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ARTICLE TYPE

Asymptotic behavior of a three-species food chain model with time-varying delays

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Abstract

In this paper, a stochastic three-species food chain model with time-varying delays is focussed. The existence and the asymptotic behavior of global positive solutions to the model are discussed, and the sufficient conditions for the 1th moment practical exponential stability and the extinction of the model are given by using the Razumikhin technique and Lyapunov method.

KEYWORDS:
Three-species food chain model; Stochastic perturbations; Time-varying delays; Existence; Stability and extinction.

1 | INTRODUCTION

Predator-prey theory is traced from its origins in the Malthus-Verhulst logistic equations, and was modified by Lotka-Volterra equation to the predator-prey equation[1,2,3]. Some scholars have pointed out that the population model of two species can not accurately describe the real world, and a large number of key behaviors can only be expressed by the population model of three or more species[4,5]. Many literatures have been written on the dynamical problems of system persistence, extinction and global stability of equilibrium point[6,7,8,9]. The classic three-species food chain model with two competing prey populations and one predator population is represented as follows:

$$\begin{cases} dx_1(t) = x_1(t)\left[r_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t)\right]dt, \\ dx_2(t) = x_2(t)\left[r_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)\right]dt, \\ dx_3(t) = x_3(t)\left[-r_3 + a_{31}x_1(t) + a_{32}x_2(t) - a_{33}x_3(t)\right]dt, \\ x_i(0) = x_i^0, i = 1,2,3. \end{cases} \tag{1}$$

where $x_i(t)$ is the population size of the ith species at time t; $a_{ij}(i,j = 1,2,3)$ and $r_i(i = 1,2,3)$ are all positive constants.

In the real world, it is a usual phenomenon that one predator feeds on some competing prey in nature. However, the growth of population in nature is always affected by environmental stochastic perturbations which should be considered in the process of mathematical modeling[10]. In addition, time delays can not be ignored in biological models[11,12]. Time delay is a common problem in many practical systems, which will lead to system performance degradation and even instability. Therefore, the stability of time-delay systems has been one of the hot topics in recent twenty years[13]. It is worth noting that in many practical situations such as image encryption, neural computation, mechanics and biological models, the delay may be variable and cannot be accurately observed, and the boundary of the delay may be a transcendental unknown or even unbounded[17]. Due to the change of the time-delay systems, research into the dynamics of the corresponding time-delay systems is gradually known by people very difficult. In order to describe the predator-prey system accurately and reveal the evolution of the system, it is necessary to introduce sufficient time-varying delays into the system under consideration. To the best of our knowledge, few scholars have considered practical exponential stability of the following stochastic three-species food chain model with time-varying delays:

$$\begin{cases} dx_1(t) = x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t-\tau(t))]dt + \sigma_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)[r_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t-\tau(t))]dt + \sigma_2 x_2(t)dB_2(t), \\ dx_3(t) = x_3(t)[-r_3 + a_{31}x_1(t-\tau(t)) + a_{32}x_2(t-\tau(t)) - a_{33}x_3(t)]dt + \sigma_3 x_3(t)dB_3(t), \\ x_i(\theta) = \phi_i(\theta), -\tau_0 \le \theta \le 0, t \ge 0, 0 \le \tau(t) \le \tau_0, i = 1,2,3. \end{cases} \tag{2}$$

where $x_1(t)$, $x_2(t)$ and $x_3(t)$ stands for population size at time t of the two independent prey populations and one predator population, respectively. $r_i > 0(i = 1,2,3)$ denotes the intrinsic growth rate of the corresponding population at time t. The coefficients a_{11}, a_{22} and a_{33} represent the density correlation coefficients of two prey populations and one predator population, respectively. The coefficients a_{13}, a_{23} are the capturing rate of the predator. a_{12}, a_{21}, a_{31} and a_{32} stand for the rate of conversion of nutrients into the reproduction of the predator. $a_{ij}(i,j = 1,2,3)$ are all positive constants. $\sigma_i(i = 1,2,3)$ are the coefficients of effects of environmental stochastic perturbations on two prey populations and one predator population, respectively. $\tau(t) > 0$ represents the time-varying delay, $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \phi_3(\theta))^T \in U$, $U = C([-\tau_0, 0], R^3_+)$ represent the space of all the continue function. The Brownian motions $B_i(t), i = 1,2,3$ defined on a complete probability space $(\Omega, \mathcal{F}_t, (\mathcal{F}_t)_{t\ge 0}, P)$ with a filtration $(\mathcal{F}_t)_{t\ge 0}$ are independent to each other.

In this paper, we aim to investigate well-posedness and asymptotics about the model (2). The organization of the paper are as follows. (i) Some definitions and lemmas are given. (ii) We discuss the existence of global positive solutions to the model (2). (iii) Using the Razumikhin technique and Lyapunov method, the 1th moment practical exponential stability and the extinction to the model (2) are studied.

2 | PRILIMARY

Consider the stochastic functional differential equations

$$\begin{cases} dx(t) = f(t, x_t)dt + g(t, x_t)dB(t), \\ x(t_0) = \phi(0), x(t_0 + \theta) = \phi(\theta), x_t = x(t+\theta), -\tau_0 \le \theta \le 0, t \ge t_0, \end{cases} \tag{3}$$

where
$\phi \in C([-\tau_0, 0], R^n)$, $x \in R^n$, $x_t \in L^p_{F_t}([-\tau_0, 0], R^n)$, $f : L^p_{F_t}(R_+ \times ([-\tau_0, 0], R^n)) \to R^n$, $g : L^p_{F_t}(R_+ \times ([-\tau_0, 0], R^n)) \to R^{n\times m}$.

Definition 1. 1. If $p > 0$ and $\lim_{t\to+\infty} E|x(t)|^p = 0$, a.s., then the model (3) is said to be pth moment extinction.
2. If $p > 0$ and $\lim_{t\to+\infty} E|x(t)|^p > 0$, a.s., then the model (3) is said to be pth moment persistence.
3. For $p > 0$, the model (3) is said to be pth moment practical exponential stability. If there exist positive constants $D_1 > 0$, $D_2 \ge 0$ and $\lambda > 0$ such that

$$E|\boldsymbol{x}(t)|^p \le D_1 E|\boldsymbol{\phi}|^p_C e^{-\lambda(t-t_0)} + D_2, \quad t \ge t_0. \tag{4}$$

Especially, when $D_2 = 0$, the model (3) is said to be pth moment exponential stability.

Lemma 1. (see[14]) Assume that there exists a function $V \in C^{1,2}([t_0 - \tau_0, \infty) \times R^n; R_+)$ and constants $p > 0$, $c > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 \ge 0$, $\lambda > 0$, $\rho \ge 0$, $\delta \ge 0$ and $\gamma > 0$ such that

$$c_1|x|^p \le V(t,x) \le c_2|x|^p + c_3, \ for\ (t,x) \in [t_0 - \tau_0, \infty) \times R^n. \tag{5}$$

$$ELV(t,\phi) \le cEV(t,\phi(0)) + \delta, \ for\ t \ge t_0, \phi \in C([-\tau_0, 0], R^n). \tag{6}$$

$$EV(t+\theta,\phi) \le qEV(t,\phi(0)) + \rho, \ for\ t \ge t_0, \ \theta \in [-\tau_0, 0], q \ge \gamma e^{\lambda\tau_0}. \tag{7}$$

Then, the system (3) is pth moment practical exponential stability. That is,

$$E|\boldsymbol{x}(t)|^p \le \frac{c_2\gamma e^{\lambda\tau_0}}{c_1} E|\boldsymbol{\phi}|^p_C e^{-\lambda(t-t_0)} + \frac{\gamma}{c_1}, \quad t \ge t_0. \tag{8}$$

(see theorem 3.1 in Caraballo et.al[14]).

Remark 1. When parameter $c > 0$, the practical exponential stability of the system (3) depends on parameters p, c_1, c_2, q, γ, λ and delay constant τ_0. It is not about paraments $c_3 \geq 0$, $\delta \geq 0$ and $\rho \geq 0$. The pth moment practical exponential stability means the pth moment persistence for ecosystems.

Lemma 2. Assume that there exists a function $V \in C^{1,2}([t_0 - \tau_0, \infty) \times R^n; R_+)$ and constants $p > 0$, $c > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 \geq 0$, $\rho \geq 0$ and $q > 1$ such that

$$c_1|x|^p \leq V(t,x) \leq c_2|x|^p + c_3, \ for\ (t,x) \in [t_0 - \tau_0, \infty) \times R^n. \tag{9}$$

$$ELV(t,\phi) \leq -cEV(t,\phi(0)), \ \ t \geq t_0, \phi \in C([-\tau_0, 0], R^n). \tag{10}$$

$$EV(t+\theta,\phi) \leq qEV(t,\phi(0)) + \rho, \ for\ t \geq t_0, \ \theta \in [-\tau_0, 0]. \tag{11}$$

Then,

$$E|\boldsymbol{x}(t)|^p \leq \frac{c_2}{c_1} E|\boldsymbol{\phi}|_C^p e^{-\lambda(t-t_0)}, \tag{12}$$

where $\lambda = min[c, \frac{\ln q}{\tau_0}]$.

The system (3) is pth moment exponential stability.

Proof. The proof is similar to [14] and [15], here we omit it. □

In fact, the pth moment exponential stability means the pth moment extinction for ecosystems.

3 | GLOBAL POSITIVE SOLUTIONS

We use the following symbols for convenience. Let

$$\begin{aligned}
x(t) &= (x_1(t), x_2(t), x_3(t))^T, \ x(t-\tau(t)) \quad = (x_1(t-\tau(t)), x_2(t-\tau(t)), x_3(t-\tau(t)))^T, \\
R &= \left(r_1, r_2, -r_3\right), \\
J_1 &= \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}, \\
J_2 &= \begin{pmatrix} 0 & 0 & -a_{13} \\ 0 & -a_{23} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix}, \\
\sigma &= \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix},
\end{aligned}$$

$dB = (dB_1(t), dB_2(t), dB_3(t))^T$, $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \phi_3(\theta))^T$, then (2) can be rewritten as

$$\begin{cases} dx(t) = x^T(t)(R - J_1x(t) + J_2x(t-\tau(t)))dt + \sigma(x^T(t)dB(t)), \\ x(0) = \phi(0), x(\theta) = \phi(\theta), -\tau_0 \leq \theta \leq 0, t \geq 0. \end{cases} \tag{13}$$

Theorem 1. In the model (13), for any given initial condition $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \phi_3(\theta))^T$, $\phi_i(\theta) \in C([-\tau_0, 0], R^+)$, there is a unique solution $x(t)$ on $t \in R^+ = [0, \infty)$, with $x(\theta) = \phi(\theta)$, $\theta \in [-\tau_0, 0)$ and the solution will remain in R^3_+ with probability 1.

Proof. We can prove that the model (13) has a unique global solution $x(t)$ with probability 1 in R^3_+ by the same method as the lemma 2.1 in [16]. In fact, the equation (2.4) in [16] is pure differential equation, while the equation (13) in this paper is of the same type of simultaneous differential equation. It's proof is similar, so we omit. □

4 | STABILITY AND EXTINCTION

Theorem 2. Let $2a_{11}a_{22}a_{33} - a_{22}a_{31}^2 - a_{11}a_{32}^2 \neq 0$. The following the Razumikhin conditions hold

$$x_i(t-\tau(t)) < x_i(t),\ t \geq 0 (i = 1, 2, 3). \tag{14}$$

(1) When

$$\begin{aligned}&\frac{a_{11}a_{22}(r_3+a_{33})^2}{4a_{11}a_{22}a_{33} - 2a_{22}a_{31}^2 - 2a_{11}a_{32}^2} + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\ &+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) > 0,\end{aligned} \tag{15}$$

then, the system (2) is 1th moment practical exponential stability.
(2) When

$$\begin{aligned}&\frac{a_{11}a_{22}(r_3+a_{33})^2}{4a_{11}a_{22}a_{33} - 2a_{22}a_{31}^2 - 2a_{11}a_{32}^2} + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\ &+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) < 0.\end{aligned} \tag{16}$$

then, the system (2) is 1th moment extinction.

Proof. Let $x = (x_1, x_2, x_3)^T \in R_+^3$ and $|x| = x_1 + x_2 + x_3$. Define $V_i(x) = x_i + \ln(x_i + 1)(i = 1, 2, 3)$, $V(x) = \sum_{i=1}^3 V_i(x)$. Then

$$x_i \leq V_i(x) \leq 2x_i + 1(i = 1, 2, 3), for\ (t, x) \in [-\tau_0, \infty) \times R^3. \tag{17}$$

So

$$|x| \leq V(x) \leq 2|x| + 3,\ for\ (t, x) \in [-\tau_0, \infty) \times R^3. \tag{18}$$

From (14) and (18), we have

$$EV(\phi(\theta)) \leq 2E|\phi(\theta)| + 3 < 2E|\phi(0)| + 3 \leq 2V(\phi(0)) + 3,\ -\tau_0 \leq \theta \leq 0. \tag{19}$$

From $x_i(t) \geq 0(i = 1, 2, 3)$, we have

$$\begin{aligned}LV_1 &= (1 + \frac{1}{x_1+1})x_1(r_1 - a_{11}x_1 - a_{12}x_2 - a_{13}x_3(t-\tau(t))) - \frac{1}{2}\frac{x_1^2}{(1+x_1)^2}\sigma_1^2 \\ &< (x_1+1)(r_1 - a_{11}x_1 - a_{12}x_2 - a_{13}x_3(t-\tau(t))) + \frac{1}{2}\sigma_1^2 \\ &\leq -a_{11}x_1^2 + (r_1 - a_{11})x_1 - a_{12}x_2 + r_1 + \frac{1}{2}\sigma_1^2.\end{aligned} \tag{20}$$

Similarly, we have

$$LV_2 < -a_{22}x_2^2 + (r_2 - a_{22})x_2 - a_{21}x_1 + r_2 + \frac{1}{2}\sigma_2^2. \tag{21}$$

From (14) and $x_i(t) \geq 0(i = 1, 2, 3)$, we have

$$\begin{aligned}LV_3 &< -a_{33}x_3^2 - (r_3 + a_{33})x_3 + a_{31}x_1(t-\tau(t)) + a_{32}x_2(t-\tau(t)) + a_{31}x_1(t-\tau(t))x_3 \\ &\quad + a_{32}x_2(t-\tau(t))x_3(t) - r_3 + \frac{1}{2}\sigma_3^2 \\ &< -a_{33}x_3^2 - (r_3 + a_{33})x_3 + a_{31}x_1 + a_{32}x_2 + a_{31}x_1x_3 + a_{32}x_2x_3 - r_3 + \frac{1}{2}\sigma_3^2.\end{aligned} \tag{22}$$

So

$$
\begin{aligned}
LV &= LV_1 + LV_2 + LV_3 \\
&< -a_{11}x_1^2 + (r_1 - a_{11})x_1 - a_{12}x_2 + r_1 + \frac{1}{2}\sigma_1^2 - a_{22}x_2^2 + (r_2 - a_{22})x_2 - a_{21}x_1 + r_2 + \frac{1}{2}\sigma_2^2 \\
&- a_{33}x_3^2 - (r_3 + a_{33})x_3 + a_{31}x_1 + a_{32}x_2 + a_{31}x_1x_3 + a_{32}x_2x_3 - r_3 + \frac{1}{2}\sigma_3^2 \\
&= (r_1 - (a_{11} + a_{21} - a_{31}))x_1 - \frac{1}{2}a_{11}x_1^2 + (r_2 - (a_{12} + a_{22} - a_{32}))x_2 - \frac{1}{2}a_{22}x_2^2 \\
&- \frac{1}{2}a_{11}x_1^2 + a_{31}x_1x_3 - \frac{1}{2}a_{22}x_2^2 + a_{32}x_2x_3 - a_{33}x_3^2 - (r_3 + a_{33})x_3 \\
&+ (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \\
&= -((\frac{a_{11}}{2})^{\frac{1}{2}}x_1 - (2a_{11})^{-\frac{1}{2}}(r_1 - a_{11} - a_{21} + a_{31}))^2 + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\
&- ((\frac{a_{22}}{2})^{\frac{1}{2}}x_2 - (2a_{22})^{-\frac{1}{2}}(r_2 - a_{12} - a_{22} + a_{32}))^2 + (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 \\
&- ((\frac{a_{11}}{2})^{\frac{1}{2}}x_1 - (2a_{11})^{-\frac{1}{2}}a_{31}x_3)^2 + \frac{a_{31}^2}{2a_{11}}x_3^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \\
&- ((\frac{a_{22}}{2})^{\frac{1}{2}}x_2 - (2a_{22})^{-\frac{1}{2}}a_{32}x_3)^2 + \frac{a_{32}^2}{2a_{22}}x_3^2 - a_{33}x_3^2 - (r_3 + a_{33})x_3 \\
&\le -(a_{33} - \frac{a_{31}^2}{2a_{11}} - \frac{a_{32}^2}{2a_{22}})x_3^2 - (r_3 + a_{33})x_3 + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\
&+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \\
&= -((a_{33} - \frac{a_{31}^2}{2a_{11}} - \frac{a_{32}^2}{2a_{22}})^{\frac{1}{2}}x_3 + \frac{r_3 + a_{33}}{2(a_{33} - \frac{a_{31}^2}{2a_{11}} - \frac{a_{32}^2}{2a_{22}})^{\frac{1}{2}}})^2 \\
&+ \frac{(r_3 + a_{33})^2}{4(a_{33} - \frac{a_{31}^2}{2a_{11}} - \frac{a_{32}^2}{2a_{22}})} + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\
&+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \\
&\le \frac{(r_3 + a_{33})^2}{4(a_{33} - \frac{a_{31}^2}{2a_{11}} - \frac{a_{32}^2}{2a_{22}})} + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\
&+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \\
&= \frac{a_{11}a_{22}(r_3 + a_{33})^2}{4a_{11}a_{22}a_{33} - 2a_{22}a_{31}^2 - 2a_{11}a_{32}^2} + (2a_{11})^{-1}(r_1 - a_{11} - a_{21} + a_{31})^2 \\
&+ (2a_{22})^{-1}(r_2 - a_{12} - a_{22} + a_{32})^2 + (r_1 + r_2 - r_3) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) = D. \qquad (23)
\end{aligned}
$$

Case (I). When $D > 0$, Let $D = cEV(\phi(0))$. Then, $ELV(\phi(\theta)) < cEV(\phi(0))$. From Lemma 1 , we have

$$E|\boldsymbol{x}(t)| \le 2\gamma e^{\lambda\tau_0}E|\boldsymbol{\phi}|_C e^{-\lambda t} + \gamma, \quad t \ge 0, \qquad (24)$$

where $2 \ge \gamma e^{\lambda\tau_0}$. So, the system (2) is 1th moment practical exponential stability, i.e. the system (2) is also 1th moment persistence.

Case (II). When $D < 0$, Let $|D| = cEV(\phi(0))$. Then, $ELV(\phi(\theta)) < -cEV(\phi(0))$. From Lemma 2, we have

$$E|\boldsymbol{x}(t)| \le 2E|\boldsymbol{\phi}|_C e^{-\lambda t}, \quad t \ge 0, \qquad (25)$$

where $\lambda = \min[c, \frac{ln2}{\tau_0}]$. So, the system (2) is exponential stability, i.e. the system (2) is also 1th moment extinction. □

5 | CONCLUSION

This paper investigates a class of autonomous stochastic three-species food chain model with time-varying delays. Especially, the sufficient conditions for the 1th moment practical exponential stability and the 1th moment extinction of the model are given by using the Razumikhin technique and Lyapunov method. In the future work, we're going to investigate the pth moment extinction and pth moment practical exponential stability to non-autonomous stochastic three-species food chain model with time-varying delays.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interests regarding the publication of this article.

References

1. Berryman A. The origins and evolution of predator-prey theory. Ecology, 1992.

2. Lotka A J. Elements of Physical Biology. Baltimore: Williams and Wilkins USA, 1927.

3. Volterra V. Variazioni e fluttuazioni del numero da'rindividui in specie animali conviventi. *Mem Acad Lincei Roma.* 1926; **2**: 31-113.

4. Paine R T. Food webs: road maps of interactions or grist for theoretical development?. *Ecology,* 1998; **69**: 1648-1654.

5. Pimm S L. Food webs. New York: Chapman and Hall. USA, 1982.

6. May R M. Stability and complexity in model ecosystems. Princeton:Princeton Univ. Press, 1973.

7. Ahmad S, Stamova I M. Almost necessary and sufficient conditions for survival of species, *Anal Real World Appl.* 2004; **5**: 219-229.

8. Freedman I H, Waltman P. Mathematical analysis of some three-species food chain models. *Math Biosci.* 1997; **33**: 257-276.

9. Rudnicki R. Long-time behaviour of a stochastic prey-predator model. *Stochast Process Appl.* 2009; 108: 93-107. 2009; **108**: 93-107.

10. Zhao Y, Wang L, Wang Y. The periodic solutions to a stochastic two-prey one-predator population model with impulsive perturbations in a polluted environment. *Methodol Comput Appl Probab.* (2020)

11. Mao X, Yuan C, Zou J. Stochastic differential delay equations of population dynamics. *J Math Anal Appl.* 2005; **304**: 296-320.

12. Geng J, Liu M, Zhang Y. Stability of a stochastic one-predator two-prey population model with time delays. *Commun Nonlinear Sci Numer Simulat.* 2007; **53**: 65-82.

13. Kim J H. Further improvement of Jensen inequality and application to stability of time-delayed systems. *Automatica.* 2016; **64**: 121-125.

14. Caraballo T, Hammami M A, Mchiri L. Practical exponential stability of impulsive stochastic functional differential equations. *Syst Control Lett.* 2017; **109**: 43-48.

15. Mao X. Stochastic differential equations and applications. Chichesterr, 2nd ed. UK: Horwood Publishing, 2008.

16. Lu C, Ding X. Persistence and extinction of an impulsive stochastic logistic model with infinite delay. *Osaka J Math.* 2016; **53**: 1-29.

17. Li X, Cao J. An impulsive delay inequality involving unbounded time-varying delay and applications, *IEEE T Automat Contr.* 2017; **62**: 3618-3625.

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