Ducting and Conversions of Whistler Waves in Varying Density Plasma With Boundary Conditions

UCLA Physics 180E

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1 Introduction

Whistler waves were heard as early as 1886 on long telephone lines. In 1953, Storey showed that whistlers originate from lightning discharges, propogating along field-aligned tubes of density enhancement or depletion. The first experiment to investigate whistler waves in detail was by Stenzel in 1975, in a different plasma with a much higher density but otherwise comparable plasma parameters for this experiment [1]. This experiment seeks to find out what occurs to a plasma and whistler waves when there is a density depletion or duct. In the lab a duct is made in the plasma by placing a a copper circular disc between the wave launch antenna and the plasma source. The disc is biased to collect electrons and in turn creates a density depletion [1]. Using Appleton's equation, simulations of ducted waves were compared to measured ducted waves as well as finding the polarization of these waves due to ducting.

2 Theory

The study of whistler waves in ducting will still be determined by the dispersion relation given by the approximation of Appleton's Equation

$$\eta^2 = 1 - \frac{\omega_{pe}^2}{\omega^2 \left((1 + \frac{i\nu}{\omega}) - \frac{\omega_{ce}^2 \sin^2\theta}{2\omega^2 \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)} \pm \sqrt{\frac{(\omega_{ce}^2 \sin^2\theta)^2}{4(1 - \frac{\omega_{pe}^2}{\omega^2})} + \frac{\omega_{ce}^2 cos^2\theta}{\omega^2}} \right)}$$

One of the remarkable aspects of ducted whistler waves is that the phase velocity travels at a separate angle to the group velocity and direction of propogation of the wave. The group velocity is given by $\vec{V}_{gr} = \frac{\partial\omega}{\partial\vec{k}}$ and the phase velocity is given by $\vec{V_{ph}} = \frac{\omega}{k}$. The group velocity can be rewritten as $V_{group} = \frac{c}{\frac{d}{df}(\eta f)}$, which for whistler waves becomes approximately $V_{group} =$

$2c\frac{\sqrt{f}}{f_{pe}f_{ce}\cos 2\theta}(f_{ce}\cos\theta - f)^{\frac{3}{2}}$ where θ is the angle between the group velocity and the magnetic field. It can then be shown that this angle is represented as

$$\theta_{group} = a\tan\left(\frac{\sin\theta\left(\cos\theta - 2\frac{f}{f_{ce}}\right)}{1+\cos\theta\left(\cos\theta - 2\frac{f}{f_{ce}}\right)}\right)$$

Consider the case when the wave propagation angle is small with respect to the magnetic field and there are no collisions, then appletons equation reduces to

$$\eta = \sqrt{1 - \frac{\omega_{pe}^2}{\omega(\omega - \omega_{ce}\cos\theta)}}$$

and plotting the index of refreaction surface gives conditions for wave propagation in a duct [3]. It is seen that for waves with $\omega > \frac{\omega_{ce}}{2}$ will travel along density maximums, while waves with $\omega < \frac{\omega_{ce}}{2}$ will travel in density minimums [3].

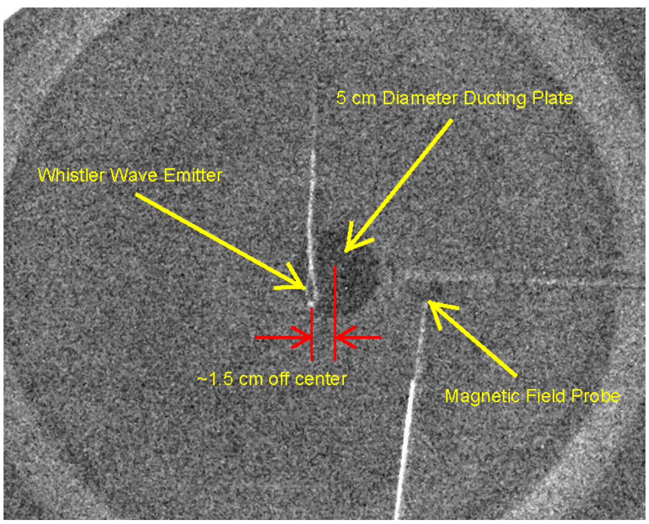

Figure 1: Experimental setup for the purpose of ducting as seen from the front (opposite the source). The copper ducting plate is placed about 1.5 cm in the negative radial direction with the whistler wave emitter taken as centered.

3 Experimental Setup

The plasma was created by an inductively coupled RF source operating at a power of 210 W with 5.5×10^{-3} torr Helium as the working gas. A voltage sweep was put across a langmuir probe 70 cm from the RF source 0.1 ms into the

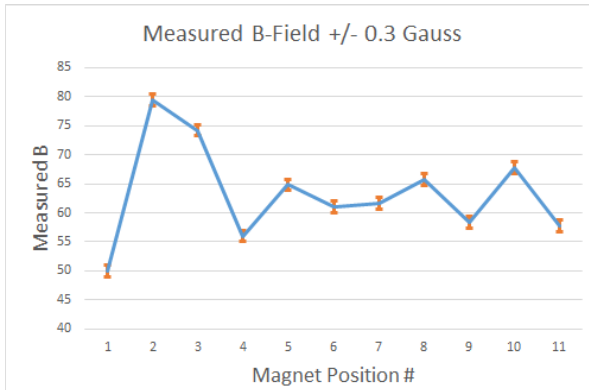

Figure 2: Magnetic field measurements taken using a Hall probe. Position 2 is just after the source along the z-axis. The magnetic field is assumed to be radially symmetric.

afterglow. A magnetic field was applied to the device through the use of two sets of four coaxial magnets created using 59.0 A and 24.5 A currents laid out around the device and resulting in a radially symmetric magnetic field through the device averaging to about 63.5 Gauss along the length. Measurements of the magnetic field using a hall probe is shown in figure 2, where position 2 is just after the source. Whistler waves were generated in the plasma using a waveform generating antenna, set to 110 MHz. To make a duct, a copper circular plate 5 cm in diameter was installed 3.9 cm distance from the source between the source and the whistler wave launch antenna. The disc is biased to collect electrons. The plasma in turn will try to retain quasi-neutrality by parting the ions. This creates a density profile and "ducts" the wave [2]. A B-dot probe was used to measure the change in the magnetic field due to the wave over an axial range of 4.8 cm - 44.8 cm from the RF source and -45^o to 45^o radially. The B-dot probe used in this experiment consists of 6 faces of a wire of one loop in a cube orientation. When the loop of wire detects a change in magnetic flux, there will be an induced voltage by faradays law.

4 Data Analysis

The measurements have been plotted in figures 3 through 5. Figure 3 shows waves launched 1.2 μs after being triggered. First waves are seen propagating towards density minimum, where the duct is located. Figure 4 shows measurements taken 2.28 μs after being triggered, were the wavefront is evident to be propagating away from duct position. The limit at which the wave will propagate towards the density maximum is when

$$\omega \geq \frac{\omega_{ce}}{2}$$

where $\omega_{ce} = \frac{qB}{m_e}$. Solving for the magnetic field and plugging in frequency $f = 2\pi\omega$:

$$B_{max} = \frac{4\pi m_e f}{q}$$

Plugging in $f = 110$ MHz, then $B_{max} = 78.6$ Gauss. Looking at figure 2, the magnetic field measurements next to the source (position 2) were measured to be 79.4 Gauss, above the maximum magnetic field. Since the magnetic field an that region was higher than critical magnetic field for 110 MHz, its reasonable to assume the waves will propagate towards the density minimum where the duct is located, as can be seen in the measurements. In figure 4, after a micro second, wave fronts are seen traveling towards the density maximum. Magnetic fields measured further away from the source than position 2 are less than B_{max}, where $\omega > \frac{\omega_{ce}}{2}$. Plotting magnetic field magnitudes for x and y, as shown in figure 5, it can be seen that the plasma is right-hand elliptically polarized [3].This would suggest that the wave front is not necessarily travelling parallel to the field lines, but is moving at an angle, which can be seen in figures 3 and 4.

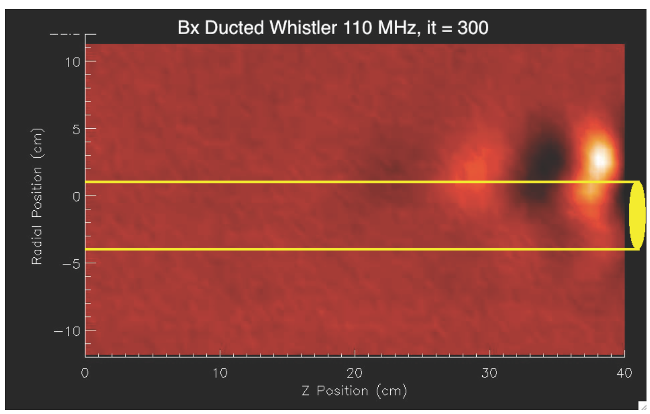

Figure 3: Ducted whistler waves 1.2 μs after being triggered. Wave fronts are seen to be travelling towards density minimum where the ducting plate is positioned. The source is located on the right($z > 40cm$). Yellow lines represent where the duct is located relative to the source.

The simulation was made by modeling Appleton's equations with a density profile. Waves were generated at different angles and summed to obtain an interference pattern. The results are shown in figure 6. Waves can clearly be seen as ducted with expected results of waves propagating along density maximum. The parameters used were similar to the experiment with frequency and magnetic field set to 110 MHz and 63.5 Gauss respectively. The density profile used is shown in figure 7. Due to errors when taking density measurements (not all channels were saved), a different density measurement had to be used. However

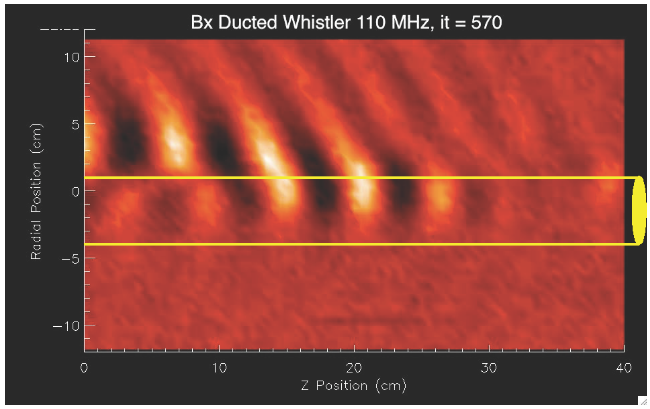

Figure 4: Ducted whistler waves 2.28 μ s after being triggered. Wave fronts are seen to be travelling towards density maximum, away from the ducting plate. In this region, the magnetic field is low enough where $\omega > \frac{\omega_{ce}}{2}$. Yellow lines represent where the duct is located relative to the source.

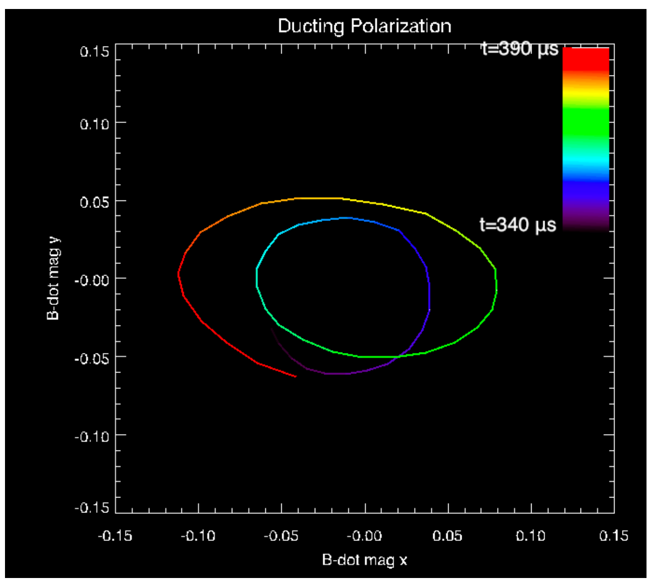

Figure 5: Time evolution of the magnitudes measured in the x-axis and y-axis. The magnetic field is taken as traveling "out of the board". It can be seen that the waves are right-hand polarized and elliptical. This was taken 5 cm away from the source along the central axis ($\theta = 0^o$).

the utilized density measurements were similar to the ones in the experiment, except the ducting plate was placed on the opposite side of the wave emmitter. The density profile was mirrored about x = 0 cm to best match measured results. Comparing the simulation to the measured results shown in figures 3 and 4, it is evident that they do not match. Though similar, the simulation does not take into account a magnetic field changing along the z-axis.

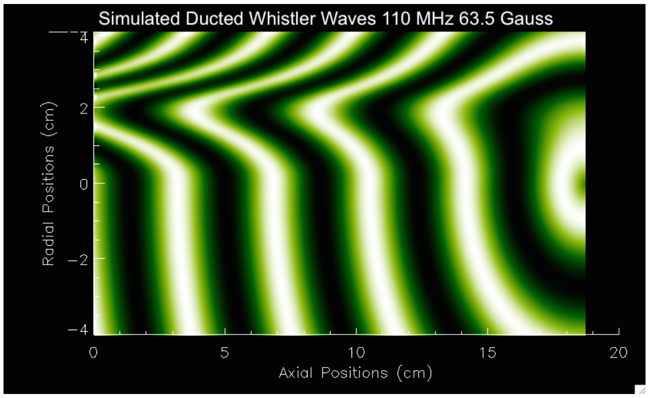

Figure 6: Simulated ducted whistler waves using parameters similar to experiment. Wave fronts are seen traveling fastest towards density maximum, as theory would suggest when $\omega > \frac{\omega_{ce}}{2}$.

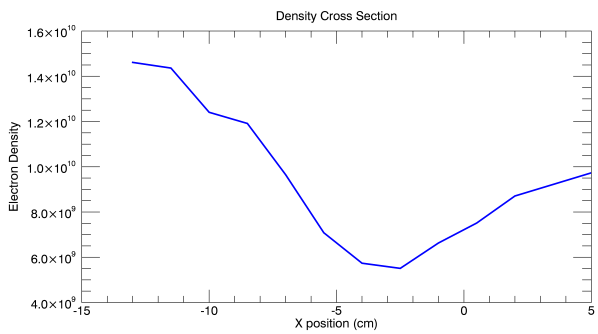

Figure 7: The density profile used for the simulation. This is a similar setup to the one used in this experiment except the ducting disk was placed on the opposite side of the source. This density profile was used due to mishandeling of density measurements during the actual experiment. The density measurements used were mirrored version about x = 0 to best match actual density profile.

5 Discussion of Results

Since B-field is higher than critical B-field next to source, waves close to source propagate along density minimum, and then work their way up to density maximum. As they travel into a region of lower magnetic field, the frequency becomes larger than $\frac{f_{ce}}{2}$, making the wavefronts travel along density maximums, as expected by theory. The simulation used did not match the results, the most probable reason because it did not take into account a changing magnetic field along the z-axis. The polarization of the ducted waves were found to be right-hand elliptical, suggesting wavefronts were propagating at an angle against the field lines, which were observed in the measurements.

6 Appendix

For Appletons's equation, the electron cyclotron frequency is given by

$$\omega_{ce} = \frac{eB}{m_e}$$

where e is the elementary charge, B is the magnetic field and m_e is the electron masst.The plasma frequency is given by

$$\omega_{pe} = \sqrt{\frac{ne^2}{m_e \epsilon_0}}$$

where n is the plasma density,and ϵ_0 is the permittivity of free space and ν is the collision frequency.

7 References

[1]-R. L. Stenzel, "Whistler Wave propagation in a large magnetoplasma", Phys. Fluids, 19, 858 (1975)

[2]-Gekelman, Walter. Week 7 notes,pg.3, 180E Lab Course, UCLA, Spring 2013

[3]-Chen, Francis F. "Chapter 4: Waves in Plasmas." Introduction to Plasma Physics. Boston, MA: Springer US, 1995.Print