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DARBOUX CURVES ON PRODUCT TIME SCALES

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Abstract. In this study, we give the structure of parameterized ruled surfaces on a product time scale. Mean and Gaussian curvatures of these surfaces are constructed on distinct time scales. Then, we obtain principal curvatures for ruled surfaces. Finally, we will clarify the issue with some examples.

Key words:Product Time Scales, Ruled Surfaces, Partial Delta Derivatives

1. INTRODUCTION

Time scale theory is a subject that's already been acquiring a lot of attention lately. It is konown that, a time scale $\mathbb{T}$ is an arbitrary, nonempty, closed subset of real numbers. Initially, Hilger investigated time scale theory in his doctorate [1]. The fundamental definitions and theorems of time scale computations are given in the textbooks of Bohner and Peterson [2]. Later, time scale theory with details for one and multi- functions provided by Bohner and Georgiev [3] together. This theory is a significant subject for biology, engineering, physics, and economics.

Many publications connected with differential geometry on $\mathbb{T}$ are examined. The essential concepts of geometry on $\mathbb{T}$ has been studied before. In [4], Bohner and Guseinov constructed the partial derivatives, tangent lines and tangent planes on $\mathbb{T}$. In 2010, Bohner and Guseinov [5] researched parameterized surfaces by time scale parameters and built an integral formula to calculate area of surfaces on $\mathbb{T}$. Actually, the main target of these studies was to unify the difference and differential geometries and to formulate the integrable geometry on $\mathbb{T}$. The number of studies for applying of differential geometry on $\mathbb{T}$ is gradually increasing (see [6], [7], [8], [9], [10], [11], [12], [13]).

In $3-$dimensional space, a continuously moving of a straight line generates ruled surface. The two variable function

$$\Phi(t,s) = \alpha(t) + s\beta(t),$$

characterizes a ruled surface generated by the family of one parameter lines $\{\alpha(t), \beta(t)\}$. Here, $\alpha(t)$ is called ruled surface directrix (base curve), and $\beta(t)$ is the director curve, [14]. These surfaces have applyings for industrial areas like mechanics, robotics. Consequently, many scientists like mechanical engineers, computer engineers and mathematicians are interested in ruled surfaces.

In Euclidean space Darboux described a family of curves on surfaces that are kept by the action of the Möbius group and share many qualities with geodesics. For a surface M and a curve on it, if the tangent plane of surface M and the tangent plane of the osculating sphere of the curve α coincide at every point of the curve, then α is called a Darboux curve. In Euclidean space, Darboux curves were firstly introduced by [15] and were generalized by [16]. For a curve α on a surface in the Euclidean 3-space, the function

$$D = < \alpha''', \mathbf{u} > = \kappa'_{\mathbf{u}} - \kappa_g \tau_g,$$

is called the Darboux function of α. Here $\mathbf{u}$ is normal vector field of surface; $\kappa_{\mathbf{u}}$, κ_g and τ_g are normal curvature, geodesic curvature and geodesic torsion. If the Darboux function is equal to zero, then the curve is called Darboux curve.

In this study, we provide essential properties of product time scales. Then, we give Darboux curve on product time scales. Finally, we categorize Darboux curves on ruled surfaces with some common product time scales.

2. BASIC CONCEPTS ON PRODUCT TIME SCALES

In this section, we gave some basic definitions and properties of product time scales.

Definition 1. *Partial delta derivative of a continuous, multi-variable function $f : \Lambda^2 \to \mathbb{R}$ is defined by*

$$\frac{\partial f}{\Delta t_j} = \lim_{\substack{s_j \to t_j \\ s_j \neq \sigma(t_j)}} \frac{f(\sigma_j(t)) - f(s)}{\sigma(t_j) - s_j},$$

where $t_j \in \mathbb{T}_j^{\kappa}$. These partial delta derivatives can be denoted also by f^{Δ^j}, $j = \overline{1,n}, n \in \mathbb{N}$. In our study, we will generally use this notation in $2-$dimensional case.

LEMMA 1. *Suppose that $f : \Lambda^2 \longrightarrow \mathbb{R}$ has mixed partial delta derivatives $\dfrac{\partial^2 f(t,s)}{\Delta_1 t \Delta_2 s}$ and $\dfrac{\partial^2 f(t,s)}{\Delta_2 s \Delta_1 t}$ in some neighbourhood of $(t_0, s_0) \in \mathbb{T}_1^{\kappa} \times \mathbb{T}_2^{\kappa}$. If these partial derivatives are continuous at (t_0, s_0), then*

$$\frac{\partial^2 f(t_0,s_0)}{\Delta_1 t \Delta_2 s} = \frac{\partial^2 f(t_0,s_0)}{\Delta_2 s \Delta_1 t}.$$

[5]

Definition 2. *$f : \Lambda^2 \to \mathbb{R}$ is called completely delta differentiable at a point $(t_1^0, t_2^0) \in \mathbb{T}_1^{\kappa} \times \mathbb{T}_2^{\kappa}$ if there is numbers A_1 and A_2 in order that*

$$\begin{aligned}
f(t_1^0,t_2^0) - f(t_1,t_2) &= A_1(t_1^0 - t_1) + A_2(t_2^0 - t_2) + \alpha_1(t_1^0 - t_1) + \alpha_2(t_2^0 - t_2),\\
f(\sigma_1(t_1^0),t_2^0) - f(t_1,t_2) &= A_1(\sigma_1(t_1^0) - t_1) + A_2(t_2^0 - t_2) + \beta_{11}(\sigma_1(t_1^0) - t_1) + \beta_{12}(t_2^0 - t_2),\\
f(t_1^0,\sigma_2(t_2^0)) - f(t_1,t_2) &= A_1(t_1^0 - t_1) + A_2(\sigma_2(t_2^0) - t_2) + \beta_{21}(t_1^0 - t_1) + \beta_{22}(\sigma_2(t_2^0) - t_2),
\end{aligned}$$

for all $(t_1,t_2) \in U_\delta(t_1^0,t_2^0)$ where $\alpha_i = \alpha_i(t_1^0,t_2^0;t_1,t_2)$ and $\beta_{ij} = \beta_{ij}(t_1^0,t_2^0;t_1,t_2)$ are equal to zero at $(t_1,t_2) = (t_1^0,t_2^0)$. [5]

Definition 3. *f:$\Lambda^2 \to \mathbb{R}$ is σ_1-completely delta differentiable at $(t_1^0,t_2^0) \in \mathbb{T}_1^{\kappa} \times \mathbb{T}_2^{\kappa}$ if it is completely delta differentiable at that point and moreover, along with the numbers A_1 and A_2 there exists also a number B independent of $(t_1,t_2) \in \Lambda^2$ such that*

$$f(\sigma_1(t_1^0), \sigma_2(t_2^0)) - f(t_1,t_2) = A_1(\sigma_1(t_1^0) - t_1) + B(\sigma_2(t_2^0) - t_2) + \gamma_1(\sigma_1(t_1^0) - t_1) + \gamma_2(t_2^0 - t_2),$$

for all $(t_1,t_2) \in V^{\sigma_1}(t_1^0,t_2^0)$ where $V^{\sigma_1}(t_1^0,t_2^0)$ is a union of some neighbourhoods of the points (t_1^0,t_2^0) and $(\sigma(t_1^0), t_2^0)$, and $\gamma_1 = \gamma_1(t_1^0,t_2^0;t_1,t_2)$, $\gamma_2 = \gamma_2(t_1^0,t_2^0;t_2)$ are equal to zero at $(t_1,t_2) = (t_1^0,t_2^0)$. σ_2-completely delta differentiable functions at a point $(t_1^0,t_2^0) \in \mathbb{T}_1^{\kappa} \times \mathbb{T}_2^{\kappa}$ can be defined analogously. [5]

3. Structures of Darboux Curve and Ruled Surfaces on Product Time Scales

Let $\Omega \subset \Lambda^2$ and $\varphi, \psi, \chi : \Omega \to \mathbb{R}$ be continuous functions with time scale topology. Here, we consider opens as the sets whose closures are open in standard real topology. We refer the readers who want to go further into the topic to [17]. Consider the $xyz-$space, i.e., the set of all ordered triples (x,y,z) of real numbers x,y and z. Each such triple determines a point of the space, and the numbers x,y and z are the coordinates of that point. Now let's express how the concept of surface is explained on product time scale.

Definition 4. ***L**et S be a closed subset of $\mathbb{R}^3$. S is a surface if for each point P in S, there is a neighbourhood A of P and a function $\phi : U \to S$, where U is a closed set in $\mathbb{R}^2$ and an open set on time scale topology which satisfies following conditions:*

i) $\phi : U \to \mathbb{R}^3$ is $\Delta-$differentiable and

$$\frac{\partial \phi(t,s)}{\Delta_1 t} \times \frac{\partial \phi(t,s)}{\Delta_2 s} \neq 0,$$

for all $(t,s) \in U$ where $\times$ denotes the cross product. That is, ϕ is $\Delta-$ regular.

ii) $\phi(U) = S \cap A$ and $\phi : U \to \phi(U)$ is a homeomorphism where $\phi : U \to S$ is a surface patch. S is called a smooth surface if there exists a surface patch such that $P \in \phi(U)$ for all points P in S. [18]

There are detailed [18] studies in the literature regarding different properties of surfaces on time scales. For instance, Atmaca et al. presented metric properties of surfaces on time scales. Samanci and Caliskan [11] viewed the extent curves and surfaces on time scales.

Definition 5. *Let $\alpha_i(t_1), \beta_j(t_1) \in C^\infty(\Lambda^2)$ $i,j = \overline{1,3}$ where $C^\infty(\Lambda^2)$ denotes the space of all continuous functions which are completely delta differentiable on Λ^2 and $\alpha : [a,b] \subset \mathbb{T}_1 \to \mathbb{R}^3$, $\beta : [a,b] \subset \mathbb{T}_2 \to \mathbb{R}^3$ be given curves in the parametric forms $\alpha(t) = (\alpha_1(t_1), \alpha_2(t_1), \alpha_3(t_1))$, $\beta(t_1) = (\beta_1(t_1), \beta_2(t_1), \beta_3(t_1))$, respectively. Then,*

$$\phi : \Lambda^2 \to \mathbb{R}^3,\ (t_1,t_2) \to \phi(t_1,t_2) = \alpha(t_1) + t_2\beta(t_1),$$

is called ruled surface. If ϕ is Δ- regular, i.e. $\frac{\partial\phi}{\Delta_1 t} \times \frac{\partial\phi}{\Delta_1 t} \neq 0$ everywhere in $\mathbb{R}^3$, the normal vector field of ϕ is defined by

$$\mathbf{U} = \frac{\alpha^{\Delta_1}(t_1) \times \beta(t_1) + t_2\beta^{\Delta_1}(t_1) \times \beta(t_1)}{\|\alpha^{\Delta_1}(t_1) \times \beta(t_1) + t_2\beta^{\Delta_1}(t_1) \times \beta(t_1)\|}. \tag{3.1}$$

*(see [18], [4], [10], [11], [12], [20]**)**.*

Here, we give some properties of darboux curve with arc lenght and ruled surfaces on Λ^2.

THEOREM 1. *Let $\alpha(t_1)$ and $\beta(t_1)$ be two curves on Λ^2 and*

$$\phi : \Lambda^2 \to \mathbb{R}^3, \phi(t_1,t_2) = \alpha(t_1) + t_2\beta(t_1), \tag{3.2}$$

be a ruled surface on Λ^2. Then $\alpha(t_1)$ is a darboux curve on $\phi(t_1,t_2)$ if

$$\det(\alpha^{\Delta_1^3}(t_1), \alpha^{\Delta_1}(t_1), \beta(t_1)) + t_2 \det(\alpha^{\Delta_1^3}(t_1), \beta^{\Delta_1}(t_1), \beta(t_1)) = 0. \tag{3.3}$$

Proof. To prove the given theorem, we need to use definition of partial delta derivative on product time scales. Using partial delta derivatives of (3.1) with respect to t_1 and t_2 yields

$$\phi^{\Delta_1}(t_1,t_2) = \alpha^{\Delta_1}(t_1) + t_2\beta^{\Delta_1}(t_1), \tag{3.4}$$

$$\phi^{\Delta_2}(t_1,t_2) = \beta(t_1). \tag{3.5}$$

Then, the unit normal vector field of the surface ϕ is (3.1). So, if $\alpha(t_1)$ is a Darboux curve,

$$< \alpha^{\Delta_1^3}(t_1), \frac{\alpha^{\Delta_1}(t_1) \times \beta(t_1) + t_2\beta^{\Delta_1}(t_1) \times \beta(t_1)}{\|\alpha^{\Delta_1}(t_1) \times \beta(t_1) + t_2\beta^{\Delta_1}(t_1) \times \beta(t_1)\|} >= 0. \tag{3.6}$$

Finally the proof is complete. □

COROLLARY 1. *Let $\alpha(t_1)$ and $\beta(t_1)$ be two curves on Λ^2 and*

$$\phi : \Lambda^2 \to \mathbb{R}^3, \phi(t_1,t_2) = \alpha(t_1) + t_2\beta(t_1),$$

be a ruled surface on Λ^2. Then $\beta(t_1)$ is a darboux curve on $\phi(t_1,t_2)$ if

$$\det(\beta^{\Delta_1^3}(t_1), \alpha^{\Delta_1}(t_1), \beta(t_1)) + t_2 \det(\beta^{\Delta_1^3}(t_1), \beta^{\Delta_1}(t_1), \beta(t_1)) = 0. \tag{3.7}$$

EXAMPLE 1. Let $\alpha(t_1) = (a\cos t_1, a\sin t_1, bt_1)$ and $\beta(t_1) = ((1+h)^{t_1/h}, (1-h)^{t_1/h}, \sqrt{2}t_1)$ be two curves on $h\mathbb{Z} \times h\mathbb{Z}$. Then, we have a *ruled surface* on $h\mathbb{Z} \times h\mathbb{Z}$

$$\varphi(t_1,t_2) = (a\cos t_1 + t_2(1+h)^{t_1/h}, a\sin t_1 + t_2(1-h)^{t_1/h}, bt_1 + \sqrt{2}t_1t_2). \tag{3.8}$$

From derivatives of the curve $\alpha(t_1)$ according to t_1, we have

$$\alpha^{\Delta_1^3}(t_1) = (a\sin t_1, -a\cos t_1, 0). \tag{3.9}$$

The unit normal vector field of the surface $\varphi(t_1,t_2)$is

$$\mathbf{U} = \frac{1}{\sqrt{A}}(\sqrt{2}a\cos t_1 + b(1-h)^{t_1/h}, \sqrt{2}a\sin t_1 + b(1+h)^{t_1/h}, a\sin t_1(1-h)^{t_1/h} - a\cos t_1(1+h)^{t_1/h}), \tag{3.10}$$

where

$$A = 2a^2 + 2\sqrt{2}ab(\cos t_1(1-h)^{t_1/h} + \sin t_1(1+h)^{t_1/h}) + b^2((1-h)^{2t_1/h} + (1+h)^{2t_1/h}) + a^2(\sin^2 t_1(1-h)^{2t_1/h} + \cos^2 t_1(1+h)^{2t_1/} \tag{3.11}$$

Then, if $\alpha(t_1)$ is Darboux curve on the surface $\varphi(t_1,t_2)$, we have

$$ab(\sin t_1(1-h)^{t_1/h} - \cos t_1(1+h)^{t_1/h}) = 0. \tag{3.12}$$

From (3.11), following conditions are satisfied;

i) $a = 0$ or $b = 0$.

ii) $\tan t_1 = \dfrac{(1+h)^{t_1/h}}{(1-h)^{t_1/h}}$.

EXAMPLE 2. Let $\varphi(t_1,t_2)$ be given with the parameterization (3.8) on $h\mathbb{Z} \times h\mathbb{Z}$. Then, if $\beta(t_1)$ is a Darboux curve on $\varphi(t_1,t_2)$, we have $a = 0$ or $\tan t_1 = \dfrac{(1+h)^{t_1/h}}{(1-h)^{t_1/h}}$.

EXAMPLE 3. Let $\alpha(t_1) = (a\cos t_1, a\sin t_1, bt_1)$ and $\beta(t_1) = ((1+h)^{t_1/h}, (1-h)^{t_1/h}, \sqrt{2}t_1)$ be two curves on $\mathbb{Z} \times \mathbb{R}$. Then, we have a *ruled surface* on $\mathbb{Z} \times \mathbb{R}$

$$\psi(t_1,t_2) = (a\cos t_1 + 2^{t_1}t_2, a\sin t_1, bt_1 + 2t_1t_2). \tag{3.13}$$

Then, if $\alpha(t_1)$ is Darboux curve on the surface $\varphi(t_1,t_2)$ on $\mathbb{Z} \times \mathbb{R}$, we have

i) $a = 0$ or $b = 0$.

ii) $t_1 = \dfrac{\pi}{2} + 2k\pi, k \in \mathbb{R}$.

b) For the ruled surface which is parameterized (3.13) on $\mathbb{Z} \times \mathbb{R}$, if $\beta(t_1)$ is Darboux curve on the surface

$\varphi(t_1,t_2)$ on $\mathbb{Z}\times\mathbb{R}$, we have $a=0$ or $t_1=\dfrac{\pi}{2}+2k\pi, k\in\mathbb{R}$.

4. A CLASSIFICATION OF RULED SURFACES WITH DARBOUX CURVE ON SOME COMMON PRODUCT TIME SCALES

Here, we examine properties of Darboux curves on ruled surfaces in a special case

$$\chi:\Lambda^2\to\mathbb{R}^3,\ (t_1,t_2)\to\phi(t_1,t_2)=\alpha(t_1)+t_2\alpha^{\Delta_1}(t_1). \tag{4.1}$$

where $\alpha(t_1)$ is a curve on Λ^2. If $\chi(s,t)$ is $\Delta-$ regular, the normal vector field of χ is defined by

$$\mathbf{U}=\frac{\alpha^{\Delta_1}(t_1)\times\alpha^{\Delta_1^2}(t_1)+t_2\alpha^{\Delta_1^2}(t_1)\times\alpha^{\Delta_1}(t_1)}{\left\|\alpha^{\Delta_1}(t_1)\times\alpha^{\Delta_1^2}(t_1)+t_2\alpha^{\Delta_1^2}(t_1)\times\alpha^{\Delta_1}(t_1)\right\|}. \tag{4.2}$$

THEOREM 2. *Let $\alpha(t_1)$ and $\beta(t_1)$ be two curves on Λ^2 and*

$$\phi:\Lambda^2\to\mathbb{R}^3,\phi(t_1,t_2)=\alpha(t_1)+t_2\alpha^{\Delta_1}(t_1),$$

be a ruled surface on Λ^2. Then $\alpha(t_1)$, is a Darboux curve on $\phi(t,s)$ if

$$\det(\alpha^{\Delta_1^3}(t_1),\alpha^{\Delta_1^2}(t_1),\alpha^{\Delta_1}(t_1))=0. \tag{4.3}$$

Proof. To prove the given theorem, we need to use definition of partial delta derivative on product time scales. Using partial delta derivatives of $\psi(t,s)$ with respect to t and s yields

$$\chi^{\Delta_1}(t_1,t_2)=\alpha^{\Delta_1}(t_1)+t_2\alpha^{\Delta_1^2}(t_1), \tag{4.4}$$

$$\chi^{\Delta_2}(t_{1,}t_2)=\alpha^{\Delta_1}(t_1), \tag{4.5}$$

Then, the unit normal vector field of the surface ϕ is (3.1). So, if $\alpha(t_1)$ is a Darboux curve,

$$<\alpha^{\Delta_1^3}(t_1),\frac{\alpha^{\Delta_1}(t_1)\times\alpha^{\Delta_1^2}(t_1)+t_2\alpha^{\Delta_1^2}(t_1)\times\alpha^{\Delta_1}(t_1)}{\left\|\alpha^{\Delta_1}(t_1)\times\alpha^{\Delta_1^2}(t_1)+t_2\alpha^{\Delta_1^2}(t_1)\times\alpha^{\Delta_1}(t_1)\right\|}>=0. \tag{4.6}$$

Finally, the proof is complete. □

COROLLARY 2. *Let $\alpha(t_1)$ and $\beta(t_1)$ be two curves on Λ^2 and*

$$\phi:\Lambda^2\to\mathbb{R}^3,\phi(t_1,t_2)=\alpha(t_1)+t_2\alpha^{\Delta_1}(t_1),$$

be a ruled surface on Λ^2. Then $\beta(t_1)$ is a Darboux curve on $\phi(t,s)$ if

$$\det(\beta^{\Delta_1^3}(t_1),\alpha^{\Delta_1^2}(t_1),\alpha^{\Delta_1}(t_1))=0. \tag{4.7}$$

EXAMPLE 1. Let

$$\varphi(t_1,t_2)=(a\cos t_1-at_2\sin t_1,a\sin t_1+at_2\cos t_1,b(t_1+t_2)), \tag{4.8}$$

be a ruled surface on $\Lambda^2=h\mathbb{Z}\times h\mathbb{Z}$ or $\Lambda^2=\mathbb{Z}\times\mathbb{R}$, $\alpha(t_1)=(a\cos t_1,a\sin t_1,bt_1)$ on $h\mathbb{Z}\times h\mathbb{Z}$ or $\mathbb{Z}\times\mathbb{R}$. Then,.if $\alpha(t_1)$ is a Darboux curve, $a=0$ or $b=0$.

5.CONCLUSION

In this paper, firstly we gave some definition and properties of time scales. Then, we investigate Darboux curve with some properties and examples on product time scales.

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