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RESEARCH ARTICLE

# Fault Estimation And Tolerant Anti-disturbance Switching Control for Switched Systems with Its Application to Circuit Systems

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**Summary**

This study proposes a fault estimation and tolerant anti-disturbance switching control (ADSC) approach for the switched systems subject to the system fault and multiple disturbances. The fault dose not required to have a model.The disturbances contain the modeled unmeasurable part and the unmodeled measurable part. First, a composite switching estimator is constructed to simultaneously estimate the unavailable system state, fault and modeled disturbance. Then, by means of the estimator, a switching controller is developed to tolerant the fault and complement the modeled disturbance. Further, under the dwell time relevant switching signals, criteria are established to ensure the fault estimation performance and robustness property for the switched systems. Finally, via implementing the presented fault estimation and tolerant ADSC scheme on a switched circuit system to regulate the charge and flux, the reasonability of the established result is verified.

**KEYWORDS:**
switched system, fault estimation, fault tolerant, multiple disturbances

## 1 | INTRODUCTION

Switched systems are peculiar hybrid systems featured by multi-modes and switching. The switching behavior is described by switched system and a function specified as the switching index. Numerous practical dynamics have features of multi-modes and switching, such as power electronics [1,2], aero-engine systems (see [3]-[5]), circuit systems [6,7] and so on. Lots of efforts have been cast into the investigation of switched systems with a growing number of results attained. To list a few, the dissipativeness [8], event-triggered (see [9]-[15]), adaptiveness [16] and so on.

In practice, while a control system work, it may suffer from various unexpected factors. Disturbances and failures are two typical unwelcome characteristics which usually cause a reduction on reliability and security of the practical processes.

On the one hand, to deal with the faults in the systems, many fault tolerant control (FTC) schemes have been established. During the process of fault tolerant, fault estimation occupies a vital position. Lots of contributions have been devoted to fault estimation. For the different occurrence parts of the system failure, different fault estimators have been developed for estimating the faults. The robust estimator [17,18], sliding mode estimator [19], descriptor estimator [20], adaptive estimator [21,22] are the well developed fault estimators. It is necessary to point out that a matching constraint assumption is adapted in the construction of many fault estimators see [23] for example. Luckily, [24] presented a relaxtion on the matching constraint and then attained the fault tolerant of a liner system. However, the considered system is non-switched. For the existing investigations on the fault estimation and FTC issues of switched systems in [25], this matching constraint still exists. This constitutes one motivation of this paper.

On the other hand, multiple types of disturbances often simultaneously exist as soon as a control system operates in practice. Recently, several kinds of effective disturbance estimators have been developed, like the composite estimator[26], extended estimator[27] and equivalent input estimator[28 29], to achieve the elimination or suppression of the disturbances. Notice that the composite estimator is favorable due to its perfect compensation of the unmeasurable modeled disturbance. The composite estimator technique has been successfully employed to combine with other control techniques to pursue the corresponding system performance with a good disturbance suppression property. For switched systems, to mention a few, the composite estimator approach has been combined with the $H_\infty$ control scheme[30], the bumpless transfer control strategy (see[31]-[34]) and the adaptive control way (see[35]-[37]) to obtain the better robustness, bumpless transfer performance and the adaptive property, respectively. For the FTC issue of switched systems, the study of composite anti-disturbance control is quite few.[38 39] addressed the FTC issue for the switched stochastic system with the mismatched unmodeled disturbance instead of the modeled disturbance and the fault occur at the actuator. Notice that the process fault is more general than the actuator fault. Therefore, it is more general and interesting to make an investigation on the FTC issue for the switched systems with the process fault and multiple disturbances.

Via this paper, the fault estimation based as fault tolerant ADSC problem of switched systems encountered by both process fault and multiple disturbances are studied. By designing the disturbance estimator, fault estimator without the matching constraint, the unknown disturbance and fault are estimated. Then, according to the observers, the fault tolerant anti-disturbance switching controller is established to pursue the practical stability of the switched systems under the dwell-time relevant switching index. Finally, the reasonability of the developed fault tolerant ADSC control scheme is verified by a switched circuit model. Different from the existing investigations, three features of this paper is worth noticed.

**i)** The matching limitation used in the fault estimation process of switched systems is taken away. This renders that a wider range of faults can be mastered.

**ii)** The multiple disturbances are suppressed or compensated while addressing the fault. The disturbances include the modeled disturbances and unmodeled disturbances. This coincides with practice better.

**iii)** The criterion is formed to ensure that the established fault tolerant ADSC scheme can force the switched systems to be stable.

**Structure.** Section 2 introduces the formulation of the fault estimation and tolerant ADSC issue. The fault tolerant ADSC technique is developed in Section 3. Section 4 verifies the effectiveness of the established fault estimator and tolerant ADSC strategy. The conclusions are obtained by Section 5.

**Notation.** $L_2[t_0,\infty)$ suggests the set of square integrable vector functions on $[t_0,\infty)$. $diag\{A,B\}$ represents a diagonal matrix with $A$ and $B$ specifying elements on the diagonal. $sym(A)$ denotes $A+A^T$.

## 2 | PROBLEM FORMULATION

We take the following switched system

$$\begin{aligned} \dot{x}(t) &= J_{\theta(t)}x(t) + L_{\theta(t)}[u(t)+\Psi_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\mathbb{f}(t), \\ y(t) &= N_{\theta(t)}x(t) + O_{\theta(t)}\Psi_2(t), \end{aligned} \tag{1}$$

into account, $x(t)\in R^n$, $u(t)\in R^w$ and $y(t)\in R^q$ are the system state, control input and control output, respectively, $\Psi_1(t)\in R^\omega$ and $\Psi_2(t)\in R^p$ are the disturbance input, $\mathbb{f}(t)\in R^r$ indicates the fault signal, $\theta(t)$ represents switching logic (SL). For convenience , we exploit the sequence

$$\{x_0;(q_0,t_0),(q_1,t_1),\cdots,(q_n,t_n),\cdots|q_n\in\Xi, n\in N\}$$

to express the switching index $\theta(t)$, where $x_0$, $t_0$, $t_n$ refer to the incipient state, incipient point and $n$th switching point, respectively, $\Xi=\{1,2,\dots m\}$ refers to the amount of the subsystems, and $\theta(t)=q_n$ specifies that the $q_n$th submodel activates.

In the system (1), the state $x(t)$, the disturbance $\Psi_1(t)$ and the fault $f(t)$ are pre-unknown, $\Psi_1(t)$ comes from the following model

$$\begin{aligned} \dot{\xi}(t) &= P_{\theta(t)}\xi(t) + Q_{\theta(t)}\Psi_3(t), \\ \Psi_1(t) &= R_{\theta(t)}\xi(t), \end{aligned} \tag{2}$$

in which $\xi(t)\in R^h$ refers to the state, $\Psi_3(t)\in R^r$ refers to the disturbance input satisfying the same property as $\Psi_2(t)$, namely, $\Psi_2(t)$ and $\Psi_3(t)$ all belong $L_2[0,\infty)$.

For the system (1), the following assumptions and definition are needed.

**Assumption 1.** $S_i$ is the full rank matrix, namely, $\mathrm{rank}(S_i) = r$, $i \in \Xi$.

**Assumption 2.** For any complex value $\chi$ having non-negative real part, it yields that

$$\mathrm{rank}\begin{bmatrix} J_i - \chi I & S_i \\ N_i & 0 \end{bmatrix} = \mathrm{n} + \mathrm{rank}(S_i).i \in \Xi.$$

**Assumption 3.** $\mathrm{Rank}(L_i, S_i) = \mathrm{rank}(L_i)$,$i \in \Xi$.

**Assumption 4.** The unknown time-varying fault $f(t)$ satisfies $\|\dot{f}(t)\| \leq \aleph$ with $\aleph \geq 0$ being a specified constant.

**Definition 1.** [40]For any SL $\theta(t)$ and time $t_b > t_a \geq 0$, if the quantity of switchings for the SL $\theta(t)$ during the interval $[t_a, t_b)$ ensures

$$\rho_\theta(t_a, t_b) \leq \frac{t_b - t_a}{s}$$

then, the invariable $s$ is the average dwell time of the SL $\theta(t)$.

The control aim is to estimate the unknown state $x(t)$, fault $\mathbb{f}(t)$ and disturbance $\Psi_1(t)$,and then design a control scheme to tolerant the fault $\mathbb{f}(t)$, suppress the disturbance $\Psi_1(t)$,$\Psi_2(t)$ and $\Psi_3(t)$.

First, in order to capture the disturbance $\Psi_1(t)$, we design the following observer

$$\begin{aligned} &\dot{\eta}(t) = (P_{\theta(t)} + \Lambda L_{\theta(t)} R_{\theta(t)})[\eta(t) - \Lambda x(t)] + \Lambda[J_{\theta(t)}x(t) + L_{\theta(t)}u(t) + S_{\theta(t)}\mathbb{f}(t)], \\ &\hat{\xi}(t) = \eta(t) - \Lambda x(t), \\ &\hat{\Psi}_1(t) = R_{\theta(t)}\hat{\xi}(t), \end{aligned} \tag{3}$$

where $\hat{\Psi}_1(t)$ represents the estimation of the disturbance $\Psi_1(t)$ , $\Lambda$ is the observer gain to be yielded.

Then, for purpose of getting the state of $x(t)$ and the fault $\mathbb{f}(t)$, we introduce the following variable

$$r(t) = \mathbb{f}(t) - W x(t), \tag{4}$$

and constructing the following observer

$$\begin{aligned} &\dot{\hat{x}}(t) = J_{\theta(t)}\hat{x} + L_{\theta(t)}[u(t) + \hat{\Psi}_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\hat{\mathbb{f}}(t) + T_{\theta(t)}[y(t) - \hat{y}(t)], \\ &\dot{\hat{r}}(t) = -W S_{\theta(t)}\hat{r}(t) - W\{J_{\theta(t)}\hat{x}(t) + L_{\theta(t)}[u(t) + \hat{\Psi}_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)} W \hat{x}(t)\}, \\ &\hat{y}(t) = N_{\theta(t)}\hat{x}(t) + O_{\theta(t)}\Psi_2(t), \\ &\hat{\mathbb{f}}(t) = \hat{r}(t) + W\hat{x}(t), \end{aligned} \tag{5}$$

among them, $\hat{x}(t)$, $\hat{r}(t)$, $\hat{y}(t)$, $\hat{\mathbb{f}}(t)$ are the estimation of $x(t)$, $r(t)$, $y(t)$ and $\mathbb{f}(t)$, respectively, $W$ is a matrix to be specified satisfying $W = \upsilon_i S_i^T, i \in \Xi$ , $\upsilon$ is a scalar to be solved.

Further, definite the estimation errors

$$\begin{aligned} &e_x(t) = x(t) - \hat{x}(t), e_r(t) = r(t) - \hat{r}(t), \\ &e_{\mathbb{f}}(t) = \mathbb{f}(t) - \hat{\mathbb{f}}(t), e_\xi(t) = \xi(t) - \hat{\xi}(t), \end{aligned}$$

thus, the error dynamics are attained as

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= J_{\theta(t)}x(t) + L_{\theta(t)}[u(t) + \Psi_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\mathbb{f}(t) - \{J_{\theta(t)}\hat{x}(t) + L_{\theta(t)}[u(t) + \hat{\Psi}_1(t)] \\ &\quad + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\hat{\mathbb{f}}(t) + T_{\theta(t)}[y(t) - \hat{y}(t)]\} \\ &= (J_{\theta(t)} - T_{\theta(t)}N_{\theta(t)})e_x(t) + S_{\theta(t)}e_{\mathbb{f}}(t) + L_{\theta(t)}R_{\theta(t)}e_\xi(t) \\ &= (J_{\theta(t)} - T_{\theta(t)}N_{\theta(t)} + S_{\theta(t)}W)e_x(t) + S_{\theta(t)}e_r(t) + L_{\theta(t)}R_{\theta(t)}e_\xi(t), \end{aligned} \tag{6}$$

$$\begin{aligned}
\dot{e}_\xi(t) &= \dot{\xi}(t) - \dot{\hat{\xi}}(t) \\
&= P_{\theta(t)}\xi(t) + Q_{\theta(t)}\Psi_3(t) - [\dot{\eta}(t) - \Lambda\dot{x}(t)] \\
&= P_{\theta(t)}\xi(t) + Q_{\theta(t)}\Psi_3(t) - \{(P_{\theta(t)} + \Lambda L_{\theta(t)}R_{\theta(t)})[\eta(t) - \Lambda x(t)] + \Lambda[J_{\theta(t)}x(t) + L_{\theta(t)}u(t) + S_{\theta(t)}\mathbb{f}(t)] \\
&= (P_{\theta(t)} + \Lambda L_{\theta(t)}R_{\theta(t)})e_\xi(t) + Q_{\theta(t)}\Psi_3(t) + \Lambda M_{\theta(t)}\Psi_2(t), \\
\dot{e}_r(t) &= \dot{r}(t) - \dot{\hat{r}}(t) \\
&= \dot{\mathbb{f}}(t) - W\{J_{\theta(t)}x(t) + L_{\theta(t)}[u(t) + \Psi_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\mathbb{f}(t)\} + WS_{\theta(t)}\hat{r}(t) \\
&\quad + W\{J_{\theta(t)}\hat{x}(t) + L_{\theta(t)}[u(t) + \hat{\Psi}_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}W\hat{x}(t)\} \\
&= \dot{\mathbb{f}}(t) - (WJ_{\theta(t)} + WS_{\theta(t)}W)e_x(t) - WL_{\theta(t)}R_{\theta(t)}e_\xi(t) - WS_{\theta(t)}e_r(t).
\end{aligned}$$

Next, according to Assumption 3 and the estimators (3), (5), we can design the following controller

$$u(t) = -\hat{\Psi}_1(t) + Y_{\theta(t)}\hat{x}(t) - L^*_{\theta(t)}S_{\theta(t)}\hat{\mathbb{f}}(t), \tag{7}$$

where $Y_i, i \in \Xi$ are the controller gain need to be selected.

By substituting the control input (7) into (5), the dynamics of the whole closed-loop system can be obtained within

$$\begin{aligned}
\dot{\hat{x}}(t) &= J_{\theta(t)}\hat{x}(t) + L_{\theta(t)}[-\hat{\Psi}_1(t) + Y_{\theta(t)}\hat{x}(t) - L^*_{\theta(t)}S_{\theta(t)}\hat{\mathbb{f}}(t) \\
&\quad + \hat{\Psi}_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\hat{\mathbb{f}}(t) + T_{\theta(t)}[y(t) - \hat{y}(t)] \\
&= (J_{\theta(t)} + L_{\theta(t)}Y_{\theta(t)})\hat{x}(t) + M_{\theta(t)}\Psi_2(t) + T_{\theta(t)}N_{\theta(t)}e_x(t).
\end{aligned} \tag{8}$$

Combining (2) and (8), we attain the augmented system

$$\begin{aligned}
\dot{\alpha}(t) &= \Sigma_{1\theta(t)}\alpha(t) + \Sigma_{2\theta(t)}\Psi(t) + \Sigma_{3\theta(t)}\dot{\mathbb{f}}(t), \\
y(t) &= \Gamma_{1\theta}\alpha(t) + \Gamma_{2\theta}\Psi(t),
\end{aligned} \tag{9}$$

where

$$\alpha(t) = \begin{bmatrix} \hat{x}^T(t) \; e_x{}^T(t) \; e_r{}^T(t) \; e_\xi{}^T(t) \end{bmatrix}^T \Sigma_{2i} = \begin{bmatrix} M_i & 0 \\ 0 & 0 \\ 0 & 0 \\ \Lambda M_i & Q_i \end{bmatrix}, \Sigma_{3i} = \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \end{bmatrix},$$

$$\Sigma_{1i} = \begin{bmatrix} J_i + L_iY_i & T_iN_i & 0 & 0 \\ 0 & J_i - T_iN_i + S_iW & S_i & L_iR_i \\ 0 & -WJ_i - WS_iW & -WS_i & -WL_iR_i \\ 0 & 0 & 0 & P_i + \Lambda L_iR_i \end{bmatrix}, \Gamma_{1i} = \begin{bmatrix} N_i \; N_i \; 0 \; 0 \end{bmatrix}, \Gamma_{2\theta} = \begin{bmatrix} O_i \; 0 \end{bmatrix}$$

Based on the augmented system (9), the control aim is reformulated as

i) If the disturbance $\Psi(t)$ and $\mathbb{f}(t)$ are absent, then the system (9) is asymptotical stable.

ii) If the disturbance $\Psi(t)$ is absent, the system (9) is practical stable.

iii) If the disturbance $\Psi(t)$ and the fault $\mathbb{f}(t)$ all exist, then the following $L_2$ gain property is ensured:

$$\int_0^\infty e^{-\delta_0 t}y^T(t)y(t)dt \le \beta \int_0^\infty \Psi^T(t)\Psi(t)dt \tag{10}$$

where $\beta$ is the $L_2$-gain level, $\delta_0$ and $\beta$ are given positive constant.

If the above control aim is reached, then the fault estimation and tolerant ADSC issue of the system (1) is solvable. The structure of the fault estimation and tolerant ADSC scheme is depicted by Fig. 1.

## 3 | MAIN RESULT

In this part, the fault estimation and tolerant ADSC scheme is designed for the system (1) to force the fault tolerant and multiple disturbances alleviation.

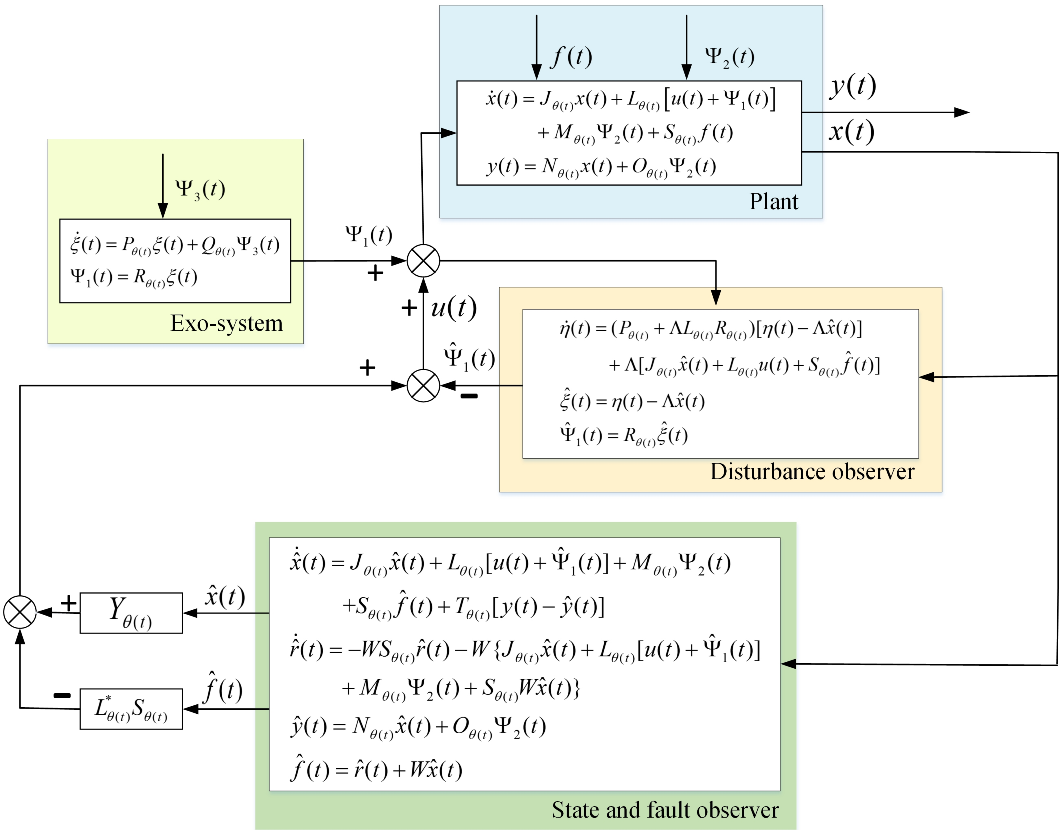


**Figure 1** The structure of the fault estimation and tolerant anti-disturbance switching control scheme.

**Theorem 1.** Consider the system (9) with the given scalars $\delta_0 > 0$, $\beta > 0$, $\mu \geq 1$. Assume that the positive definite matrices $H_i$, matrices $Y_i$, $T_i$, $\Lambda$ can be founded to ensure

$$\begin{bmatrix} \Sigma_{1i}^T H_i + H_i\Sigma_{1i} + \Gamma_{1i}^T\Gamma_{1i} + cH_i & H_i\Sigma_{2i} + \Gamma_{1i}^T\Gamma_{2i} \\ * & \Gamma_{2i}^T\Gamma_{2i} - \beta I \end{bmatrix} < 0, \tag{11}$$

$$H_i < \mu H_j, i, j \in \Xi, \tag{12}$$

then under the estimators (3), (5) and the dwell-time relevant SL $\theta(t)$ ensuring

$$s_d > s_d^* = \frac{\ln\mu}{\delta}, \delta \in \left(0, \delta_0\right), \tag{13}$$

the fault estimation and tolerant ADSC issue can be solved by the controller (7).

For $\theta(t) = i$, take

$$V_i(t) = \alpha^T(t)H_i\alpha(t), \tag{14}$$

as the Lyapunov function of the $i$th subsystem for the system (9).

Thus, it generates that

$$\begin{aligned} \dot{V}_i(t) &= \dot{\alpha}^T(t)H_i\alpha(t) + \alpha^T(t)H_i\dot{\alpha}(t) \\ &= 2\alpha^T(t)H_i\Sigma_{1i}\alpha(t) + 2\alpha^T(t)H_i\Sigma_{2i}\Psi(t). \end{aligned} \tag{15}$$

Futher, it yields that

$$
\begin{aligned}
&\dot{V}_i(t) + y^T(t)y(t) - \beta\Psi^T(t)\Psi(t) + cV_i(t) \\
&= \begin{bmatrix} \alpha(t) \\ \Psi(t) \end{bmatrix}^T \begin{bmatrix} \iota_{11} & H_i\Sigma_{2i} + \Gamma_{1i}^T\Gamma_{2i} \\ * & \Gamma_{2i}^T\Gamma_{2i} - \beta I \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \Psi(t) \end{bmatrix} + cV_i(t), \\
&= \begin{bmatrix} \alpha(t) \\ \Psi(t) \end{bmatrix}^T \begin{bmatrix} \iota_{11} + cH_i & H_i\Sigma_{2i} + \Gamma_{1i}^T\Gamma_{2i} \\ * & \Gamma_{2i}^T\Gamma_{2i} - \beta I \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \Psi(t) \end{bmatrix}
\end{aligned} \tag{16}
$$

where

$$\iota_{11} = \Sigma_{1i}^T H_i + H_i\Sigma_{1i} + \Gamma_{1i}^T\Gamma_{1i}.$$

From (11), we have

$$\dot{V}_i(t) + y^T(t)y(t) - \beta^2\Psi^T(t)\Psi(t) + cV_i(t) \leq 0. \tag{17}$$

When $\Psi(t) = 0$ from (11), it is not hard to render that

$$\dot{V}_i(\alpha(t)) \leq -\delta_0 V_i(\alpha(t)). \tag{18}$$

According to (12), we deduce

$$V_i(\alpha(t)) \leq \mu V_j(\alpha(t)),\ \forall i, j \in \Xi. \tag{19}$$

Combing (18) and (19), we are clear that

$$\dot{V}_i(\alpha(t)) \leq \mu^{\rho_\theta(0,t)} e^{-\delta_0 t} V(0) = e^{\rho_\theta(0,t)\ln\mu - \delta_0 t} V(0), \tag{20}$$

where $V(t_k)$ represents the value of the Lyapunov function at the switching point $t_k$, $k = 0, 1, \ldots, \rho_\theta(0,t)$. Further, due to $\rho_\theta(0,t) \leq s/s_d^*$ holds for any $s > 0$. It follows from (13) that

$$\rho_\theta(0,s)\ln\mu \leq \delta s. \tag{21}$$

Thus, we have

$$V_i(\varepsilon(t)) \leq e^{-(\delta_0-\delta)t} V(0). \tag{22}$$

This renders that the augmented system (9) is globally asymptotically stable when $\Psi(t) = 0$.

When $\Psi(t) \neq 0$, from (17) and (18), we deduce that

$$
\begin{aligned}
&V_i(\alpha(t)) \\
&\leq V_i(\alpha(t_{\rho_\theta(0,t)}))e^{-\delta_0(t - t_{\rho_\theta}(0,t))} + \int_{t_{\rho_\theta(0,t)}}^{t} e^{-\delta_0(t-s)}\Upsilon(s)ds \\
&\leq \mu V_i(\alpha(t^-_{\rho_\theta(0,t)}))e^{-\delta_0(t - t_{\rho_\theta(0,t)})} + \int_{t_{\rho_\theta(0,t)}}^{t} e^{-\delta_0(t-s)}\Upsilon(s)ds \\
&\leq \mu[V_i(\alpha(t_{\rho_\theta(0,t)-1}))e^{-\delta_0(t_{\rho_\theta(0,t)} - t_{\rho_\theta(0,t)-1})} + \int_{t_{\rho_\theta(0,t)-1}}^{t_{\rho_\theta(0,t)}} e^{-\delta_0(t_{\rho_\theta(0,t)}-s)}\Upsilon(s)ds]e^{-\delta_0(t - t_{\rho_\theta(0,t)})} + \int_{t_{\rho_\theta(0,t)}}^{t} e^{-\delta_{0(t-s)}}\Upsilon(s)ds \\
&\leq \cdots\cdots \\
&\leq \mu^{\rho_\theta(0,t)} e^{-\delta_0(t-t_0)} V_i(\alpha(t_0)) + \mu^{\rho_\theta(0,t)} \int_{t_0}^{t_1} e^{-\delta_{0(t-s)}}\Upsilon(s)ds + \mu^{\rho_\theta(0,t)-1} \int_{t_1}^{t_2} e^{-\delta_{0(t-s)}}\Upsilon(s)ds + \cdots + \mu^0 \int_{t_{\rho_\theta(0,t)}}^{t} e^{-\delta_{0(t-s)}}\Upsilon(s)ds \\
&= \mu^{\rho_\theta(t_0,t)} e^{-\delta_0(t-t_0)} V_i(\alpha(t_0)) + \int_{t_0}^{t} \mu^{\rho_\theta(s,t)} e^{-\delta_0(t-s)}\Upsilon(s)ds \\
&= e^{-\delta_0(t-t_0)+\rho_\theta(t_0,t)\ln\mu} V_i(\alpha(t_0)) + \int_{t_0}^{t} e^{-\delta_0(t-s)+s(s,t)\ln\mu}\Upsilon(s)ds,
\end{aligned} \tag{23}
$$

where

$$\Upsilon(t) = -y^T(t)y(t) + \beta^2\Psi^T(t)\Psi(t).$$

According to (23), letting $t_0 = 0$, under zero initial condition, we derive

$$\int_0^t e^{-\delta_0(t-s)+s(s,t)\ln\mu}\Upsilon(s)ds \leq 0. \tag{24}$$

Then, multiplying both sides of (24) by $e^{-\rho_\theta(0,t)\ln\mu}$, we can get

$$\int_0^t e^{-\delta_0(t-s)-\rho_\theta(0,s)\ln\mu}y^T(s)y(s)ds \leq \int_0^t e^{-\delta_0(t-s)-\rho_\theta(0,s)\ln\mu}\gamma^2\Psi^T(s)\Psi(s)ds. \tag{25}$$

From (21), we derive

$$\int_0^t e^{-\delta_0(t-s)-\delta_0 s}y^T(s)y(s)ds \leq \int_0^t e^{-\delta_0(t-s)}\gamma^2\Psi^T(s)\Psi(s)ds. \tag{26}$$

Integrating from $t : 0 \to \infty$ on two sides of the above inequality, we can deduce

$$\int_0^\infty e^{-\delta_0 s}y^T(s)y(s)ds \leq \gamma^2\int_0^\infty \Psi^T(s)\Psi(s)ds.$$

Based on Definition 2, we know that the $L_2$-gain property (10) holds.

*Remark 1.* Theorem 1 forms a criterion via which the fault estimation and tolerant ADSC issue of the system (1) is addressed in the case that the maintain condition is not required. Different from the fault estimation issue studied in[28], here, the multiple disturbances are considered which is more coincide with practice.

Next, we offer the design process of the controller (7) to solve the fault estimation and tolerant ADSC issue for the system (1).

**Theorem 2.** Consider the system (9). Suppose that for the given scalar $\mu > 1$, if there exist the positive define matrixes $Z_{1i}$, $Z_{3i}$ and matrix $\Lambda$ such that the relations

$$\begin{bmatrix} \sigma_{11i} & \partial_{12i} & 0 & 0 & \partial_{15i} & 0 & Z_{1i}N_i^T \\ * & \partial_{22i} & \partial_{23i} & L_iR_i & N_i^TO & 0 & 0 \\ * & * & \partial_{33i} & \partial_{34i} & 0 & 0 & 0 \\ * & * & * & \partial_{44i} & \Lambda M_i & Q_i & 0 \\ * & * & * & * & O_i^TO_i - \beta I & 0 & 0 \\ * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{27}$$

$$Z_{1i} < \mu Z_{1j}, i, j \in \Xi, \tag{28}$$

$$Z_{3i} < \mu Z_{3j}, i, j \in \Xi \tag{29}$$

hold,where

$$\begin{aligned}
\sigma_{11i} &= \Omega_i L_i^T + Z_{1i}J_i^T + J_iZ_{1i} + L_i\Omega_i^T + cZ_{1i},\\
\partial_{12i} &= T_iN_i + Z_{1i}N_i^TN_i,\\
\partial_{15i} &= Z_{1i}N_i^TO_i + M_i,\\
\partial_{22i} &= J_i^T - N_i^TT_i^T + \upsilon S_iS_i^T + J_i - T_iN_i + \upsilon S_iS_i^T + N_i^TN_i + cI,\\
\partial_{23i} &= -\upsilon J_i^TS_i - \upsilon^2 S_iS_i^TS_i + S_iZ_{3i},\\
\partial_{33i} &= -\upsilon Z_{3i}S_i^TS_i - \upsilon S_i^TS_iZ_{3i} + cZ_{3i},\\
\partial_{34i} &= -\upsilon S_i^TL_iR_i,\\
\partial_{44i} &= P_i^T + P_i + R_i^TL_i^T\Lambda^T + \Lambda L_iR_i + cI,
\end{aligned}$$

then, the controller (7) is a solution to the fault tolerant ADSC issue for the system (9). Furthermore, the controller gain can be solved by $Y_i = \Omega_i^T Z_{1i}^{-1}$, $i \in \Xi$.

According to Theorem 1, if the constraints (11) and (12) are ensured, then the fault tolerant ADSC scheme for the system (9) is solved by the controller (7).

We firstly show that the condition (27) holds if and only if (11) holds. Let $H_i^{-1} = diag\{Z_{1i}, I, Z_{3i}, I\}$ and multiply $diag\{H_i^{-1}, I\}$ on both sides of (11), we can generate

$$\begin{bmatrix} \wp_{11} & \Sigma_{2i} + H_i^{-1}\Gamma_{1i}^T\Gamma_{2i} \\ \Sigma_{2i}^T + \Gamma_{2i}^T\Gamma_{1i}H_i^{-1} & \Gamma_{2i}^T\Gamma_{2i} - \beta I \end{bmatrix} < 0, \tag{30}$$

where

$$\wp_{11} = H_i^{-1}\Sigma_{1i}^T + \Sigma_{1i}H_i^{-1} + H_i^{-1}\Gamma_{1i}^T\Gamma_{1i}H_i^{-1} + \varepsilon\Sigma_{3i}\Sigma_{3i}^T + CH_i^{-1}.$$

Replacing $\Sigma_{1i}, \Sigma_{2i}, \Gamma_{1i}, \Gamma_{2i}$ in (30) by their specified expressions generates

$$\begin{bmatrix} \partial_{11i} & \partial_{12i} & 0 & 0 & \partial_{15i} & 0 \\ * & \partial_{22i} & \partial_{23i} & L_iR_i & N_i^TO_i & 0 \\ * & * & \partial_{33i} & \partial_{34i} & 0 & 0 \\ * & * & * & \partial_{44i} & \Lambda M_i & Q_i \\ * & * & * & * & O_i^TO_i - \beta I & 0 \\ * & * & * & * & * & -\beta I \end{bmatrix} < 0, \tag{31}$$

where

$$\partial_{11i} = Z_{1i}Y_i^TL_i^T + Z_{1i}J_i^T + J_iZ_{1i} + L_iY_iZ_{1i} + Z_{1i}N_i^TN_iZ_{1i} + cZ_{1i},$$

Letting $Z_{1i}Y_i^T = \Omega_i$ and applying Schur complement lemma to (31), we get (27). This implies that the constraint (27) can ensure (11). On the other hand, multiplying $diag\{H_i^{-1}, I\}$ on two sides of (12) generates (4) and (29). Thus, it can be concluded that the controller (7) can cope with the fault tolerant ADSC problem of the system (9).

*Remark 2.* Via Theorem 2, the linear matrix inequality restraints are formed to serving for the computation of the fault estimation and tolerant anti-disturbance switching controller (7).

## 4 | VERIFICATION

This section illustrates the effectiveness of proposed fault estimation and tolerant ADSC method. Take the switched RLC circuit of[39] for example. The switched RLC circuit model is described by Fig. 2 and formulated by

$$\dot{x}(t) = J_{\theta(t)}x(t) + L_{\theta(t)}[u(t) + \Psi_1(t)] + M_{\theta(t)}\Psi_2(t) + S_{\theta(t)}\mathfrak{f}(t), \tag{32}$$

where $x^T(t) = [q^T(t) \ \ \phi_L^T(t)]$ with $q(t)$ denoting the charge across the capacitor $C_i, i = 1, 2$ and $\phi_L(t)$ denoting the flux across the inductance $L$,

$$J_i = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C_i} & -\frac{R}{L} \end{bmatrix},$$

$C_i$, $R$, and $L$ are the capacitance, resistance and inductance, respectively, $\Psi_i(t), i = 1, 2$ are the disturbance input, $\Psi_1(t)$ is generated by the model (2), $\Psi_2(t) = \sin(t \times \exp(-2.3t))$, $f(t) = 4 \times \sin(t \times \exp(-2.3t))$ is the fault input.

Set $R = 5, L = 2, C_1 = 3, C_2 = 4$, the parameters of the model (32) are listed as follows:

$$L_1 = \begin{bmatrix} -0.8 \\ -0.6 \end{bmatrix}, L_2 = \begin{bmatrix} -0.6 \\ -0.8 \end{bmatrix}, M_1 = \begin{bmatrix} 0.5 & -0.5 \\ -0.3 & 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.2 & -0.6 \\ 0.1 & -0.2 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0 & 0.9 \\ 0.3 & 0.1 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & 1.8 \\ 0.6 & 0.2 \end{bmatrix}, N_1 = \begin{bmatrix} -0.1 & 0 \end{bmatrix}, N_2 = \begin{bmatrix} -0.8 & 0 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 0.1 & -0.5 \end{bmatrix}, O_2 = \begin{bmatrix} 0 & 0.6 \end{bmatrix}, P_1 = \begin{bmatrix} -0.2 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}, P_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}, Q_2 = \begin{bmatrix} -0.2 \\ -0.6 \end{bmatrix}, R_1 = \begin{bmatrix} -0.1 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} -0.6 & 6.6 \end{bmatrix}.$$

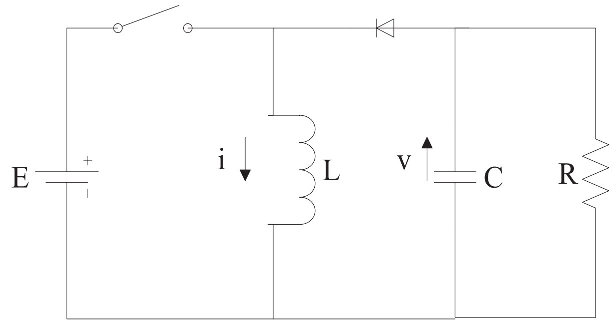


**Figure 2** Switched circuit.

The task is to regulate the charge and flux of the switched RLC circuit model (32) with the disturbance $\Psi_1(t)$ and fault $f(t)$ estimated.

Let $\mu = 1.001$. By solving the constraints of Theorem 2, we derive

$$\upsilon_1 = 4, \upsilon_2 = 2,$$

$$\Omega_1 = 10^7 \times \begin{bmatrix} 2.9472 \\ -6.6961 \end{bmatrix}, \Omega_2 = 10^8 \times \begin{bmatrix} 1.2032 \\ -2.3383 \end{bmatrix},$$

$$Z_{11} = \begin{bmatrix} 7.3178 \times 10^3 & 739.2282 \\ 739.2282 & 7.7021 \times 10^7 \end{bmatrix}, Z_{12} = \begin{bmatrix} 7.3133 \times 10^3 & 738.8242 \\ 738.8242 & 7.6977 \times 10^7 \end{bmatrix},$$

$$Z_{31} = \begin{bmatrix} 6.7583 & -11.4838 \\ -11.4838 & 480.5620 \end{bmatrix}, Z_{32} = \begin{bmatrix} 6.7583 & -11.4838 \\ -11.4838 & 480.5624 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 0.1557 & -3.5088 \\ -4.5851 & 44.1965 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 4.0275 \times 10^3 & -0.9080 \end{bmatrix}, Y_2 = \begin{bmatrix} 4.0300 \times 10^3 & -0.9086 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} -1.4019 \times 10^8 \\ -26.4413 \end{bmatrix}, T_2 = \begin{bmatrix} -1.6429 \times 10^7 \\ 129.1762 \end{bmatrix}.$$

The simulation results are displayed by Figs. 3-7. Fig. 3 shows the SL $\theta(t)$. The state estimation of the switched RLC circuit model (32) is given in Fig. 4, which indicates that the unmeasurable state has been estimated by observed. The estimation errors of the state $x(t)$, disturbance $\Psi_1(t)$ and fault $f(t)$ are depicted by Figs. 5-7. From Figs. 5-7, we can see that all the estimation errors (4) are bounded. Therefore, we can claim that the fault estimation and tolerant ADSC scheme is effective in estimating the charge and flux of the switched RLC circuit model (32).

## 5 | CONCLUSIONS

In this investigation, a fault estimation and tolerant ADSC strategy has been developed for the switched systems subject to unmeasurable system state, disturbance and fault. First, the unmeasurable state, fault and disturbance have been estimated by co-design of a switching observer. In the observer,by virtue of an auxiliary variable, the traditional observer matching relation

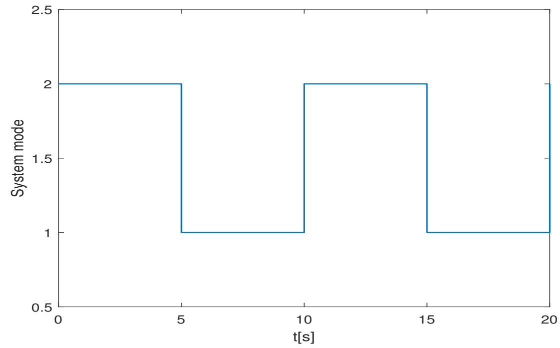


**Figure 3** Switching signal.

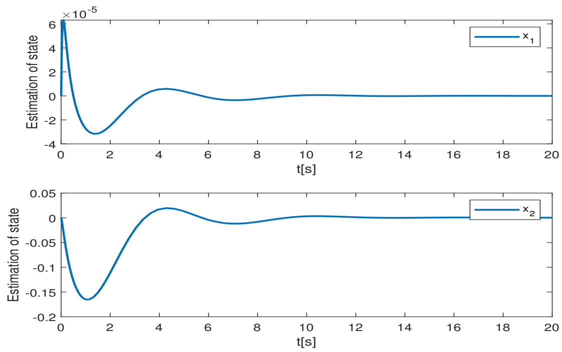


**Figure 4** Estimation of state.

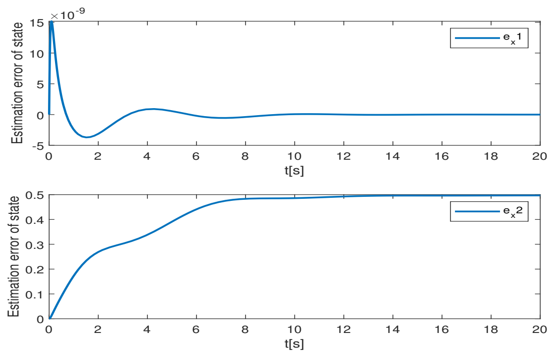


**Figure 5** Estimation error of state.

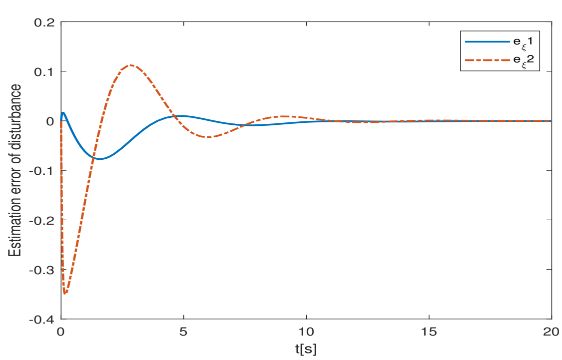


**Figure 6** Estimation error of disturbance.

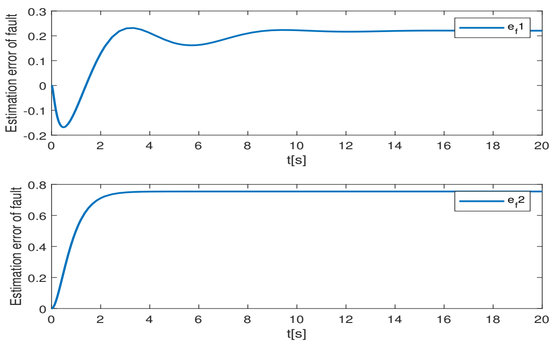


**Figure 7** Estimation error of fault.

in the fault estimation issue of switched systems has been relaxed. Third, under the developed estimator and controller, the condition has been formed to capture the solvability of the fault estimation and tolerant ADSC issue for the switched systems. In the end, by regulating the charge and flux of a switched circuit system, the effectiveness of the established fault estimation and tolerant ADSC method has been explained.

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## CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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