We illuminate this idea with many applications and improvements. We also compare the Lyapunov stability theory with Hamiltonian and Lagrangian systems in the sense of conservative and dissipative systems. Then, we provide new explicit stability and passivity results with minimum criteria.

Keywords: nonlinear LRC circuits; Lyapunov stability; nonlinear differential equations; passivity

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1. Introduction

Stability theory dates back almost to the birth of the theory of differential equations. Poincare, Lagrange, Hamilton, and Lyapunov were the first to study the stability theory of dynamical systems [1, 2]. Their investigations were based on the predictability of solutions to differential equations. The Poincaré regular solution is simply a Lyapunov stable motion [2]. Hamiltonian and Lagrangian systems correspond to conservative systems (exact differential equations), but engineering systems often have damping [3]. The Lyapunov stability theory can be applicable to both conservative systems (undamped case) and dissipative systems (damped case) or applicable to arbitrary differential equations. The quantum advance in the stability theory of dynamical systems is due to the Russian mathematician A.M. Lyapunov (1892) [1]. In this paper, we will illuminate the stability theory of dynamical systems with LRC circuit systems that may be concrete models. For example, the first nuclear reactors were modeled by analog computers (electrical models) before the reactors themselves were built mechanically [4] because electrical models are accurate, safe, and inexpensive, and circuit elements can be found easily. In this connection, we study the qualitative behavior of the network

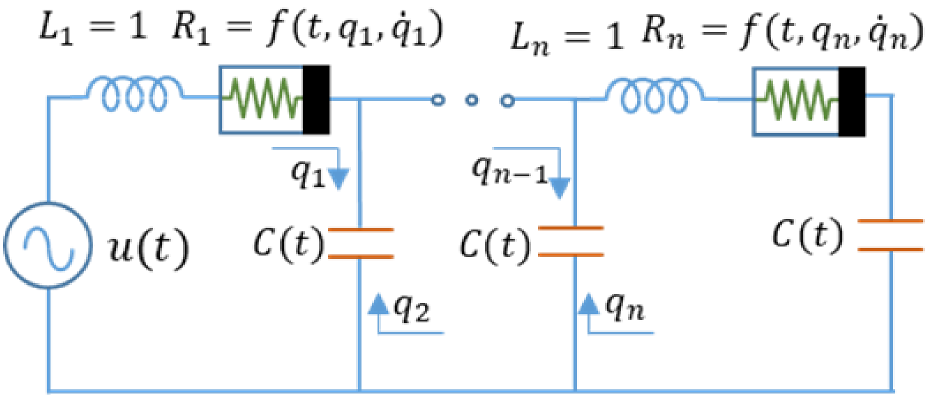

Fig. 1. Time varying nonlinear *LRC* circuit

which with the following arguments generate the differential equation

$$\ddot{x} + f(t, x, \dot{x})\, \dot{x} + b(t)\, g(x) = u(t)\,, \tag{1}$$

where $t \in \mathfrak{R}^+$, $\mathfrak{R}^+ = [\, t_0, \infty)$, $t_0 \geq 0$ denotes time, $x \in \mathfrak{R}^n$ is the state variable which represents the motion of the charges q_i, f and b $(= c^{-1},\ \det c \neq 0)$ are continuous $n \times n$ diagonal matrix functions denote the dissipative terms (resistance in ohm) and bounded storage elements (capacitance in $(farad)^{-1}$) of the above circuit, respectively. g is an $n-$ vector function denotes the storage charge on the plate of the time-varying capacitors with $g(0) = 0$, and all $L = 1$ henry, while $u \in \mathfrak{R}^m$ is called the input voltage or control function in volt. In the real Euclidean space $\Re^n$, the usual norm denoted by $\|\cdot\|$, the symbol $\langle\ ,\ \rangle$ stands for the scalar product, $\lambda_i(\cdot), (i = 1, 2, \dots, n)$ are the eigenvalues of the $n \times n$ matrix $(\cdot)$.

Second-order differential equations have ample applications in many scientific investigations. Therefore, (1) and its special forms are very important problems in mathematical control theory [5], circuit theory [6], and especially in the application of the power transmission line model (with $G \approx 0$, usually) [7]. In this context, we can find a large number of excellent works in the literature discussing qualitative behavior of second-order differential equations [1-5], [8 -14]. The qualitative behaviors of the mentioned nonlinear systems have been extracted by Lyapunov's direct method. This is still one of the most efficient methods to investigate the qualitative behaviors of nonlinear dynamical systems [15,16]. The genius of the method is that it can be applicable to any differential equation. But Hamilton and Lagrange principles are not so flexible. They are only applicable to exact differential equations (conservative systems). So, the method is an indispensable tool in the qualitative analysis of nonlinear systems.

The connection between stability and passivity is founded by the proposed Lyapunov stability theorem [17]. The term *passivity* is a fundamental feature of the dissipative dynamical systems [18-20]. Electrical networks, viscoelastic systems, and thermodynamic systems with their external sources are representative examples of dissipative systems. The key basis in developing the dissipativity hypothesis for general dynamical systems was introduced by Willems [20]. Dissipativity can be described in terms of a dissipation inequality including the energy function (*or storage function*) and the power input (*or supply rate*) of the system under consideration. The dissipativity theory gives a fundamental framework for the stability and passivity analysis for dynamical systems.

Stored energy or entropy is also a fundamental property of storage energy or the Lyapunov function. A dynamical system is dissipative if it involves a damping term; otherwise, the system is conservative. The time derivative of storage or the Lyapunov function of a conservative system is zero, since the energy of the system is constant along the system orbits. But for a dissipative system the derivative is equal to the negative value of the dissipated power in the system when the input of the system is zero (isolated, or undisturbed system). For example, the time derivative (directional) of the energy function $E(t)$ of a simple series unforced LRC circuit is

$$\dot{E}(t) = -R\,I^2, \tag{2}$$

where R is the resistance and I is the current of the circuit. The above signification with (2) not exist in the recent literature ([1], [8-14]). Because the researchers usually have difficulties constructing a suitable Lyapunov function for a given system. In literature, the general idea is that there exists no a general approach to construct such functions. But for the first- and second-order differential equations, we may overcome this problem by giving physical meanings to these equations with various LRC circuit systems or extracting these equations from various LRC circuits. Then, we construct the storage function of each system from ***the power-energy relationship***. So, in this paper, we will improve some well-known studies [8-11] with suitable tools below Theorem 1.

The stability of the following differential equations (with their arguments) has been investigated in [8], [9], [10], and [11], respectively,

$$\ddot{x} + a(t)f(x,\dot{x})\dot{x} + b(t)g(x) = p(t,x,\dot{x}) \tag{a1}$$

$$V_0 = \frac{1}{2}y^2 + b(t)\int_0^x g(\xi)\,d\boldsymbol{\xi} + k, \tag{a2}$$

where k is a positive constant,

$$\dot{V}_0 = -a(t)f(x,y)y^2 + p(t,x,y)y + \dot{b}(t)\int_0^x g(\xi)\,d\boldsymbol{\xi}; \tag{a3}$$

$$\ddot{x} + \dot{x} + p(t)\,g_1(x) + q(t)\,g_2(x) = 0, \tag{b1}$$

$$V(t,x,y) = \frac{1}{2}y^2 + p(t)\,G_1(x) + q(t)\,G_2(x), \tag{b2}$$

where $G_i(x) = \int_0^x g_i(\xi)d\xi \quad (i = 1, 2)$,

$$\dot{V}(t,x,y) = \dot{p}(t)G_1(x) - p(t)xg_1(x) + \dot{q}(t)G_2(x) - q(t)xg_2(x); \qquad \text{(b3)}$$

$$\ddot{x} + a(t)b(x) = 0, \qquad (c1)$$

$$V(t,x,y) = \frac{1}{2}y^2 + a(t)\int_0^x b(s)\,ds, \qquad (c2)$$

$$\dot{V}(t,x,y) = \dot{a}(t)\int_0^x b(s)\,ds \le 0; \qquad (c3)$$

and

$$\ddot{X} + A(t)\,F\,(X) = 0, \qquad \text{(d1)}$$

$$V_0 = \frac{1}{2}\,\langle\, Y, Y\,\rangle + \int_0^1 \langle\, A(t)\,F\,(\sigma\,X, X)\,\rangle\,d\sigma\,, \qquad \text{(d2)}$$

$$\dot{V}_0 = \int_0^1\int_0^1 \langle \sigma_1 \dot{A}(t) J_F(\sigma_1\sigma_2 X)X, X\rangle d\sigma_2\sigma_1 \le 0, \qquad \text{(d3)}$$

where $t \in \Re^+, X \in \Re^n, A(t)$ is a symmetric $n \times n$ -matrix and $F: \Re^n \to \Re^n$.

The above differential equations are given without any physical meaning. So, the given Lyapunov function for each equation is not the suitable tool. Hence, the obtained stability results of the above studies are very complex. Therefore, we will develop a new approach that adds vitality to the given systems so that they will be obtained from a set of $\{LRC\}$ circuit systems. Then, we construct the real energy function of each system from the storage elements of the system. So, this approach is a systematic way that (this after) may play an important role in the stability theory of the dynamical systems. Further, by the way, we realized that for first and second order unforced physical systems the directional derivative of all the energy (Lyapunov) functions are confirmed by (2). This standardization is not existing in the literature that can also be applicable to higher order systems.

2. Preliminaries

A dynamical system is examined as a theoretic mathematical model that maps inputs (excitations, causes) into outputs (responses, effects) by a set of intermediate variables (state variables). So, we usually encounter the following dynamical system in the advanced theory of nonlinear oscillation:

$$\dot{x}(t) = f\big(t, x(t), u(t)\big), \;\; x(0) = x_0, \tag{3}$$

where $t \in \Re^+,\;\; \Re^+ = [t_0,\; \infty),\; t_0 \geq 0, x \in \Re^n$ is the state vector, $f(t, x, u) \in C^1$ $(\Re^+ \times \Re^n \times \Re^m,\; \Re^n)$ is Lipschitz, $f(t, 0, 0) = 0$ while $u \in (\Re^m,\; \Re^m)$ is the input or forcing function. Assume that the existence and uniqueness solution hold for the initial value problem (3), and let the measured output of (3) is $y(t) = \dot{x}(t)$.

For the energy or Lyapunov function $L_1(t) = L_1(t, x) \in C^1$ $(\Re^+ \times \Re^n,\; \Re^+)$ of (3), we define the derivative $\dot{L}_1(t)$ of $L_1(t)$ along the motions of (3) as

$$\frac{d}{dt} L_1\big(t, x(t)\big) = \dot{L}_1(t).$$

Now, we can define some properties of the energy or Lyapunov functions [1] as the following.

Definition 1 A function α $(\Re^+, \Re^+)$ is of *class* $\boldsymbol{\mathcal{K}}$ if it is continuous on $[0,\; \infty)$, monotonically increasing, and $\alpha(0) = 0$. A class $\boldsymbol{\mathcal{K}}$ function $\alpha(r)$ belongs to class $\boldsymbol{\mathcal{K}}_\infty$ if $\alpha(r) \to \infty$ as $r \to \infty$.

Definition 2 A function $L_1(t, x) \in C^1$ $(\Re^+ \times \Re^n,\; \Re^+)$ is said to be positive definite function if there exists a function α of *class* $\boldsymbol{\mathcal{K}}$ *such that*

$$\alpha(\|x\|) \leq L_1(t, x),\;\; \forall t \geq 0, \;\;\; \forall x \in \Re^n.$$

L_1 is decrescent if there exists a function β of *class* $\boldsymbol{\mathcal{K}}$ such that

$$L_1(t, x) \leq \beta(\|x\|),\; \forall t \geq 0,\; \forall x \in \Re^n.$$

If a Lyapunov fiction $\mathrm{L}_1(\mathrm{t}, \mathrm{x})$ is decrescent, then

$$\dot{L}_1\big(t, x(t)\big) < 0.$$

L_1 is a radially unbounded function with the property that

$$\alpha(\|x\|) \to \infty \quad\; as \;\;\; \|x\| \to \infty.$$

Definition 3 ([13] paraphrased) System (1) is passive if there exists a positive definite function $L(t) = L(t, x, y) \in C^1$ $(\Re^+ \times \Re^2,\; \Re^+)$ such that *(i)* $\dot{L}(t) \leq r(u, y)$.

Moreover, it is lossless if

(ii) $\dot{L}(t) = r(u, y)$,

in this case, (1) is a conservative system (there is no a damping term, that is, $f(t, x, \dot{x}) = 0$). (3) is a dissipative system which has at least one damping term h. Consequently, the system is strongly passive if

(iii) $\dot{L}(t) + h(t, x, y) \leq r(u, y)$

where $y(t) = \dot{x}(t)$, $h \in C(\Re^+ \times \Re^2, \Re^+)$ is some positive definite function, $r \in C(\Re \times \Re, \Re)$ and

$$r(t) = \big(u(t), y(t)\big) = \sum_{i=1}^{m} u_i y_i, \ (i = 1, \dots, m)$$

is the supply rate function of (3) defined for any admissible u and y satisfy

$$\int_0^t \|r(s)\| ds < \infty, \forall\, t \geq 0.$$

Lemma 1 ([15] paraphrased) Let $f(t, x, y)$ be a positive definite real symmetric $n \times n$ matrix function for all $(t, x, y) \in \Re^+ \times \Re^{2n}$. Then,

$$0 < \inf_{t \in \Re^+,\ (x,\ y) \in \Re^{2n}} \lambda_{min}\ [f(t, x, y)] = f^- \ ;$$

and

$$\operatorname{sub}_{t \in \Re^+,\ (x,\ y) \in \Re^{2n}} \lambda_{max}\ [f(t, x, y)] = f^+,$$

where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ are the least and greatest eigenvalues of a symmetric matrix, respectively.

3. Main results

The equivalent system of (1) is given by

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -f(t, x, y)y - b(t)g(x) + u(t). \end{aligned} \tag{4}$$

From the storage elements (inductors and capacitors, respectively) of the network in Fig. 1, we can construct the following real energy or the suitable Lyapunov function $L_2 \in C^1\,(\mathfrak{R}^+ \times \mathfrak{R}^{2n},\ \mathfrak{R}^+)$ such as

$$L_2(t) = L_2(t,x,y) = \frac{1}{2}\langle y,y\rangle + \sum_{i=1}^{n}\Big(\int_0^{x_i} b_i\,(t)\,g(s_i)\,ds_i\Big) \tag{5}$$

where $ds_i = \dot{x}_i(t)dt$, $b_i\,(t) = b(t)\ (i = 1,\dots,n)$, x and y are the state vectors of the above network, and

$$x_i\ g(x_i) > 0\ \text{ for }\ x_i \neq 0, (i = 1,\dots,n),$$

where

$$G(x_i) = \int_0^{x_i} g(s_i)\ ds_i, \quad G(x_i) \to \infty\ \ as\ \ |x_i| \to \infty,\ \ (i = 1,\dots,n).$$

From Lemma 1, let define

$$0 < \inf_{t\,\in\,\mathfrak{R}^+} \lambda_{min}\ [b(t)] = b^-,$$

and

$$\operatorname*{sub}_{t\,\in\,\mathfrak{R}^+} \lambda_{max}\ [b(t)] = b^+,$$

where $b^-\ and\ b^+$ are positive, real constants. So, after some rearrangements, we have

$$\frac{1}{2}\|y\|^2 + b^-\big(\textstyle\sum_{i=1}^{n} G(x_i)\big) \le L_2(t) \ \le\ \frac{1}{2}\|y\|^2 + b^+\,\big(\textstyle\sum_{i=1}^{n} G(x_i)\big). \tag{6}$$

Consequently, L_2 is confirmed by Definition 2, and the discussion on the storage function [20].

Theorem 1 The isolated equilibrium solution $x(t) = 0$ of (2) with $u = 0$ is globally and asymptotically stable if

(i) $f^- > 0$;

(ii) $g(0) = 0$;

(iii) $x_i\ g(x_i) > 0,\ \ x_i \in \mathfrak{R} \neq 0,\ \ (i = 1,\dots,n)$;

(iv) $L_2 \to \infty\ \ as\ \ \|x\|^2 + \|y\|^2 \to \infty$.

Proof Now, we show that the time derivative of (5) along the trajectories of (4) is negative definite. Hence, we have

$$\dot{L}_2\,(t) = \langle\, y, \dot{y}\,\rangle\, + \langle b_i\,(t)\; g\;(x_i), \dot{x}_i \rangle\,.$$

It follows from (4) that

$$\dot{L}_2\,(t)\;=\;-\,\langle\, y, f(t,x,y)\; y\,\rangle - \langle\; b\;(t)\; g(x), y\;\rangle\; + \langle b_i\;(t)\; g\;(x_i),\;\; y_i\rangle. \tag{7}$$

The last two terms in (7) are equal in magnitude. Then, with the assumption of Lemma 1, it follows that

$$\dot{L}_2(t) = -\langle\, y, f(t,x,y) y\,\rangle \leq -f^-\; \|y\|^2 < 0. \tag{8}$$

(8) is the dissipated power of (4).

Theorem 1 implies that $\dot{L}_2 < 0$ on $\Re^+ \times \Re^{2n}$ for $y \neq 0, L_2(\infty) = 0$ and L_2 is radially unbounded such that $L_2(t,x,y) \to \infty$ as $\|x\|^2 + \|y\|^2 \to \infty$. The set S where $\dot{L}_2 = 0$ is $\{0,0\}$. $\{0,0\}$ is the only invariant subset of S. Thus, the isolated $(u = 0)$ equilibrium solution $\big(x(t),\; y(t)\big) = (0{,}0)$ of (4) is globally and asymptotically stable. That is, the isolated system is lossless at infinity due to $u(t) =\; 0$. Hence, it matches real-world applications.

3.1. Comparisons, discussions and improvements

(A) Theorem 1 improves the result given in [8] such as:

$(a1)$ may represent a LRC circuit (dissipative) system with $L = 1$, $R = a(t)f(x,\dot{x}\,)$, and $C = 1/a(t)$, $(a(t) \neq 0)$ with electric charge $b(x)$.The equivalent system of $(a1)$ is

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -a(t)f(x,y)y - b(t)g(\,x)^{\boldsymbol{\cdot}} \end{aligned} \tag{$a4$}$$

The natural energy function for $(a1)$ must be

$$V(t,x,y) = \tfrac{1}{2}y^2 + \int_0^{x(t)} b(t)g(s)\,ds. \tag{$a5$}$$

Since, (a5) has been constructed from the physical meaning of (a1) by power- energy relationship. The time derivative of $(a5)$ with respect to $(a4)$ is

$$\dot{V}(t,x,y) = -a(t)f(x,y)y^2. \tag{a6}$$

$(a6)$ is the dissipated power of (a1) and verified by (2). When comparing $(a2)$ with $(a5)$, and $(a3)$ with $(a6)$: we conclude that $(a2)$ is not a consistent tool (Lyapunov function) for

$(a1)$, and we ask how it is constructed. In the sense of physical meaning $(a1)$ does not match with $(a2)$.

(B) Theorem 1 improves Theorem 2.2 given in [9] such as:

$(b1)$ is a special form of (1) for $n = 1$. Now, the actual energy function for $(b1)$ must be in the form of (5) such that

$$V(t,x,y) = \frac{1}{2}y^2 + \int_0^{x(t)}[p(t)g_1(s) + q(t)g_2(s)]\,ds, \qquad \text{(b4)}$$

where $ds = \dot{x}(t)dt$, $p > 0$ and $q > 0$ are continuous functions on $[0,\infty)$, g_1, g_2 are continuous functions on $\Re$, satisfying (A_1) of [9].

Then, the time derivative of (b4) along the solutions of (b1) gives

$$\dot{V}(t) = -y^2 < 0. \qquad \text{(b5)}$$

In fact, the coefficient of $\dot{x}$ in (b1) is 1. (b5) is confirmed with (2) due to the suitable tool. The comparisons between (b2) and (b4), between (b3) and (b5) give our improvement.

(C) Theorem 1 improves the paper (Theorem 1, 2, 3, 4, 5) given in [10] (here we only compare our Theorem 1 with Theorem1 in [10]) such as:

$(c1)$ may represent a LC circuit (conservative) system with $L = 1$, and $C = 1/a(t)$, $(a(t) \neq 0)$ with electrical charge $b(x)$. The equivalent system for $(c1)$ is

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -a(t)b(x)^{\cdot} \end{aligned} \qquad (c4)$$

The natural energy function of $(c4)$ must be

$$V(t,x,y) = \frac{1}{2}y^2 + \int_0^{x(t)} a(t)b(s)\,ds = M_0, \ \forall\, t \geq 0, \qquad (c5)$$

where M_0 is a positive constant. (c5) has been constructed from the physical meaning of (c1). The time derivative of $(c5)$ with respect to $(c4)$ is

$$\dot{V}(t,x,y) = -a(t)b(x)y + a(t)b(x)y = 0. \qquad (c6)$$

A conservative system does not work as a dissipative system, for convince, compare $(c3)$ with $(c6)$. Thus, we conclude that $(c2)$ is not a consistent tool for $(c1)$.

(D) Theorem 1 improves the unforced system given in [11] such as

Obviously, (1) generalizes $(d1)$ when $A(t)$ is a diagonal matrix, for $f(t, x, \dot{x}) = 0$ in (1), we have $(d1)$. Therefore, $(d1)$ is a conservative system since there is no a damping term in it (see (c1)). In the sense of physical meaning, $(d1)$ consists of a set of inductors and capacitors. So, its energy function must be in the form of (5) but equal to a constant such as

$$L_{d1}(t) = L_{d1}(t, X, Y) = \frac{1}{2}\langle Y, Y\rangle + \sum_{i=1}^{n}\left(\int_0^{x_i(t)} a_i\,(t)\, s_i\, ds_i\right) = M_1,\ \forall t \geq 0, \qquad \text{(d5)}$$

Where M_1 is a positive constant. The time derivative of (d4) along the solutions of $(d1)$ is

$$\dot{L}_{d1}(t) = -\langle A(t)\, X, Y\,\rangle + \langle a_i\,(t)\, x_i, y_i\rangle = 0. \qquad \text{(d5)}$$

$\dot{V}_0$ contradicts with (2), but (d5) is verified by (2). So, V_o does not work with $(d1)$. As a result, the zero solution of $(d1)$ does not occur since the solutions are oscillating. In other words, $(d1)$ is only marginally stable.

Theorem 1 can also be applicable to linear systems and first order systems. Thus, Theorem 1 improves many examples in the books [1] and [12-14] that based on Lyapunov approach.

The strong passivity result or the boundedness of the motions of (4) with the input function $u(t)$ is explained next.

Theorem 2 In addition to the hypothesis of Theorem 1, if there exists an admissible continuous input function $\|u(t)\| = u(t)$ satisfying the integral inequality

$$\int_{t_0}^{t} u(s)\, ds \leq K < \infty, \quad \forall\, t > t_0 \geq 0,$$

where $K > 0$ is a constant, every motion of (4) is bounded as $t \to \infty$ or the system (4) is strongly passive due to the energy dissipator element $f(\cdot) > 0$.

Proof Now from the total derivative of L_2 we write

$$\dot{L}_2(t) \leq -f^-\, \|y\|^2 + uy.$$

Then

$$\dot{L}_2(t) + f^-\, \|y\|^2 \leq uy. \qquad (9)$$

Inequality (9) is a special form of (iii) of Definition 3. This implies that (4) is strongly passive. Furthermore,

$$\dot{L}_2(t) \le u(t)(1 + \|y\|^2).$$

From (6), we get

$$\dot{L}_2(t) \le u(t) + 2u(t)L_2(t) . \quad (10)$$

First, integrating (10) from 0 to t (≥ 0), then using Theorem 2, we have

$$\dot{L}_2(t) \le K + 2\int_0^t L_2(s)u(s)\, ds.$$

Then, the Gronwall inequality with Theorem 2 yields

$$L_2(t) \le K\, exp\,(2K). \quad (11)$$

Finally, from (6) and (11), we have

$$\frac{1}{2}\|y\|^2 \le L_2(t) \le K\, exp\,(2K).$$

Hence, the function L_2 is bounded in magnitude. This implies that system (4) is strongly passive. That is, all the solutions or motions $(x(t), y(t))$ of (4) are also bounded.

Consequently, this study based on allowing the stability and passivity theories to be well understood in physical terms.

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