**THE PROBLEM OF DETERMINING THE SPEED OF SOUND AND MEMORY OF ANISOTROPIC MEDIUM**

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**Abstract.** We consider the problems of simultaneously identifying two unknowns. The wave propagation velocity and the memory of the layered medium will be determined. For their determination, two observations are used for how the boundary of the domain fluctuates. Main results are stability estimates and uniqueness theorems for the problems under consideration.

# 1 Introduction and statement of the problem

Many important materials used in modern technologies (such as nanotechnology) are viscoelastic and anisotropic. Viscoelastic materials have the properties of viscosity and elasticity upon deformation. Some mathematical models in the field of nanotechnology are contained, for example, in articles [3, 23] (see also references in them). In mathematical modeling of processes taking place in viscoelastic materials, there is a so-called system with memory, whose behavior is not completely determined by the state at the moment, but depends on the systems entire history, and therefore, describes an integro-differential equation that contains the corresponding integral with respect to the time variable.

A study of inverse problems for hyperbolic integro-differential equations and systems are the subject of research by many authors. Among the problems that are closer to the present work can be identified [8, 9]. In work [27], the inverse problem for a second order hyperbolic equation with an integral member of convolution type with respect to one-dimensional time - dependent memory function of the medium and solution of the direct problem is investigated. By Fourier's method, this problem is reduced to solving the Volterra integral equations with respect to the unknown functions of the time-dependent variable. In papers [20, 35] (see also references therein) the problem of determining the multidimensional kernel in viscoelasticity equation for an inhomogeneous isotropic medium is investigated. In [21, 22], the problem of the one-dimensional kernel reconstruction from viscoelasticity equation in the bounded and unbounded domains has been studied. The theorems for the global unique solvability of these problems in the class of continuous functions with weighted norms were proved. The basic feature inherent in [7, 17-19] and this paper is to use a boundary-localized and/or a fixed point of the spatial domain source, for the initiation of the physical process of wave propagation. Finally, we recall that the papers [10-13] are concerned with the problems of kernel determination from integro-differential equations with an integral of the convolution type. In the present paper, the approach of the works [8, 9] will be used. Besides, the works [2, 10-21, 38-40] that studied 1D and 2D inverse problems the system of integro-differential equations of a viscoelastic porous medium.

Regarding the problems devoted to determining a sub-integral function, belonging to hyperbolic equations, see works [10, 11]. In the work [10] the problem of finding out the memory, belonging to a three-dimensional wave equation with delta function at the right side is investigated.

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Further, in work [11] this problem is generalized in the case of a hyperbolic equation of the second order with a constant main part and variable coefficients at minor derivatives. Similar problems with distributed sources of disturbance can be found in works [25, 32]. In the article [25] the problem of determining a one-dimensional coefficient of wave distribution velocity and forms of impulse sources according to the information (0.3) is investigated. It turns out that to solve this problem it is sufficient to give the Fourier image of the function $g(x,t)$ in two different values of the transformation parameter. In this article we investigate the determination of two functions of one variable, one of which is under the integral sign similar to the method of the work [36].

The main feature, appropriate to other works and to the present work, is the use of a source localized on the boundary of the considered space domain; this source initiates the physical process of wave transmission. This feature essentially increases the meaning of the investigation for applications. Using Karchevsky and Fat'yanova [29-32] one can become familiar with numerical methods for solving these problems.

In this paper, we simultaneously determine the wave propagation velocity and the memory of the layered medium from the two-dimensional integro-differential wave equation with variable coefficient [8]

$$u_{tt} - \Lambda u - \bar{b}(z)u + u_t = \int_0^t k(t-\tau)\Lambda u(x,z,\tau)d\tau, \tag{0.1}$$

at conditions

$$u\Big|_{t<0} \equiv 0, \tag{0.2}$$

$$\Big[u_z(x,z,t) + \int_0^t k(t-\tau)u_z(x,z,\tau)d\tau\Big]_{z=+0} = \delta(x)\delta'(t) + \delta(x)\theta(t)f(t), \tag{0.3}$$

where $t \in \mathbb{R}$, $(x,z) \in \mathbb{R}^2_+ := \{(x,z) \in \mathbb{R}^2 : z > 0\}$, $\delta(\cdot)$ is the Dirac delta function, $\theta(\cdot)$ is the Heaviside function, $\Lambda$ is differential operator, which has the form

$$\Lambda u = \mu(z)\triangle u + \bar{a}(z)u_z,$$

in which $\triangle$ is two-dimensional Laplacian in the variable $(x,z)$ and $\bar{a}(z)$, $\bar{b}(z)$, $f(t)$ are given and continuous functions. In these equations, the coefficient $\mu(z)$ is a positive function of the class $C^2(\mathbb{R}_+)$, $\mathbb{R}_+ := \{y \in \mathbb{R} : z > 0\}$, and $k(t)$, $f(t)$ are continuous functions for $t \in \mathbb{R}$.

Given functions $\mu(z)$, $\bar{a}(z)$, $\bar{b}(z)$, $f(t)$, $k(t)$ the problem of finding a function $u(x,z,t)$, satisfying (in a generalized sense) equalities (0.1), (0.2), (0.3) is called the direct problem. The boundary condition simulates an instantaneous wave excitation source located at the point $x = 0$, $z = 0$. Suppose that the solution to this problem is given on the boundary of the domain: $\mathbb{R}^2_+ \times \mathbb{R}$

$$u\Big|_{z=+0} = g(x,t), \quad (x,t) \in \mathbb{R}^2_+. \tag{0.4}$$

The inverse problem is to determine two functions $\mu(z)$, $k(t)$ for a given functions $a(z)$, $b(z)$, $f(t)$, $g(x,t)$.

## 2 Preliminaries

We define the bilinear integral operator $L$ as follows:

$$L[k(t), u(x,z,t)] = u(x,z,t) + \int_0^t k(t-\tau)u(x,z,\tau)d\tau.$$

Sometimes, to reduce the notation, we will not indicate the dependence of functions on variables in the operator $L$ implying the dependence of the first function on the variables $t$; and the second, on $(x, z, t)$.

Denote by $\tilde{u} = F[u](\nu, z, t)$ the Fourier transform of the function $u(x, y, t)$ with respect to the variable $x$:

$$\tilde{u}(\nu, z, t) = \frac{1}{\sqrt{2\pi}} \int\limits_{\mathbb{R}} u(x, z, t) e^{i\nu x} dx.$$

Given the functions $\mu(z)$, $\bar{a}(z)$, $\bar{b}(z)$, $f(t)$, $k(t)$ the problem (0.1)-(0.3) is correctly set and it has a unique solution $u(x, y, t)$ possessing a compact support for any finite $t$. Equations (0.1), (0.2), (0.3) with respect to the function $u(x, z, t)$ can be written as

$$\frac{\partial^2 \tilde{u}}{\partial t^2} = \Big(\mu(z)\frac{\partial^2}{\partial z^2} + \bar{a}(z)\frac{\partial}{\partial z} - \nu^2 \mu(z)\Big) L[k, \tilde{u}] + \bar{b}(z)\tilde{u} - \tilde{u}_t, \ (\nu, z, t) \in \mathbb{R}^2_+ \times \mathbb{R}, \tag{1.1}$$

$$\tilde{u}\Big|_{t<0} \equiv 0, \quad \frac{\partial}{\partial z} L[k, \tilde{u}]\Big|_{z=+0} = \delta'(t) + \theta(t) f(t). \tag{1.2}$$

Let us introduce the new variable

$$y = \int\limits_0^z \frac{ds}{\sqrt{\mu(s)}}. \tag{1.3}$$

The function $z = l(y)$ defined by this equality is monotonic and defines a one-to-one correspondence between $y$ and $z$ given by (1.3). Denote $c(y) := \sqrt{\mu\big(l(y)\big)}$ and introduce the new function $\tilde{u}(\nu, z, t) = \bar{u}(\nu, y, t)$. Then inverse problem (1.1)-(1.2), (0.4) in terms of the newly introduced function $\bar{u}(\nu, y, t)$ and the variable $y$ will assume the form

$$\frac{\partial^2 \bar{u}}{\partial t^2} = \Big(\frac{\partial^2}{\partial y^2} + \frac{a(y) - c'(y)}{c(y)}\frac{\partial}{\partial y} - \nu^2 c^2(y)\Big) L[k, \bar{u}] + b(y)\bar{u} - \bar{u}_t, \quad (y, t) \in \mathbb{R}^2_+, \ \nu \in \mathbb{R}, \tag{1.4}$$

$$\bar{u}\Big|_{t<0} \equiv 0, \quad \frac{\partial}{\partial y} L[k, \bar{u}]\Big|_{y=+0} = \delta'(t) + \theta(t) f(t), \tag{1.5}$$

$$\bar{u}\Big|_{y=0} = \tilde{g}(\nu, t), \ t \in \mathbb{R}_+. \tag{1.6}$$

in which

$$a(y) = \bar{a}(z), \quad b(y) = \bar{b}(z), \quad \tilde{g}(\nu, t) = \frac{1}{\sqrt{2\pi}} \int\limits_{\mathbb{R}} g(x, t) e^{i\nu x} dx. \tag{1.7}$$

We now transform integro-differential equation (1.4) such that, first, there are no derivatives of the function $\bar{u}$ with respect to $y$ in the integrand and, second, the coefficients of $\bar{u}_y$ and $\bar{u}_t$ in terms outside the integral are equal to zero. These requirements can be satisfied by introducing a new function $v$ by the formula

$$v(\nu, y, t) = \Big[\bar{u}(\nu, y, t) + \int\limits_0^t k(t-\tau)\bar{u}(\nu, y, \tau) d\tau\Big] \sqrt{\frac{c(0)}{c(y)}} \cdot e^{\frac{1}{2}\int\limits_0^y \frac{a(s)}{c(s)} ds} \cdot e^{(1-k(0))t/2}. \tag{1.8}$$

It is easy to verify by direct calculation that the function $\bar{u}$ is then expressed in terms of $v$ as

$$\bar{u}(\nu, y, t) = \Big[e^{(k(0)-1)t/2} v(\nu, y, t) + \int\limits_0^t r(t-\tau) e^{(k(0)-1)\tau/2} v(\nu, y, \tau) d\tau\Big] \cdot e^{-\frac{1}{2}\int\limits_0^y \frac{a(s)}{c(s)} ds} \cdot \sqrt{\frac{c(y)}{c(0)}}, \tag{1.9}$$

where

$$r(t) = -k(t) - \int_0^t k(t-\tau)r(\tau)d\tau. \tag{1.10}$$

Let us introduce the notations

$$r_{00} := -r'(0) + \frac{r^2(0)}{4} - \frac{r(0)}{2} + \frac{1}{4}, \quad c_0 := \frac{c'(0) - a(0)}{2c(0)}.$$

With the new functions $\bar{u}(\nu, y, t)$ and $r(t)$, we rewrite Eqs. (1.4)-(1.6) in the form

$$\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial y^2} + H(\nu, y)v(\nu, y, t) - \int_0^t h(t-\tau)v(\nu, y, \tau)d\tau + b(y)\int_0^t p(t-\tau)v(\nu, y, \tau)d\tau, \tag{1.11}$$

$$v\Big|_{t<0} \equiv 0, \tag{1.12}$$

$$\Big(\frac{\partial v}{\partial y} + c_0 v\Big)\Big|_{y=+0} = \delta'(t) - \frac{1 + r(0)}{2}\delta(t) + \theta(t)f_0(t), \tag{1.13}$$

$$v\Big|_{y=+0} = \tilde{g}_0(\nu, t) + \int_0^t k_0(t-\tau)\tilde{g}_0(\nu, \tau)d\tau, \tag{1.14}$$

in which

$$H(\nu, y) := r_{00} + q_0(y) - \nu^2 q_1(y) + b(y),$$

$$q_0(y) := \frac{1}{4c^2(y)}\Big(2c(y)[c''(y) - a(y)] - 3c'^2(y) + 4c'(y)a(y) - \big(a(y)\big)^2\Big), \quad q_1(y) = c^2(y),$$

$$\tilde{g}_0(\nu, t) := e^{(1+r(0))t/2}\tilde{g}(\nu, t), \quad k_0(t) := e^{(1+r(0))t/2}k(t), \quad f_0(t) := e^{(1+r(0))t/2}f(t),$$

$$h(t) := e^{(1+r(0))t/2}r''(t) + e^{(1+r(0))t/2}r'(t), \quad p(t) := e^{(1+r(0))t/2}r(t).$$

Under the above assumption on smoothness of the function $c(y)$, it is obviously that $q_0(y) \in C(\mathbb{R})$, $q_1(y) \in C^2(\mathbb{R})$. In (1.13), we use the equality $k(0) = -r(0)$, which follows from (1.10).

From the theory of hyperbolic equations it follows that the solution of direct problem (1.11)-(1.13) is identically zero so that $v(\nu, y, t) = 0$ for all $y > t > 0, x \in \mathbb{R}$, because (1.11)-(1.13) is an initial boundary value problem with the zero initial data and some boundary condition concentrated on $y = 0$, $t = 0$, $\nu \in \mathbb{R}$. The following is hold:

**Lemma 1.1.** *The solution of direct problem (1.11)-(1.13) is represented as*

$$v(\nu, y, t) = -\delta(t-y) + \theta(t-y)\hat{v}(\nu, y, t). \tag{1.15}$$

*such that the regular function $\tilde{v}(\nu, y, t)$ satisfies the following equations in the domain $t > y > 0$*

$$v_{tt} = v_{yy} + H(\nu, y)v(\nu, y, t)+$$

$$+h(t-y) - b(y)p(t-y) - \int_0^{t-y} h(\tau)v(\nu, y, t-\tau)d\tau + b(y)\int_0^{t-y} p(\tau)v(\nu, y, t-\tau)d\tau, \tag{1.17}$$

$$v\Big|_{t=y+0} = \beta(\nu, y) = \beta_0 - \frac{1}{2}\int_0^y H(\nu, \xi)d\xi, \quad \nu \in \mathbb{R}, \ y \in \mathbb{R}_+, \tag{1.18}$$

$$(v_y + c_0 v)\Big|_{y=0} = f_0(t), \quad t \in \mathbb{R}_+. \tag{1.19}$$

Note that $v = \hat{v}$ for $t > y > 0$. Therefore, at further consideration of the direct and inverse problems in the domain $t > y > 0$, the sign "hat"over $v$ will be omitted.

Proof. Insert (1.15) into (1.11)-(1.13) and use the method of isolating singularities [6, pp. 611- 629]. Putting $\beta(\nu, y) = \tilde{v}(\nu, y, y+0)$. Substituting expression (1.15) into (1.11) and equating the coefficients for the same singularities, we find that $\beta(\nu, y)$ satisfies the ordinary differential equation

$$2\beta_y(\nu, y) + H(\nu, y) = 0,$$

with the initial condition $\beta(\nu, 0) = \beta_0 = \frac{1}{2}\big(r(0) + 1 - 2c_0\big)$. Solving this equation, we find

$$\beta(\nu, y) = \beta_0 - \frac{1}{2}\int\limits_0^y H(\nu, s)ds.$$

Moreover, substituting expression (1.15) into (1.13), then we obtain the condition (1.19). The proof of the lemma is complete.

Note that, by (1.14) and the above-introduced notation, $\tilde{g}_0(\nu, t)$ has the form

$$\tilde{g}_0(\nu, t) = -\delta(t - y) + \theta(t - y)\bar{g}(\nu, t),\ (\nu, t) \in \mathbb{R}^2_+, \tag{1.20}$$

where the function $\bar{g}(\nu, t)$ with respect to the argument $t$ satisfies some smoothness conditions, which will be discussed below. In this regard, the additional information (1.14) for the function $v$ looks like

$$v\Big|_{y=0} = \bar{g}_{00}(\nu, t),\ t \in \mathbb{R}_+, \tag{1.21}$$

where

$$\bar{g}_{00}(\nu, t) = \bar{g}(\nu, t) - k_0(t) + \int\limits_0^t k_0(t - \tau)\bar{g}(\nu, \tau)d\tau.$$

Now we narrow down the data of the problem, assuming that the function $\bar{g}(\nu, t)$ (hence the function $\tilde{g}(\nu, t)))$ is known only for two values of $\nu_1$, $\nu_2$ such that $\nu_1^2 \neq \nu_2^2$. Thus, the inverse problem (0.1)-(0.4) has been reduced to the problem of determining the functions $c(y)$, $k(t)$ from the relations (1.17)-(1.19) if the solution of the direct problem is known for $\nu = \nu_i$, $i = 1, 2$ and is given by the equalities (1.21). It turns out that with these data the functions $c(y)$, $k(t)$ are uniquely determined. After finding the function $c(y)$, the function $l(z)$, defined the correspondence between the variables $y$ and $z$, as follows from (1.3), is found by the formula

$$l(y) = \int\limits_0^y c(\xi)d\xi,$$

and $\sqrt{\mu(z)} = c(l^{-1}(z))$. Due to the fact that equation (1.17) describes a wave process propagating with a velocity equal to unity $v(\nu, 0, t)$ for $t \in [0, T]$ and for a fixed $\nu$ depends from function $c(y)$ and its derivatives (through function $H(\nu, y)$) only on the interval $[0, T/2]$, and from $k(t)$ on $[0, T]$. Therefore, it is natural to expect that the inverse is also true: the function $c(y)$ on each interval $[0, T/2]$ and the function $k(t)$ on $[0, T]$ are determined by $\bar{g}(\nu, t)$ only on interval $[0, T]$. It turns out that such a local character of the dependence of $c(y)$, $k(t)$ on $\bar{g}(\nu, t)$ really takes place, and it, of course, is reflected in the results that in what follows, we intend to prove.

# 3 Properties of the direct problem solution

We study the properties of the direct problem solution (1.17)-(1.19).

**Lemma 2.1.** *Let $(a(y), b(y)) \in C[0,T/2]$, $c(y) \in C^2[0,T/2]$, $f(t) \in C[0,T]$, $k(t) \in C^2[0,T]$ for some fixed $T > 0$. Then, for each fixed value of the parameter $\nu$ the solution of problem (1.17)-(1.19) for*

$$(y,t) \in D_T, \quad D_T = \big\{(y,t) : 0 \le y \le t \le T - y\big\}$$

*belongs to the functional class $C^1(D_T)$ and for it the following estimate is hold:*

$$\|v\|_{C^1(D_T)} \le d\Big(\|a(y)\|_{C[0,T/2]} + \|b(y)\|_{C[0,T/2]} + \|c(y)\|_{C^2[0,T/2]}+$$

$$+\|f(t)\|_{C[0,T/2]} + \|k(t)\|_{C^2[0,T/2]}\Big), \tag{2.1}$$

*where $d$ depends only on $T$, $\nu$, $\|a(y)\|_{C[0,T/2]}$, $\|b(y)\|_{C[0,T/2]}$, $\|c(y)\|_{C^2[0,T/2]}$ and $\|k(t)\|_{C^2[0,T/2]}$. Also function*

$$\psi(\nu_1,\nu_2,t) = v_t(\nu_1,0,t) - v_t(\nu_2,0,t)$$

*for any fixed $\nu_j$, $j = 1,2$ is a function of the class $C^1[0,T]$.*

Proof. Using the equalities

$$\Big(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial t^2}\Big)v(\nu,y,t) = \Big(\frac{\partial}{\partial t} - \frac{\partial}{\partial y}\Big)(v_t + v_y) = \Big(\frac{\partial}{\partial t} + \frac{\partial}{\partial y}\Big)(v_t - v_y),$$

for $v_{tt} - v_{yy}$ from relations (1.17)-(1.19) for $(y,t) \in D_T$ by integration along the corresponding characteristics of first-order differential operators, we obtain the equalities

$$(v_t + v_y)(\nu,y,t) = -\frac{1}{2}H\Big(\nu, \frac{y+t}{2}\Big) + \int\limits_{y}^{(y+t)/2} \Big[H(\nu,\xi)v(\nu,\xi,t+y-\xi)+$$

$$+h(t+y-2\xi) - b(\xi)p(t+y-2\xi)-$$

$$-\int\limits_0^{t+y-2\xi} h(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau + b(\xi)\int\limits_0^{t+y-2\xi} p(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau\Big]d\xi, \tag{2.2}$$

$$v(\nu,0,t) = \beta_0 e^{c_0 t} - \frac{1}{2}\int\limits_0^t e^{c_0(t-\tau)}H(\nu,\frac{\tau}{2})d\tau - \int\limits_0^t e^{c_0(t-\tau)}f_0(\tau)d\tau+$$

$$+\int\limits_0^t e^{c_0(t-\tau)}\int\limits_0^{\tau/2}\Big[H(\nu,\xi)v(\nu,\xi,\tau-\xi) + h(\tau-2\xi) - b(\xi)p(\tau-2\xi)-$$

$$-\int\limits_0^{\tau-2\xi} h(\alpha)v(\nu,\xi,\tau-\xi-\alpha)d\alpha + b(\xi)\int\limits_0^{\tau-2\xi} p(\alpha)v(\nu,\xi,\tau-\xi-\alpha)d\alpha\Big]d\xi d\tau, \tag{2.3}$$

$$(v_t - v_y)(\nu,y,t) = -2f(t-y) + 2c_0 v(\nu,0,t-y) - \frac{1}{2}H\Big(\nu, \frac{t-y}{2}\Big)+$$

$$+\int\limits_0^{(t-y)/2}\Big[H(\nu,\xi)v(\nu,\xi,t-y-\xi)+h(t-y-2\xi)-b(\xi)p(t-y-2\xi)-$$

$$-\int\limits_0^{t-y-2\xi}h(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau+b(\xi)\int\limits_0^{t-y-2\xi}p(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau\Big]d\xi+$$

$$+h(t-y)y-p(t-y)\int\limits_0^y b(\xi)d\xi+\int\limits_0^y\Big[H(\nu,\xi)v(\nu,\xi,t-y+\xi)-$$

$$-\int\limits_0^{t-y}h(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau+b(\xi)\int\limits_0^{t-y}p(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau\Big]d\xi. \tag{2.4}$$

From (2.2), (2.4), we find the equations for $v_t$, $v_y$, $v$:

$$v_t(\nu,y,t)=-f(t-y)+c_0v(\nu,0,t-y)-\frac{1}{4}H\Big(\nu,\frac{y+t}{2}\Big)-\frac{1}{4}H\Big(\nu,\frac{t-y}{2}\Big)+$$

$$+\frac{1}{2}\Big(h(t-y)y-p(t-y)\int\limits_0^y b(\xi)d\xi\Big)+$$

$$+\frac{1}{2}\int\limits_0^{(t-y)/2}\Big[H(\nu,\xi)v(\nu,\xi,t-y-\xi)+h(t-y-2\xi)-b(\xi)p(t-y-2\xi)-$$

$$-\int\limits_0^{t-y-2\xi}h(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau+b(\xi)\int\limits_0^{t-y-2\xi}p(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau\Big]d\xi+$$

$$+\frac{1}{2}\int\limits_0^y\Big[H(\nu,\xi)v(\nu,\xi,t-y+\xi)-\int\limits_0^{t-y}h(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau+$$

$$+b(\xi)\int\limits_0^{t-y}p(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau\Big]d\xi+\frac{1}{2}\int\limits_y^{(y+t)/2}\Big[H(\nu,\xi)v(\nu,\xi,t+y-\xi)+$$

$$+h(t+y-2\xi)-b(\xi)p(t+y-2\xi)-\int\limits_0^{t+y-2\xi}h(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau+$$

$$+b(\xi)\int\limits_0^{t+y-2\xi}p(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau\Big]d\xi, \tag{2.5}$$

$$v_y(\nu,y,t)=f(t-y)-c_0v(\nu,0,t-y)-\frac{1}{4}H\Big(\nu,\frac{t+y}{2}\Big)+\frac{1}{4}H\Big(\nu,\frac{t-y}{2}\Big)-$$

$$-\frac{1}{2}\int\limits_0^{(t-y)/2}\Big[H(\nu,\xi)v(\nu,\xi,t-y-\xi)+h(t-y-2\xi)-b(\xi)p(t-y-2\xi)-$$

$$
-\int\limits_0^{t-y-2\xi} h(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau+
$$

$$
+b(\xi)\int\limits_0^{t-y-2\xi} p(\tau)v(\nu,\xi,t-y-\xi-\tau)d\tau\Big]d\xi-\frac{1}{2}\int\limits_0^{y}\Big[H(\nu,\xi)v(\nu,\xi,t-y+\xi)-
$$

$$
-\int\limits_0^{t-y} h(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau+
$$

$$
+b(\xi)\int\limits_0^{t-y} p(\tau)v(\nu,\xi,t-y+\xi-\tau)d\tau\Big]d\xi+\frac{1}{2}\int\limits_y^{(y+t)/2}\Big[H(\nu,\xi)v(\nu,\xi,t+y-\xi)+
$$

$$
+h(t+y-2\xi)-b(\xi)p(t+y-2\xi)-\int\limits_0^{t+y-2\xi} h(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau+
$$

$$
+b(\xi)\int\limits_0^{t+y-2\xi} p(\tau)v(\nu,\xi,t+y-\xi-\tau)d\tau\Big]d\xi, \tag{2.6}
$$

$$
v(\nu,y,t)=\beta_0-\frac{1}{2}\int\limits_0^{y}H(\nu,\xi)d\xi-\int\limits_0^{t-y}f(\tau)d\tau+c_0\int\limits_0^{t-y}v(\nu,0,\tau)d\tau-
$$

$$
-\frac{1}{4}\int\limits_y^{t}\Big[H\Big(\nu,\frac{y+t}{2}\Big)+H\Big(\nu,\frac{t-y}{2}\Big)\Big]d\tau+\frac{1}{2}\int\limits_0^{t-y}\Big(h(\tau)y-p(\tau)\int\limits_0^{y}b(\xi)d\xi\Big)d\tau+
$$

$$
+\frac{1}{2}\int\limits_y^{t}\int\limits_0^{(\tau-y)/2}\Big[H(\nu,\xi)v(\nu,\xi,\tau-y-\xi)+h(\tau-y-2\xi)-b(\xi)p(\tau-y-2\xi)-
$$

$$
-\int\limits_0^{\tau-y-2\xi}h(\alpha)v(\nu,\xi,\tau-y-\xi-\alpha)d\alpha+b(\xi)\int\limits_0^{\tau-y-2\xi}p(\alpha)v(\nu,\xi,\tau-y-\xi-\alpha)d\alpha\Big]d\xi d\tau+
$$

$$
+\frac{1}{2}\int\limits_y^{t}\int\limits_0^{y}\Big[H(\nu,\xi)v(\nu,\xi,\tau-y+\xi)+\int\limits_0^{\tau-y}h(\alpha)v(\nu,\xi,\tau-y+\xi-\alpha)d\alpha-
$$

$$
-b(\xi)\int\limits_0^{\tau-y}p(\alpha)v(\nu,\xi,\tau-y+\xi-\alpha)d\alpha\Big]d\xi d\tau+\frac{1}{2}\int\limits_y^{t}\int\limits_y^{(y+\tau)/2}\Big[H(\nu,\xi)v(\nu,\xi,\tau+y-\xi)+
$$

$$
+h(\tau+y-2\xi)-b(\xi)p(\tau+y-2\xi)-\int\limits_0^{\tau+y-2\xi}h(\alpha)v(\nu,\xi,\tau+y-\xi-\alpha)d\alpha+
$$

$$+b(\xi)\int\limits_0^{\tau+y-2\xi} p(\alpha)v(\nu,\xi,\tau+y-\xi-\alpha)d\alpha\Big]d\xi d\tau. \tag{2.7}$$

Equation (2.7) is the integral equation of Volterra type in the domain $D_T$ and it has the unique continuous solution. It follows from equalities (2.5), (2.6) that this solution is continuously differentiable in $D_T$. Substituting the expression for $v(\nu,0,t)$, defined by formula (2.3) into equations (2.5)-(2.7) and constructing the method of successive approximations for these equations by the usual scheme, which has factorial convergence by argument $t$, it is easy to establish the validity of the estimate (2.1) in the domain $D_T$. Using equality (2.5), we compose the function $\psi(\nu_1,\nu_2,t)$:

$$\psi(\nu_1,\nu_2,t)=c_0\big(v(\nu_1,0,t)-v(\nu_2,0,t)\big)+\frac{1}{2}(\nu_1^2-\nu_2^2)q_1\Big(\frac{t}{2}\Big)+$$

$$+\int\limits_0^{t/2}\Big[H(\nu_1,\xi)v(\nu_1,\xi,t-\xi)-H(\nu_2,\xi)v(\nu_2,\xi,t-\xi)-\int\limits_0^{t-2\xi}h(\tau)\Big(v(\nu_1,\xi,t-\xi-\tau)-$$

$$-v(\nu_2,\xi,t-\xi-\tau)\Big)d\tau+b(\xi)\int\limits_0^{t-2\xi}p(\tau)\big(v(\nu_1,\xi,t-\xi-\tau)-v(\nu_2,\xi,t-\xi-\tau)\big)d\tau\Big]d\xi. \tag{2.8}$$

The right-hand side of this equality belongs to the class $C^2[0,T]$. Therefore $\psi(\nu_1,\nu_2,t)\in C^2[0,T]$, for any fixed $\nu_i$, $i=1,2$.

# 4 Reducing the inverse problem to an equivalent system of integral equations

As consequence of the Lemma 2.1, we note that under the conditions of lemma 2.1, the function $\bar{g}(\nu,t))$, entering in equality (1.21), for each fixed $\nu$ belongs to the class $C^2[0,T]$, and the function $G(t)\equiv\bar{g}(\nu_1,t)-\bar{g}(\nu_2,t)$ belongs to the class $C^2[0,T]$.

**Lemma 3.1.** *Suppose that the function $\tilde{g}_0(\nu,t)$ has the structure (1.20) $\bar{g}(\nu,t)\in C^2[0,T]$ for any fixed $\nu$, and*

$$\bar{g}(\nu_1,t)-\bar{g}(\nu_2,t)\equiv G(t)\in C^2[0,T].$$

*In addition, let the function $\bar{g}_t(\nu,0)$ be increase in $\nu$, $\nu\in\mathbb{R}$, $f(t)\in C[0,T]$. Then the inverse problem (1.17)-(1.19), (1.21) in the domain $D_T$ is equivalent to the problem of finding the functions $v$, $v_t$, $c(y)$, $c'(y)$, $q_0(y)$, $k_0(t)$, $k_0'(t)$, $k_0''(t)$, $h(t)$, $p(t)$ from the following closed system of the following integral equations:*

$$c(y)=c(0)+\int_0^y c'(\xi)d\xi, \tag{3.1}$$

$$c'(y)=c'(0)+\int_0^y a(\xi)d\xi+\int_0^y\Big[2c(\xi)q_0(\xi)+$$
$$+\frac{3(c'(\xi))^2-4c'(\xi)a(\xi)+a^2(\xi)}{2c(\xi)}\Big]d\xi, \tag{3.2}$$

$$q_0(y)=-r_{00}-b(y)+\nu_1^2q_1(y)-2f(2y)+2c_0\Big[\bar{g}(\nu_1,y)-k_0(y)+$$

$$+\int_0^y k_0(\tau)\bar{g}(\nu_1,y-\tau)d\tau\Big]-\bar{g}_t(\nu_1,y)+k_0'(y)-\bar{g}(\nu_1,0)k_0(y)-\int_0^y k_0(\tau)\bar{g}_t(\nu_1,y-\tau)d\tau+$$

$$+\int_0^y \Big[H(\nu_1,\xi)v(\nu_1,\xi,2y-\xi)+h(2y-2\xi)-$$

$$-b(\xi)p(2y-2\xi)-\int_0^{2(y-\xi)} h(\tau)v(\nu_1,\xi,2y-\xi-\tau)d\tau+$$

$$+b(\xi)\int_0^{2(y-\xi)} p(\tau)v(\nu_1,\xi,2y-\xi-\tau)d\tau\Big]d\xi, \quad y\in[0,T/2], \tag{3.3}$$

$$k_0(t)=-r(0)+\Big(r_{00}-\frac{r^2(0)}{4}+\frac{1}{4}\Big)t+\int\limits_0^t (t-\tau)k_0''(\tau)d\tau, \tag{3.4}$$

$$k_0'(t)=r_{00}-\frac{r^2(0)}{4}+\frac{1}{4}+\int\limits_0^t k_0''(\tau)d\tau, \tag{3.5}$$

$$k_0''(t)=-c_0\bar g_t(\nu_1,t)+\bar g_{tt}(\nu_1,t)+(c_0+\bar g(\nu_1,0))k_0'(t)+\frac{1}{4}(\nu_1^2-\nu_2^2)q_1'(t/2)+$$

$$+(c_0\bar g(\nu_1,0)-\bar g_t(\nu_1,0))k_0(t)+\int\limits_0^t k_0(\tau)(c_0\bar g_t(\nu_1,t-\tau)-\bar g_{tt}(\nu_1,t-\tau))d\tau+$$

$$+H(\nu_1,t/2)v(\nu_1,t/2,t/2)-H(\nu_2,t/2)v(\nu_2,t/2,t/2)+\int\limits_0^{t/2}\Big[H(\nu_1,\xi)v_t(\nu_1,\xi,t-\xi)-$$

$$-H(\nu_2,\xi)v_t(\nu_2,\xi,t-\xi)-\big(h(t-2\xi)-b(\xi)p(t-2\xi)\big)\big(v(\nu_1,\xi,\xi)-v(\nu_2,\xi,\xi)\big)-$$

$$-\int\limits_0^{t-2\xi}(h(\tau)-b(\xi)p(\tau))(v_t(\nu_1,\xi,t-\xi-\tau)-v_t(\nu_2,\xi,t-\xi-\tau))d\tau\Big]d\xi, \tag{3.6}$$

$$h(t)=-k_0''(t)-r_{00}k_0(t)-\int\limits_0^t k_0(\tau)h(t-\tau)d\tau, \tag{3.7}$$

$$p(t)=-k_0(t)-\int\limits_0^t k_0(t-\tau)p(\tau)d\tau, \quad t\in[0,T], \tag{3.8}$$

*where*

$$c(0)=\sqrt{\frac{2G'(0)}{\nu_1^2-\nu_2^2}}, \quad c'(0)=a(0)+c(0)\big[1-r(0)-2\bar g(\nu,0)\big]. \tag{3.9}$$

$$r(0)=1-2c_0-2c^2(0)\bar g(\nu,0),$$

$$r'(0)=-\frac{1}{2}H(\nu,0)+c^2(0)\big(c_0\bar g(\nu,0)-\bar g_t(\nu,0)\big)+r(0)\Big[r(0)+c^2(0)\bar g(\nu,0)+$$

$$+c_0-1\Big]-f_0(0). \tag{3.10}$$

Proof. At the beginning we establish of validity the equalities (3.9) and (3.10). Indeed, setting in equation (2.3) $t = 0$ and using conditions (1.21), we find

$$\bar{g}(\nu_j, 0) = \frac{1}{2}\big(1 - r(0) - 2c_0\big), \quad j = 1, 2. \tag{3.11}$$

In particular, from this we conclude that $\bar{g}|_{t=0}$ does not depend on $\nu$. Further, from equation (2.8) for $t = 0$, we obtain

$$G'(0) = \frac{1}{2}\big(\nu_1^2 - \nu_2^2\big)c^2(0).$$

Hence, by virtue of the positivity of the number $G'(t) = \bar{g}_t(\nu_1, 0) - \bar{g}_t(\nu_2, 0)$, we arrive at the first equality (2.7). Taking into account the first equality (1.15), from (2.1) we obtain the second equality (3.9). Since $\bar{g}|_{t=0}$ does not depend on $\nu$, then the second formula from (3.9) uniquely determines $c'(0)$.

We impose the continuity condition for the functions $v(\nu, y, t)$, $v_y(\nu, y, t)$ at $y = t = 0$. From relations (1.18), (1.19) and (1.21) we can easily express $r(0)$ and $r'(0)$ in terms of (3.10). To obtain the last equality for $r'(0)$, we use the relation $k'(0) = -r'(0) + r^2(0)$, which follows from (1.10). Further, we assume that instead of $r(0)$ and $r'(0)$, their values are substituted in $H(\nu, y)$.

Continuing the proof of the Lemma 3.1. Equation (2.5) is derived from the relations (1.17)-(1.19). Equation (2.7) is obtained from equation (2.6) by integration over $t$ from the point $(0, t)$ to the point $(y, t)$ on the plane of variables $(\xi, \tau)$. In turn, equation (2.5) satisfies equalities (1.17)-(1.20). Further, in equation (2.5) we set $y = 0$ and use condition (1.21) for $\nu = \nu_1$. Then, after simple transformations, we obtain equality (3.3). In this equation, for definiteness, we put $\nu = \nu_1$. In fact, the result of the calculation should not depend from the choice of the parameter $\nu$. To obtain equality (3.6), we use relation (2.8), which is derived from equation (2.5) using the conditions (1.21). Noting that $\psi(\nu_1, \nu_2, t) \equiv \bar{g}_{00t}(\nu_1, t) - \bar{g}_{00t}(\nu_2, t)$, we differentiate equality (2.5) with respect to the variable $t$. Then we have (3.6). The other equalities in the lemma 3.1 are given for the closure of the system of equations. They can be obtained from the definitions of the functions $h(t)$, $p(t)$ and $k_0(t)$ and equality (1.10). In satisfying the conditions of lemma 3.11, we state the equivalence of system of integral equations (3.1)-(3.8) and inverse problem (1.17)-(1.19), (1.21) in the ordinary way [see. 22]. The lemma is proved.

The system of integral equations (2.5), (2.7), for $\nu = \nu_j$, $j = 1, 2$ and (3.1)-(3.8) is closed in the domain $D_T$ and defines the unique continuous functions $v$, $v_t$, $c(y)$, $c'(y)$, $q_0(y)$, $k_0(t)$, $k_0'(t)$, $k_0''(t)$, $h(t)$, $p(t)$ for sufficiently small $T$. Without dwelling on the theorem of local unique solvability of the problem, we pass to the results characterizing stability estimates and unique solvability for an arbitrary $T > 0$.

**4. Proof of the main results.** Denote by $\Psi(s_0, d_0)$ the set of pairs of functions $\{c(y),\, k(t)\}$, satisfying for some $T > 0$ the following conditions:

$$0 < s_{00} \le c(y), \quad \|c(y)\|_{C^2[0,T/2]} \le s_0, \quad \|k(t)\|_{C^2[0,T]} \le d_0.$$

Besides,

$$\|a(y)\|_{C[0,T/2]} \le a_0, \quad \|b(y)\|_{C[0,T/2]} \le b_0, \quad \|f(t)\|_{C[0,T]} \le f_0,$$

where $a_0$, $b_0$, $f_0$ are known numbers.

**Theorem 4.1.** *Let $(c^{(1)}, k^{(1)}) \in \Psi(s_0, d_0)$, $(c^{(2)}, k^{(2)}) \in \Psi(s_0, d_0)$ be solutions of inverse problem (1.17)-(1.21) with data*

$$\Big(\bar{g}^{(1)}(\nu_j, t),\, a^{(1)}(y),\, b^{(1)}(y),\, f^{(1)}(t)\Big), \quad \Big(\bar{g}^{(2)}(\nu_j, t),\, a^{(2)}(y),\, b^{(2)}(y),\, f^{(2)}(t)\Big), \quad j = 1, 2,$$

*respectively. Then there is a positive constant $M$, depending on $\nu_1$, $\nu_2$, $s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$ such that the estimate*

$$\|k^{(1)}(t)-k^{(2)}(t)\|_{C^2[0,T]}+\|c^{(1)}(y)-c^{(2)}(y)\|_{C^2[0,T/2]}\leq M\lambda, \tag{4.1}$$

*is fulfilled, where*

$$\lambda=\sum_{j=0}^{2}\|\bar{g}^{(1)}(\nu_j,t)-\bar{g}^{(2)}(\nu_j,t)\|_{C^2[0,T]}+$$

$$+\|a^{(1)}(y)-a^{(2)}(y)\|_{C[0,T/2]}+\|b^{(1)}(y)-b^{(2)}(y)\|_{C[0,T/2]}+\|f^{(1)}(t)-f^{(2)}(t)\|_{C[0,T/2]}.$$

Theorem 4.1 obviously implies the following uniqueness theorem for any $T>0$.

**Theorem 4.2.** *The functions $c^{(i)}(y)\in C^2[0,T/2]$, $k^{(i)}(t)\in C^2[0,T]$ and $\bar{g}^{(i)}(\nu_j,t)$, $a^{(i)}(y)$, $b^{(i)}(y)$, $f^{(i)}(t)$, $i=1,2$, $j=1,2$ have the same meaning as in Theorem 4.1. If at the same time*

$$\bar{g}^{(1)}(\nu_j,t)=\bar{g}^{(2)}(\nu_j,t),\ j=1,2,\ a^{(1)}(y)=a^{(2)}(y),\ b^{(1)}(y)=b^{(2)}(y),\ f^{(1)}(t)=f^{(2)}(t),$$

*for $t\in[0,T]$, then*

$$c^{(1)}(y)=c^{(2)}(y),\quad y\in[0,T/2],\quad k^{(1)}(t)=k^{(2)}(t),\quad t\in[0,T].$$

Proof. Suppose that the functions $a^{(i)}$, $b^{(i)}$, $f^{(i)}$, $\bar{g}^{(i)}(\nu_j,t)$, $c^{(i)}$, $k^{(i)}$, $i,j=1,2$, are as defined in Theorem 4.1. The solution of problem (1.17)-(1.19) for $a=a^{(i)}$, $b=b^{(i)}$, $f=f^{(i)}$, $c=c^{(i)}$, $h=h^{(i)}$, $p=p^{(i)}$, $k_0=k_0^{(i)}$, $\nu=\nu_j$ denote by $v^{(ij)}(y,t)$, $i,j=1,2$. Introduce also notations

$$\tilde{b}(y)=b^{(1)}-b^{(2)},\quad \tilde{f}(t)=f^{(1)}-f^{(2)},\quad \tilde{c}(y)=c^{(1)}-c^{(2)},$$

$$\tilde{k}_0(t)=k_0^{(1)}-k_0^{(2)},\quad \tilde{\bar{g}}_{00}(\nu_j,t)=\bar{g}_{00}^{(1)}-\bar{g}_{00}^{(2)},\quad \tilde{l}(y)=l^{(1)}-l^{(2)},\quad \tilde{h}(t)=h^{(1)}-h^{(2)},$$

$$\tilde{p}(t)=p^{(1)}-p^{(2)},\quad \tilde{G}(t)=G^{(1)}-G^{(2)},\quad \tilde{v}^{(j)}(y,t)=v^{(1j)}(y,t)-v^{(2j)}(y,t),\quad j=1,2.$$

Let $q_0^{(i)}(y)$, $q_1^{(i)}(y)$, $H^{(ij)}(y)=r_{00}^{(i)}+q_0^{(i)}(y)-\nu_j^2q_1^{(i)}(y)+b^{(i)}(y)$, $\tilde{H}^{(j)}(y)=H^{(1j)}-H^{(2j)}$ and

$$c_0^{(i)}=\frac{(c^{(i)})'(0)-a^{(i)}(0)}{2c^{(i)}(0)},\ r_{00}^{(i)}=-(r^{(i)})'(0)+\frac{(r^{(i)})^2(0)}{4}-\frac{r^{(i)}(0)}{2}+\frac{1}{4},\ \beta_0^{(i)}=\frac{1}{2}\Big(r^{(i)}(0)-2c_0^{(i)}+1\Big),$$

be auxiliary functions and number corresponding to the functions $c^{(i)}(y)$. Let us write out the corresponding integral relations for the newly introduced functions. From equalities (2.7) and (2.5) it follows that

$$\tilde{v}^{(j)}(y,t)=\tilde{\beta}_0-\frac{1}{2}\int_0^y\tilde{H}^{(j)}(\xi)d\xi-\int_0^{t-y}\tilde{f}(\tau)d\tau+\tilde{c}_0\int_0^{t-y}\bar{g}_{00}(\nu_j,\tau)d\tau+c_0^{(2)}\int_0^{t-y}\tilde{\bar{g}}_{00}(\nu_j,\tau)d\tau-$$

$$-\frac{1}{4}\int_y^t\Big[\tilde{H}^{(j)}\Big(\frac{\tau+y}{2}\Big)+\tilde{H}^{(j)}\Big(\frac{\tau-y}{2}\Big)\Big]d\tau+\frac{1}{2}\int_0^{t-y}\Big(\tilde{h}(\tau)y-\tilde{p}(\tau)\int_0^y b^{(1)}(\xi)d\xi-$$

$$-p^{(2)}(\tau)\int_0^y\tilde{b}(\xi)d\xi\Big)d\tau+\frac{1}{2}\int_y^t\int_0^{(\tau-y)/2}\Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi,\tau-y-\xi)+H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi,\tau-y-\xi)+$$

$$+\tilde{h}(\tau-y-2\xi)-b^{(1)}(\xi)\tilde{p}(\tau-y-2\xi)-\tilde{b}(\xi)p^{(2)}(\tau-y-2\xi)-$$

$$-\int\limits_0^{\tau-y-2\xi}\Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau-y-\xi-\alpha)+\tilde{h}(\alpha)v^{(2j)}(\xi,\tau-y-\xi-\alpha)\Big)d\alpha+$$

$$+\int\limits_0^{\tau-y-2\xi}\Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau-y-\xi-\alpha)+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+$$

$$+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(2j)}(\xi,\tau-y-\xi-\alpha)\Big)d\alpha\Big\}d\xi d\tau+\frac{1}{2}\int\limits_y^t\int\limits_0^y\Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi,\tau-y+\xi)+$$

$$+H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi,\tau-y+\xi)+\int\limits_0^{\tau-y}\Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau-y+\xi-\alpha)+\tilde{h}(\alpha)v^{(2j)}(\xi,\tau-y+\xi-\alpha)\Big)d\alpha-$$

$$-\int\limits_0^{\tau-y}\Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau-y+\xi-\alpha)+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(2j)}(\xi,\tau-y+\xi-\alpha)\Big)d\alpha\Big\}d\xi d\tau+$$

$$+\frac{1}{2}\int\limits_y^t\int\limits_y^{(\tau+y)/2}\Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi,\tau+y-\xi)+H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi,\tau+y-\xi)+\tilde{h}(\tau+y-2\xi)-$$

$$-b^{(1)}(\xi)\tilde{p}(\tau+y-2\xi)-\tilde{b}(\xi)p^{(2)}(\tau+y-2\xi)-\int\limits_0^{\tau+y-2\xi}\Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau+y-\xi-\alpha)+$$

$$+\tilde{h}(\alpha)v^{(2j)}(\xi,\tau+y-\xi-\alpha)\Big)d\alpha+\int\limits_0^{\tau+y-2\xi}\Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,\tau+y-\xi-\alpha)+$$

$$+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(2j)}(\xi,\tau+y-\xi-\alpha)\Big)d\alpha\Big\}d\xi d\tau, \tag{4.2}$$

$$\tilde{v}_t^{(j)}(y,t)=-\tilde{f}(t-y)+\tilde{c}_0\bar{g}_{00}^{(1)}(\nu_j,t-y)+c_0^{(2)}\tilde{\bar{g}}_{00}(\nu_j,t-y)-\frac{1}{4}\Big[\tilde{H}^{(j)}\Big(\frac{t+y}{2}\Big)+$$

$$+\tilde{H}^{(j)}\Big(\frac{t-y}{2}\Big)\Big]+\frac{1}{2}\int\limits_0^{(t-y)/2}\Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi,t-y-\xi)+H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi,t-y-\xi)+$$

$$+\tilde{h}(t-y-2\xi)-b^{(1)}(\xi)\tilde{p}(t-y-2\xi)-\tilde{b}(\xi)p^{(2)}(t-y-2\xi)-$$

$$-\int\limits_0^{t-y-2\xi}\Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,t-y-\xi-\alpha)+\tilde{h}(\alpha)v^{(2j)}(\xi,t-y-\xi-\alpha)\Big)d\alpha+$$

$$+\int\limits_0^{t-y-2\xi}\Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi,t-y-\xi-\alpha)+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)\times$$

$$\times v^{(2j)}(\xi, t-y-\xi-\alpha)\Big)d\alpha\Big\}d\xi + \frac{1}{2}\int_0^y \Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi, t-y+\xi)+$$

$$+H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi, t-y+\xi) - \int_0^{t-y} \Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi, t-y+\xi-\alpha)+$$

$$+\tilde{h}(\alpha)v^{(2j)}(\xi, t-y+\xi-\alpha)\Big)d\alpha + \int_0^{t-y} \Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi, t-y+\xi-\alpha)+$$

$$+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(2j)}(\xi, t-y+\xi-\alpha)\Big)d\alpha\Big\}d\xi+$$

$$+\frac{1}{2}\int_y^{(t+y)/2} \Big\{\tilde{H}^{(j)}(\xi)v^{(1j)}(\xi, t+y-\xi) + H^{(2j)}(\xi)\tilde{v}^{(j)}(\xi, t+y-\xi) + \tilde{h}(t+y-2\xi)-$$

$$-b^{(1)}(\xi)\tilde{p}(t+y-2\xi) - \tilde{b}(\xi)p^{(2)}(t+y-2\xi) - \int_0^{t+y-2\xi} \Big(h^{(1)}(\alpha)\tilde{v}^{(j)}(\xi, t+y-\xi-\alpha)+$$

$$+\tilde{h}(\alpha)v^{(2j)}(\xi, t+y-\xi-\alpha)\Big)d\alpha + \int_0^{t+y-2\xi} \Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(j)}(\xi, t+y-\xi-\alpha)+$$

$$+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(2j)}(\xi, t+y-\xi-\alpha)\Big)d\alpha\Big\}d\xi, \quad j=1,2, \tag{4.3}$$

Note that in these equalities

$$\tilde{\beta}_0 = \frac{1}{2}(\tilde{r}(0)-2\tilde{c}_0), \quad \tilde{c}_0 = \frac{c^{(2)}(0)\tilde{c}'(0) - \tilde{c}(0)(c^{(2)})' - \tilde{a}(0)c^{(2)}(0) + a^{(2)}(0)\tilde{c}(0)}{2c^{(1)}(0)c^{(2)}(0)}. \tag{4.5}$$

Using equalities (3.1)-(3.8) we find

$$\tilde{c}(y) = \tilde{c}(0) + \int_0^y \tilde{c}'(\xi)d\xi, \tag{4.5}$$

$$\tilde{c}'(y) = \tilde{c}'(0) + \int_0^y \tilde{a}(\xi)d\xi + \int_0^y \Big[2(\tilde{c}(\xi)q_0^{(1)}(\xi) + c^{(2)}(\xi)\tilde{q}_0(\xi)) - \frac{2\tilde{c}'(\xi)}{c^{(1)}(\xi)}+$$

$$+\frac{a^{(1)}(\xi)+a^{(2)}(\xi)}{2c^{(2)}(\xi)}\tilde{a}(\xi) - \frac{3[(c^{(1)}(\xi))']^2 - 4(c^{(2)}(\xi))' + [a^{(1)}(\xi)]^2}{2c^{(1)}(\xi)c^{(2)}(\xi)}\tilde{c}(\xi)\Big]d\xi, \tag{4.6}$$

$$\tilde{q}_0(y) = -\tilde{r}_{00} - \tilde{b}(y) + \nu_1^2\tilde{q}_1(y) - 2\tilde{f}(2y) + 2\tilde{c}_0\Big[\bar{g}^{(1)}(\nu_1, y) - k_0^{(1)}(y)+$$

$$+\int_0^y k_0^{(1)}(\tau)\bar{g}^{(1)}(\nu_1, y-\tau)d\tau\Big] + 2c_0^{(2)}\Big[\tilde{\bar{g}}(\nu_1, y) - \tilde{k}_0(y) + \int_0^y \big(\tilde{k}_0(\tau)\bar{g}^{(1)}(\nu_1, y-\tau)+$$

$$+k_0^{(2)}(\tau)\tilde{\bar{g}}(\nu_1, y-\tau)\big)d\tau\Big] - \tilde{\bar{g}}_t(\nu_1, y) + \tilde{k}_0'(y) - \tilde{\bar{g}}(\nu_1, 0)k_0^{(1)}(y) - \bar{g}^{(2)}(\nu_1, 0)\tilde{k}_0(y)-$$

$$-\int_0^y \Big(\tilde{k}_0(\tau)\bar{g}_t^{(1)}(\nu_1,y-\tau)+k_0^{(2)}(\tau)\bar{\tilde{g}}_t(\nu_1,y-\tau)\Big)d\tau+\int_0^y \Big\{\tilde{H}^{(1)}(\xi)v^{(11)}(\xi,2y-\xi)+$$

$$+H^{(21)}(\xi)\tilde{v}^{(1)}(\xi,2y-\xi)+\tilde{h}(2y-2\xi)-b^{(1)}(\xi)\tilde{p}(2y-2\xi)-\tilde{b}(\xi)p^{(2)}(2y-2\xi)-$$

$$-\int_0^{2(y-\xi)} \Big(h^{(1)}(\alpha)\tilde{v}^{(1)}(\xi,2y-\xi-\alpha)+\tilde{h}(\alpha)v^{(21)}(\xi,2y-\xi-\alpha)\Big)d\alpha+$$

$$+\int_0^{2(y-\xi)} \Big(b^{(1)}(\xi)p^{(1)}(\alpha)\tilde{v}^{(1)}(\xi,2y-\xi-\alpha)+$$

$$+\big(b^{(1)}(\xi)\tilde{p}(\alpha)+\tilde{b}(\xi)p^{(2)}(\alpha)\big)v^{(21)}(\xi,2y-\xi-\alpha)\Big)d\alpha\Big\}d\xi, \tag{4.7}$$

$$\tilde{k}_0(t)=-\tilde{r}(0)+\Big(\tilde{r}_{00}-\frac{\tilde{r}(0)}{4}(r^{(1)}(0)+r^{(2)}(0))\Big)t+\int_0^t (t-\tau)\tilde{k}_0''(\tau)d\tau, \tag{4.8}$$

$$\tilde{k}_0'(t)=\tilde{r}_{00}-\frac{\tilde{r}(0)}{4}(r^{(1)}(0)+r^{(2)}(0))+\int_0^t \tilde{k}_0''(\tau)d\tau, \tag{4.9}$$

$$\tilde{k}_0''(t)=-\tilde{c}_0\bar{g}_t^{(1)}(\nu_1,t)-c_0^{(2)}\bar{\tilde{g}}_t(\nu_1,t)+\bar{\tilde{g}}_{tt}(\nu_1,t)+(\tilde{c}_0+\bar{\tilde{g}}(\nu_1,0))\big(k_0^{(1)}(t)\big)'+$$

$$+(c_0^{(2)}+\bar{g}^{(2)}(\nu_1,0))\tilde{k}_0'(t)+\frac{1}{4}(\nu_1^2-\nu_2^2)\tilde{q}_1'\Big(\frac{t}{2}\Big)+\Big(\tilde{c}_0\bar{g}^{(1)}(\nu_1,0)+c_0^{(2)}\bar{\tilde{g}}(\nu_1,0)-\bar{\tilde{g}}_t(\nu_1,0)\Big)k_0^{(1)}(t)+$$

$$+\Big(c_0^{(2)}\bar{g}^{(2)}(\nu_1,0)-\bar{g}_t^{(2)}(\nu_1,0)\Big)\tilde{k}_0(t)+\int_0^t \Big[\Big(\tilde{c}_0\bar{g}_t^{(1)}(\nu_1,t-\tau)+c_0^{(2)}\bar{\tilde{g}}(\nu_1,t-\tau)-$$

$$-\bar{\tilde{g}}_{tt}(\nu_1,t-\tau)\Big)k_0^{(1)}(\tau)+\Big(c_0^{(2)}\bar{g}_t^{(2)}(\nu_1,t-\tau)-\bar{g}_{tt}^{(2)}(\nu_1,t-\tau)\Big)\tilde{k}_0(\tau)\Big]d\tau+$$

$$+\tilde{H}^{(1)}\Big(\frac{t}{2}\Big)\beta^{(1)}\Big(\nu_1,\frac{t}{2}\Big)+H^{(21)}\Big(\frac{t}{2}\Big)\tilde{\beta}\Big(\nu_1,\frac{t}{2},\Big)-\tilde{H}^{(2)}\Big(\frac{t}{2}\Big)\beta^{(1)}\Big(\nu_2,\frac{t}{2}\Big)-$$

$$-H^{(22)}\Big(\frac{t}{2}\Big)\tilde{\beta}\Big(\nu_2,\frac{t}{2},\Big)+\int_0^{t/2} \Big\{\tilde{H}^{(1)}(\xi)v_t^{(11)}(\xi,t-\xi)+H^{(21)}(\xi)\tilde{v}_t^{(1)}(\xi,t-\xi)-$$

$$-\tilde{H}^{(2)}(\xi)v_t^{(12)}(\xi,t-\xi)+H^{(22)}(\xi)\tilde{v}_t^{(2)}(\xi,t-\xi)-\Big(\tilde{h}(t-2\xi)-\tilde{b}(\xi)p^{(1)}(t-2\xi)-$$

$$-b^{(2)}(\xi)\tilde{p}(t-2\xi)\Big)\Big(v^{(11)}(\xi,\xi)-v^{(12)}(\xi,\xi)\Big)-\Big(h^{(2)}(t-2\xi)-b^{(2)}(\xi)p^{(2)}(t-2\xi)\Big)\times$$

$$\times\Big(\tilde{\beta}(\nu_1,\xi)-\tilde{\beta}(\nu_2,\xi)\Big)-\int_0^{t-2\xi} \Big[\Big(\tilde{h}(\tau)-\tilde{b}(\xi)p^{(1)}(\tau)-b^{(2)}(\xi)\tilde{p}(\tau)\Big)\Big(v_t^{(11)}(\xi,t-\xi-\tau)-$$

$$-v_t^{(12)}(\xi,t-\xi-\tau)\Big)+\Big(h^{(2)}(\tau)-b^{(2)}(\xi)p^{(2)}(\tau)\Big)\Big(\tilde{v}_t^{(1)}(\xi,t-\xi-\tau)-\tilde{v}_t^{(2)}(\xi,t-\xi-\tau)\Big)\Big]d\tau\Big\}d\xi, \tag{4.10}$$

$$\tilde{h}(t) = -\tilde{k}_0''(t) - \tilde{r}_{00}k_0^{(1)}(t) - r_{00}^{(2)}\tilde{k}_0(t) - \int\limits_0^t \Big(\tilde{k}_0(\tau)h^{(1)}(t-\tau) + k_0^{(2)}(\tau)\tilde{h}(t-\tau)\Big)d\tau, \qquad (4.11)$$

$$\tilde{p}(t) = -\tilde{k}_0(t) - \int\limits_0^t \Big(\tilde{k}_0(\tau)p^{(1)}(t-\tau) + k_0^{(2)}(\tau)\tilde{p}(t-\tau)\Big)d\tau, \qquad (4.12)$$

here

$$\tilde{r}_{00} = -\tilde{r}'(0) + \frac{\tilde{r}(0)}{4}\Big(r^{(1)}(0) + r^{(2)}(0)\Big) - \frac{\tilde{r}(0)}{2}. \qquad (4.13)$$

We estimate the functions of the system (4.2)-(4.13) in domain $D_T$ through value $\lambda$, defined in Theorem 4.1. The domain $D_T$ allows equivalent description

$$D_T := \Big\{(y,t): \ 0 \le y \le t \le \frac{T}{2} - \Big|\frac{T}{2} - t\Big|, \ 0 \le t \le T\Big\}.$$

Let

$$\psi(t) = \max\Big\{ \max_{0\le y\le T/2-|T/2-t|} |\tilde{v}^{(j)}(y,t)|, \max_{0\le y\le T/2-|T/2-t|} |\tilde{v}_t^{(j)}(y,t)|,$$
$$\max_{0\le y\le T/2-|T/2-t|} |\tilde{c}(y)|, \max_{0\le y\le T/2-|T/2-t|} |\tilde{c}'(y)|, \max_{0\le y\le T/2-|T/2-t|} |\tilde{q}_0(y)|,$$
$$|\tilde{k}_0(t)|, |\tilde{k}_0'(t)|, |\tilde{k}_0''(t)|, |\tilde{h}(t)|, |\tilde{p}(t)|\Big\}, \quad t \in [0,T], \ j = 1,2.$$

According to Lemma 2.1, the functions $v^{(ij)}$ are differentiable in the domain $D_T$ and satisfy the estimate

$$\|v^{(ij)}\|_{C^1(D_T)} \le m_1, \quad i,j = 1,2, \qquad (4.14)$$

with some constant $m_1$, depending only on $\nu_1$, $\nu_2$, $T, s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$. Since the functions $G^{(i)}(t)$ are traces of the functions $v^{(i1)}(y,t) - v^{(i2)}(y,t)$ for $y = 0$, $i = 1,2$, for each of which, as follows from (4.14), (2.8), an estimate similar to (4.14) holds, then for $G^{(i)}(t) = \bar{g}^{(i)}(\nu_1,t) - \bar{g}^{(i)}(\nu_2,t)$ we have the inequality

$$\|G^{(i)}(t)\|_{C^2(D_T)} \le m_2, \quad i = 1,2,$$

in which the constant $m_2$ depends on the same parameters as $m_1$. From (4.14), (1.21) it follows that the functions $\bar{g}^{(i)}(\nu_j,t)$ must be bounded by the constant $m_1$:

$$\|\bar{g}^{(i)}(\nu_j,t)\|_{C^1[0,T]} \le m_1, \quad i,j = 1,2.$$

The numbers $c^{(i)}(0)$, $(c^{(i)})'(0)$, $r^{(i)}(0)$, $(r^{(i)})'(0)$, $i = 1,2$ defined by formulae (3.9), (3.10) via $\nu_1$, $\nu_2$, $G'$, for similar reasons should be bounded by the constant $m_3$, which depends from the same parameters as $m_j$, $j = 1,2$. Therefore, for the number $c_0^{(i)}$, $\tilde{c}_0$, $\beta_0^{(i)}$, $\tilde{\beta}_0$, $r_{00}^{(i)}$, $\tilde{r}_{00}$, defined by the numbers $c^{(i)}(0)$, $(c^{(i)})'(0)$, $r^{(i)}(0)$, $(r^{(i)})'(0)$, $i = 1,2$, and equalities (4.5), (4.13), the estimates

$$\max\big\{|c_0^{(i)}|, |\beta_0^{(i)}|, |r_{00}^{(i)}|\big\} \le m_4, \ \max\big\{|\tilde{c}_0|, |\tilde{\beta}_0|, |\tilde{r}_{00}|\big\} \le \lambda m_5,$$

are valid with some constants.

$m_4$, $m_5$ depending on of the set of numbers $\nu_1$, $\nu_2$, $T, s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$. Recall that $\lambda$ is defined in Theorem 4.1. Next, the functions

$$q_0^{(i)}(y), \ q_1^{(i)}(y), \ H^{(ij)}(y),$$

defined by $r_{00}^{(i)}$, $c^{(i)}(y)$, $a^{(i)}(y)$, $b^{(i)}(y)$ and $\nu_j$, $i, j = 1, 2$, for similar reasons satisfy the inequalities

$$\|q_0^{(i)}(y)\|_{C^2[0,T/2]} \leq m_6,\ \ \|q_1^{(i)}(y)\|_{C^2[0,T/2]} \leq m_7,$$

$$\|H^{(ij)}(y)\|_{C^2[0,T/2]} \leq m_8,\ \ i, j = 1, 2,$$

where $m_6$, $m_7$, $m_8$ depend on the same parameters as the previous constants.

In view of the foregoing, we proceed to estimate in the domain $D_T$ functions $\tilde{v}^{(j)}$, satisfying the integral equations (4.2). Note that these equations, like all others, contain terms containing only known and terms with unknown functions. In equation (4.2) on the right-hand side, the first four terms depend on known functions and numbers. Therefore, these terms in totality are estimated by the value $A_1\lambda$, with constant $A_1$, depending on $m_i$, $i = 1, ..., 8$. As it is easily seen the remaining part is estimated in the domain $D_T$ by an integral of the form

$$\kappa_1 \int\limits_0^t \psi(\tau) d\tau,$$

in which $\kappa_1$ depends only on $m_i$, $i = 1, ..., 8$, which in turn depend on the parameters $\nu_1$, $\nu_2$, $T$, $s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$. Thus,

$$|\tilde{v}^{(j)}(y,t)| \leq A_1\lambda + \kappa_1 \int\limits_0^t \psi(\tau) d\tau, \quad (y,t) \in D_T,\ j = 1, 2. \tag{4.15}$$

From equation (4.5), (4.6)) we can see that the functions $\tilde{c}(y)$, $\tilde{c}'(y)$ are estimated in a similar way

$$|\tilde{c}(y)| \leq A_2\lambda + \kappa_2 \int\limits_0^t \psi(\tau) d\tau, \tag{4.16}$$

$$|\tilde{c}'(y)| \leq A_3\lambda + \kappa_3 \int\limits_0^t \psi(\tau) d\tau, \quad y \in [0, T/2]. \tag{4.17}$$

The constants $A_i$, $\kappa_i$, $i = 2, 3$ depend on the same parameters as $A_1$, $\kappa_1$.

We use the inequalities (4.16), (4.17) to estimate the functions $\tilde{q}_1(y) = \tilde{c}(y)\big[c^{(1)}(y) + c^{(2)}(y)\big]$, $\tilde{q}_1'(y) = 2\big[\tilde{c}(y)(c^{(1)}(y))' + c^{(2)}(y)\tilde{c}'(y)\big]$, occurring in nonintegral terms in the right-hand sides of equations (4.7), (4.8). Similar reasoning leads to inequalities

$$|\tilde{q}_0(y)| \leq A_4\lambda + \kappa_4 \int\limits_0^t \psi(\tau) d\tau, \quad |\tilde{k}_0(t)| \leq A_5\lambda + \kappa_5 \int\limits_0^t \psi(\tau) d\tau. \tag{4.18}$$

Analogously, we will get inequalities for $\tilde{k}_0'(t)$, $\tilde{k}_0''(t)$, $\tilde{h}(t)$, $\tilde{p}(t)$,

$$|\tilde{k}_0'(t)| \leq A_6\lambda + \kappa_6 \int\limits_0^t \psi(\tau) d\tau, \quad |\tilde{k}_0''(t)| \leq A_7\lambda + \kappa_7 \int\limits_0^t \psi(\tau) d\tau. \tag{4.19}$$

$$|\tilde{h}(t)| \leq A_8\lambda + \kappa_8 \int\limits_0^t \psi(\tau) d\tau, \quad |\tilde{p}(t)| \leq A_9\lambda + \kappa_9 \int\limits_0^t \psi(\tau) d\tau. \tag{4.20}$$

We use inequalities (4.16)-(4.20) in the estimates of the functions $\tilde{H}^{(j)}$, outside the integral terms of equations (4.3) in order to obtain an estimate for the functions $\tilde{v}_t^{(j)}(y,t)$ in the form

$$|\tilde{v}_t^{(j)}(y,t)| \le A_{10}\lambda + \kappa_{10}\int_0^t \psi(\tau)d\tau,\ j=1,2. \tag{4.21}$$

In the last inequalities, the constants $A_{i+3}$, $\kappa_{i+3}$, $i=1,..,7$ depend by means of the numbers $m_i$, $i=1,...,8$ on the parameters $\nu_1$, $\nu_2$, $T$, $s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$. From relations (4.15)-(4.21) it follows that $\psi(t)$ satisfies integral inequality

$$\psi(t) \le A\lambda + \kappa\int_0^t \psi(\tau)d\tau,$$

with the new constants $A$, $\kappa$, depending only on $\nu_1$, $\nu_2$, $T$, $s_0$, $s_{00}$, $d_0$, $a_0$, $b_0$, $f_0$. Hence, using the Gronwall inequality, we obtain the estimate (4.1).

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