A direct analysis method to global h-stability of positive Cohen-Grossberg neural networks with time-varying delays

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Abstract

In this paper, global h-stability of nonlinear positive Cohen-Grossberg neural network (PCGNN) system with time-varying delays is studied by means of a direct analysis method. By selecting the appropriate h function and determining its differential expression, global h-stability is converted into two types of known stability, that is Lagrangian exponential stability and global exponential stability. For the sake of improving the accuracy of the stability results, we spare no effort to optimize the fitting effect of the system state trajectory by changing the differential expression of the h function. In addition, two examples are given to verify the feasibility and effectiveness of this method in PCGNN.

Keywords: positive systems, positive Cohen-Grossberg neural networks, direct analysis method, global h-stability, time-varying delays.

1. Introduction

The Cohen-Grossberg neural network (CGNN) is an artificial neural network model used to simulate neuron interaction in biology. It was proposed by Grossberg and Cohen in 1982 and is mainly used for processing control, optimization, clustering and nonlinear issues. The CGNN model has strong scalability and fault tolerance, thus it has been widely applied in pattern recognition, signal processing, control system, etc[1–5]. Lu and Chen [6] studied the dynamic behavior of delayed CGNN and obtained the unique sufficient condition for non-negative balance based on the nonlinear complementary theory. They proved that the non-negative balance is globally asymptotically stable. This class of CGNN models with positive dynamic systems is called positive Cohen-Grossberg neural networks (PCGNN). In the mathematical model, the initial state of a positive dynamic system is non-negative and any trajectory generated from the non-negative initial state (such as initial position, speed, etc.) must also be non-negative [7–11]. In other words, there will be no negative or non-physically reasonable conditions in this system. Non-negativity is essential to many practical problems such as mass, energy and probability. In the ecological model, the positive dynamic system can ensure that the number of various species is always non-negative, so as to avoid the disruption of ecological balance. Hence, many scholars are attracted to studying positive dynamic systems. Zhu, Sun and Liu studied the boundary problem of homogeneous positive systems and derived the sufficient condition for the existence of a sphere in [12]. Using the new comparison technology, Hien derived the feasible conditions where all state trajectories of the system converge to the unique positive equilibrium at an exponential rate in [13]. Rami [14] solved the stability problem of linear positive systems using a direct approach and proved that positive linear systems are less susceptible to time-varying delays than ordinary linear systems, which was also explained and elaborated in [11–16]. Compared with the traditional perceptron models, PCGNN uses a nonlinear activation function with a hidden layer to handle more complex classification problems. It also introduces an anti-program algorithm to optimize weights and better process data. These advantages make it possible to solve image segmentation, cluster analysis, sound recognition, and robot trajectory planning. Therefore, it is of great significance to add a positive dynamic system to the CGNN model, and it has attracted the attention of many scholars in recent years.

It is well known that time delays exist in many CGNN models, which may cause system oscillations or divergence, thus affecting the performance and response speed of the network. Therefore, the study of the dynamic behavior of CGNNs with time-varying delays is the primary problem that researchers need to solve. Some previous work has proposed various solutions [17–20]. Liu, He, and Wu [21] established a vector non-autonomous Halanay inequality using the Ito's formula and derived sufficient conditions to ensure the stability of the system in the face of non-autonomous delayed CGNN stability problems. Xiao, Zeng and Wu [22] designed two different sliding mode controllers to solve the problem of the known delayed CGNN fixed-time synchronization. In order to solve the stability problem of mixed delay CGNN, Wang and Qi [23] used Halanay delay differential inequality and Jensen inequality to handle the integral items in the Lyaponov-Krassovsky function (LKF) to avoid the impact of delay on the system and obtained new sufficient conditions for ensuring the global Lagrangian exponential stability of CGNN. However, in a nonlinear PCGNN system with multiple time-varying delays, multiple state variables at different times will interact with each other. And this influence is complex and nonlinear, making it impossible to directly describe the relationship between them in vector matrix form. This poses a challenge for modeling and controlling of neural networks with time-varying delays. Based on this, Pinto first introduced the concept of h-stability and obtained the stability result of weakly stable systems under certain perturbations [24]. The h-system, also known as h-stability, is a special type of differential equation system, which is mainly used to study the weak stability of the system [25–28]. h-stability takes into account the sensitivity of system response to time step length and space step length, and allows stability constraints to be relaxed. Specifically, we first need to define a suitable h function and set its differential expression, which depends on both time and space steps. Next we incorporate it into the differential equation, and require that the truncation error of the differential equation's part related to the h function does not increase too rapidly. The h function indicates the growth rate or decay rate of the system solution, and also indicates the stability of the system; the smaller the value of the h function, the slower the growth rate of the solution, the more stable the system. In this way, the h-stability condition is more relaxed than the traditional exponential stability. Using the concept of t_∞ similarity, Coo and Yang studied the h-stability of nonlinear perturbation differential systems and expanded the h-stability results [29], solving the stability problem of high-order inertial neural networks with proportional delays. Wang, Wang and Wang proposed a new method for constructing LKF and obtained the global h-stability criterion. This method does not need to solve new approximations, but more accurately describes the stability characteristics of the system based on dependency and correlation [30]. Therefore, in practical applications, through the estimation and analysis of the h function, accurate results and conclusions about system stability can be further derived.

Inspired by the above, this paper adopts direct analysis method, carries on the Laplace transform to the differential equation, takes the derivative of the transform form, and obtains the analytic solution of the differential equation with time-varying delays and nonlinear terms. The obtained analytical solution is reverted back to the time domain, and the sufficient conditions for global h-stability of the system are derived. The global h-stability of nonlinear positive CGNN systems with time-varying delays is studied. And solved the problem of not being able to use the vector matrix to represent the relationship between states. Furthermore, due to the strong superiority in the auxiliary stability analysis of the h function, the effect of controller replacement can be achieved by changing the differential expression of the h function, and the global h-stability can be transformed into special conditions such as Lagrangian exponential stability and global exponential stability, highlighting the generality and flexibility of global h-stability.

Notations: N^* is a set of positive integers, a set of real numbers denoted by $\Re$, $\Re_+$ is a collection of positive real numbers, $\Re_+^m$ represents the set of positive vectors, the superscript T represents the transpose of the vector, and $\|\ \|$ represents the two norm.

2. Prepare Knowledge and Assumptions

The CGNN with time-varying delays can be described:

$$\begin{aligned} \dot{r}_\imath(t) =& c_\imath(r_\imath(t))\Big[-d\imath(r_\imath(t)) + \sum_{s=1}^{m} w_{\imath s} z_{\imath s}(r_\imath(t)) \\ &+ \sum_{s=1}^{m} l_{\imath s} Z_{\imath s}(r_\imath(t-\varrho_{\imath s}(t))) + \mu_\imath\Big] \\ r_\imath(t) =& \kappa_\imath(t), \imath = 1, 2, ..., m. \quad t \in [t_0 - \varrho, t_0], \end{aligned} \tag{1}$$

where $r_\imath(t)$ is the $\imath$th state variable, the sth neuron is activated by the $z_{\imath s}(r_\imath(t))$ function at the t moment, $z_{\imath s}$: $[t_o, +\infty) \times \Re \to \Re$, and $Z_{\imath s} : [t_o, +\infty) \times \Re \to \Re_+$ is the activation functions, and $z_{\imath s}$ and $Z_{\imath s}$ only change with the change of s. $Z_{\imath s}(r_\imath(t-\varrho_{\imath s}(t)))$ indicates that the sth neuron is activated at $t-\varrho_{\imath s}(t)$ time. And $\varrho_{\imath s}(t)$ is the delay generated during signal transmission, and assumes that $\varrho_{\imath s}(t)$ is bounded. $0<\varrho_{\imath s}(t)<\acute{\varrho}_{\imath s}$, $t_0\le t$, $\acute{\varrho}_{\imath s}$ is a real number not less than zero. $\kappa_\imath(t)$ is the initial function of $[t_0-\varrho,t_0]$ mapping to $\Re_+$, and $\acute{\varrho}=\underset{\imath,s=1,2,\ldots,m}{max}\acute{\varrho}_{\imath s}(t)$. $c_\imath(r_\imath(t))$ is a kind of amplification function. $d\imath(r_\imath(t))$ is a kind of function with appropriate behavior and bounded. $W=(w_{\imath s})_{m\times m}$ shows us how the $\imath$th neuron and the sth neuron are connected and the strength of the connection at time t. $L=(l_{\imath s})_{m\times m}$ indicates the intensity of the interconnection between neurons $\imath$ and neurons s at time $t-\varrho_{\imath s}(t)$. $\mu_\imath$ is the external constant input.

Definition 1: For any $t\ge t_0$, $\imath\in[1,m]$, when the initial value of the state variable $\kappa_\imath:[t_0-\varrho,t_0]$ is more than zero and $r_\imath(t)\ge 0$, then the system (1) is positive.

Definition 2: If the system (1) global h-stability, there is a continuous bounded positive function $h:[t_0,+\infty)\to\Re$ and a constant value $\mathfrak{k}\ge 1$, so that each solution $r(t)$ of the system (1) (including the initial function $\kappa(t)$) is satisfied with:

$$\|r(t)\|\le\mathfrak{k}\,\|\kappa\|_\varrho\,h(t)h^{-1}(t_0),\quad t\ge t_0$$

where $r(t)=[r_1(t),r_2(t),...,r_m(t)]^T$, $\kappa(t)=[\kappa_1(t),\kappa_2(t),...,\kappa_m(t)]^T$, and $\|\kappa\|_\varrho=\sup_{t\in[t_0-\varrho,t_0]}\|\kappa(t)\|$.

The assumptions needed in this article are:

Assumption Q_1: The magnification function $c_\imath(r_\imath(t))$ is bounded and positive. There are two real scalars $\underline{\epsilon_\imath}$ and $\overline{\epsilon_\imath}$ that are greater than zero, making $\underline{\epsilon_\imath}\le c_\imath(r_\imath(t))\le\overline{\epsilon_\imath}<+\infty$, $\imath\in[1,m]$.

Assumption Q_2: For any $\imath\in[1,m]$, there is $\varphi_\imath>0$, so that

$$r_\imath(t)d_\imath(r_\imath(t))\ge\varphi_\imath r_\imath^2(t)$$

Assumption Q_3: Two functions $\partial_{\imath s}(t)$ and $\delta_{\imath s}(t)$ follow:

$$\begin{cases}\partial_{\imath s}(t)>0, & \delta_{\imath s}(t)>0, \quad s\ne\imath\\ \partial_{\imath s}(t)<0, & \delta_{\imath s}(t)<0, \quad s=\imath\end{cases}$$

and $\partial_{\imath s}(t)y\ge z_{\imath s}(t,y)\ge\delta_{\imath s}(t)y$ for $y\in\Re_+$, $\imath,s\in[1,m]$, $t\ge t_0$.

Assumption Q_4: For any $y\in\Re_+$, $t\ge t_0$, there are two positive functions $\acute{\hbar}_{\imath s}(t)$ and $\grave{\hbar}_{\imath s}(t)$ such that $\acute{\hbar}_{\imath s}(t)y\ge Z_{\imath s}(t,y)\ge\grave{\hbar}_{\imath s}(t)y$, $\imath,s\in[1,m]$.

Assumption Q_5: All elements in $W=(w_{\imath s})_{m\times m}$ and $L=(l_{\imath s})_{m\times m}$ are non-negative real numbers.

3. Main Results

The activation function in CGNN is bounded, which ensures the existence and uniqueness of the CGNN equilibrium point. Through translation transformation, the equilibrium point can be panned to the origin, and $r_\imath(0)=0$.

$$\begin{aligned}\dot r_\imath(t)=&c_\imath(r_\imath(t))\Big[-d\imath(r_\imath(t))+\sum_{s=1}^{m}w_{\imath s}z_{\imath s}(r_\imath(t))\\&+\sum_{s=1}^{m}l_{\imath s}Z_{\imath s}(r_\imath(t-\varrho_{\imath s}(t)))\Big],\end{aligned}\tag{2}$$

3. 1 Positivity of system (2)

In the following, we propose a proposition about the positive nature of the system (2).

Proposition 1: For any $t\in\Re$, $t\ge 0$, $\imath,s\in[1,m]\subset N^+$, $\imath\ne s$, if the system (2) is positive, it will satify $z_{\imath\imath}(t,0)\ge 0$, $z_{\imath s}(t,y)\ge 0$ and $Z_{\imath s'}(t,y)\ge 0$, $s'\in[1,m]$.

Proof : Take any period of time interval $[t_0-\varrho,t_0]$, at this time the initial function $\kappa:[t_0-\varrho,t_0]\to\Re_+^m$, let

$$\aleph=\{t:r_s(t)<0\ \ for\ some\ s\in[1,m]\ and\ t\ge t_0\}$$

from Definition 1: $\aleph$ is an empty set. Assuming that $\aleph$ is not an empty set, then make $\hat t=\inf\aleph$, at this time $\hat t\ge t_0$. For any $\imath\in[1,m]$, when it is $\hat t\ge t$, $r_\imath\ge 0$. Take $\jmath\in[1,m]$, due to the continuity of $r_\jmath(t)$, $r_\jmath(\hat t)=0$.

According to assumption Q_1 and Q_5, we have:

$$\begin{aligned}\dot r_\jmath(\hat t)=&c_\jmath(r_\jmath(\hat t))\Big[-d\jmath(r_\jmath(\hat t))+\sum_{s=1}^{m}w_{\jmath s}z_{\jmath s}(r_\jmath(\hat t))\\&+\sum_{s=1}^{m}l_{\jmath s}Z_{\jmath s}(r_\jmath(\hat t-\varrho_{\jmath s}(\hat t)))\Big]\\=&c_\jmath(r_\jmath(\hat t))\Big[-d\jmath(r_\jmath(\hat t))+w_{\jmath\jmath}z_{\jmath\jmath}(r_\jmath(\hat t))+\sum_{\jmath\ne s}^{m}w_{\jmath s}z_{\jmath s}(r_\jmath(\hat t))\\&+\sum_{s=1}^{m}l_{\jmath s}Z_{\jmath s}(r_\jmath(\hat t-\varrho_{\jmath s}(\hat t)))\Big]\\\ge&c_\jmath(r_\jmath(\hat t))\Big[-\varphi_\imath r_\jmath(\hat t)+w_{\jmath\jmath}z_{\jmath\jmath}(0)+\sum_{\jmath\ne s}^{m}w_{\jmath s}z_{\jmath s}(r_\jmath(\hat t))\\&+\sum_{s=1}^{m}l_{\jmath s}Z_{\jmath s}(r_\jmath(\hat t-\varrho_{\jmath s}(\hat t)))\Big]\\\ge&c_\jmath(r_\jmath(\hat t))\Big[-0+0+\sum_{\jmath\ne s}^{m}w_{\jmath s}\delta_{\jmath s}(\hat t)r_\jmath(\hat t)\\&+\sum_{s=1}^{m}l_{\jmath s}\grave{\hbar}_{\jmath s}(\hat t)r_\jmath(\hat t-\varrho_{\jmath s}(\hat t))\Big]\\\ge&0\end{aligned}$$

then $\dot r_\jmath(\hat t)\ge 0$, which contradicts $r_\jmath(\hat t)=0$, so $\aleph$ is an empty set. It can be seen that it is assumed that Q_1-Q_5 ensures the positivity of system (2). Therefore, the system (1) is also positive.

3. 2 An h-dependent property of system solutions

If h is a monotonically decreasing differentiable function, and it is positive, then h^{-1} is a monotonically decreasing positive and differentiable function. $h : [t_0 - \varrho, +\infty) \to \Re$, ordering $\mathfrak{Q}(t) = \frac{\mathrm{d}h^{-1}(t)}{\mathrm{d}t}$ again, then $\mathfrak{Q}(t)$ is also positive. Let's take any solution $r(t)$ of system (1):

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}(e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(t)h^{-1}(t))\\
=&e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}[\dot{r}_\iota(t) - \partial_{\iota\iota}(t)r_\iota(t)]h^{-1}(t) + e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(t)\mathfrak{Q}(t)\\
=&e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}\Big\{c_\iota(r_\iota(t))[-d_\iota(r_\iota(t)) + w_{\iota\iota}z_{\iota\iota}(r_\iota(t))\\
&+ \sum_{\iota\neq s}^{m} w_{\iota s}z_{\iota s}(r_s(t)) + \sum_{s=1}^{m} l_{\iota s}Z_{\iota s}(r_s(t - \varrho_{\iota s}(t)))]\\
&- \partial_{\iota\iota}(t)r_\iota(t)\Big\}h^{-1}(t) + e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(t)\mathfrak{Q}(t)\\
=&e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}[-c_\iota(r_\iota(t))d_\iota(r_\iota(t)) + w_{\iota\iota}c_\iota(r_\iota(t))z_{\iota\iota}(r_\iota(t))\\
&- \partial_{\iota\iota}(t)r_\iota(t)]h^{-1}(t) + e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}\sum_{\iota\neq s}^{m} w_{\iota s}z_{\iota s}(r_s(t))h^{-1}(t)\\
&+ e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{\iota s}Z_{\iota s}(r_s(t - \varrho_{\iota s}(t)))h^{-1}(t)\\
&+ e^{-\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(t)\mathfrak{Q}(t), \quad t \geq t_0.
\end{aligned}
$$

find the points on the left and right sides of the above formula together, and for any $\varepsilon \in [t_0, t]$, $\iota \in [1, m]$, we get:

$$
\begin{aligned}
r_\iota(t) =&e^{\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(t_0)h(t)h^{-1}(t_0)\\
&+ h(t)\int_{t_0}^{t} e^{\int_{\varepsilon}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}[-c_\iota(r_\iota(\varepsilon))d_\iota(r_\iota(\varepsilon))\\
&+ w_{\iota\iota}c_\iota(r_\iota(\varepsilon))z_{\iota\iota}(r_\iota(\varepsilon)) - \partial_{\iota\iota}(\varepsilon)r_\iota(\varepsilon)]h^{-1}(t)\mathrm{d}\varepsilon\\
&+ h(t)\int_{t_0}^{t} e^{\int_{\varepsilon}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}\sum_{\iota\neq s}^{m} w_{\iota s}z_{\iota s}(r_s(\varepsilon))h^{-1}(\varepsilon)\mathrm{d}\varepsilon\\
&+ h(t)\int_{t_0}^{t} e^{\int_{\varepsilon}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{\iota s}Z_{\iota s}(r_s(\varepsilon - \varrho_{\iota s}(\varepsilon)))h^{-1}(\varepsilon)\mathrm{d}\varepsilon\\
&+ h(t)\int_{t_0}^{t} e^{\int_{\varepsilon}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} r_\iota(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon, \quad t \geq t_0.
\end{aligned} \tag{3}
$$

For the above-mentioned h function, we need to use the following assumptions:

A$_1$: There is a scalar $\chi > 0$, for any $\iota \in [1, m]$, $t \geq t_0$ to meet:

$$
\int_{t_0}^{t} e^{\int_{\varepsilon}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta} h(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \leq -\frac{\chi}{\tilde{\partial}_{\iota\iota}} e^{\int_{t_0}^{t}\partial_{\iota\iota}(\theta)\mathrm{d}\theta}
$$

where $\hat{\partial}_{\iota\iota} = \sup_{\varepsilon\geq t_0}\partial_{\iota\iota}(\varepsilon)$.

A$_2$: There are scalars $\eta \geq 1$, $\rho \geq 0$ and $\iota > 0$, so that:

$$
\frac{h(t - \varepsilon)}{h(t)} \leq (\eta + \rho\varepsilon)e^{\iota\varepsilon}, \quad t \geq t_0.
$$

3. 3 Global h-stability conditions

We continue to study an essential condition to ensure the global h-stability of the system (1).

Theorem 1: On the basis of assuming $Q_1 - Q_5$, let $h : [t_0 - \varrho, +\infty) \to \Re$ is a non-monotonic incremental differentiable function, and it is positive. If there is a vector $\bar{\bar{\mathfrak{d}}} = (\bar{\bar{\mathfrak{d}}}_\iota) \in \Re_+^m$, $\iota \in [1, m]$ make:

$$
\chi\bar{\bar{\mathfrak{d}}}_\iota + \sum_{s=1}^{m} w_{\iota s}\bar{\bar{\mathfrak{d}}}_s\hat{\partial}_{\iota s} + \sum_{s=1}^{m} l_{\iota s}\bar{\bar{\mathfrak{d}}}_s\hat{\acute{\hbar}}_{\iota s}(\eta + \rho\hat{\varrho}_{\iota s})e^{\iota\hat{\varrho}_{\iota s}} < 0, \tag{4}
$$

where $\hat{\partial}_{\iota s} = \sup_{t\geq t_0}\partial_{\iota s}(t)$ and $\hat{\acute{\hbar}}_{\iota s} = \sup_{t\geq t_0}\acute{\hbar}_{\iota s}(t)$, then

(i) There is a set of real numbers $\mathfrak{k} \geq 1$, $\iota \in [1, m]$, so that when $t \geq t_0 - \varrho$, there are:

$$
r_\iota(t) \leq \mathfrak{k}_\iota \|\kappa\|_\varrho h(t)h^{-1}(t_0), \tag{5}
$$

(ii) System (1) is global h-stability.

Proof : (i) Suppose there is a vector $\bar{\bar{\mathfrak{d}}} = (\bar{\bar{\mathfrak{d}}}_\iota) \in \Re_+^m$, $\iota \in [1, m]$ then

$$
\chi\bar{\bar{\mathfrak{d}}}_\iota + \sum_{\iota\neq s}^{m} w_{\iota s}\bar{\bar{\mathfrak{d}}}_s\hat{\partial}_{\iota s} + \sum_{s=1}^{m} l_{\iota s}\bar{\bar{\mathfrak{d}}}_s\hat{\acute{\hbar}}(\eta + \rho\hat{\varrho}_{\iota s})e^{\iota\hat{\varrho}_{\iota s}} < -\bar{\bar{\mathfrak{d}}}_\iota\hat{\partial}_{\iota\iota}
$$

it can be regarded as

$$
-\frac{\chi}{\hat{\partial}_{\iota\iota}} - \sum_{\iota\neq s}^{m} w_{\iota s}\frac{\bar{\bar{\mathfrak{d}}}_s}{\bar{\bar{\mathfrak{d}}}_\iota}\frac{\hat{\partial}_{\iota s}}{\hat{\partial}_{\iota\iota}} - \sum_{s=1}^{m} l_{\iota s}\frac{\bar{\bar{\mathfrak{d}}}_s}{\bar{\bar{\mathfrak{d}}}_\iota}\frac{\hat{\acute{\hbar}}_{\iota s}}{\hat{\partial}_{\iota\iota}}(\eta + \rho\hat{\varrho}_{\iota s})e^{\iota\hat{\varrho}_{\iota s}} < 1
$$

therefore, there is at least a constant $\mathfrak{k} \geq 1$, $\iota \in [1, m]$, so that:

$$
\Xi : \frac{1}{\mathfrak{k}_\iota} - \sum_{\iota\neq s}^{m} w_{\iota s}\frac{\hat{\partial}_{\iota s}}{\hat{\partial}_{\iota\iota}}\frac{\mathfrak{k}_s}{\mathfrak{k}_\iota} - \sum_{s=1}^{m} l_{\iota s}\frac{\hat{\acute{\hbar}}_{\iota s}(\eta + \rho\hat{\varrho}_{\iota s})e^{\iota\hat{\varrho}_{\iota s}}}{\hat{\partial}_{\iota\iota}}\frac{\mathfrak{k}_s}{\mathfrak{k}_\iota} - \frac{\chi}{\hat{\partial}_{\iota\iota}} < 1, \tag{6}
$$

for any known initial function $\kappa : [t_0 - \varrho, t_0] \to \Re_+^m$, there will be:

$$
r_\iota = \kappa_\iota(t) \leq \|\kappa\|_\varrho, \quad t_0 - \varrho \leq t \leq t_0.
$$

because h is positive, $\mathfrak{k} \geq 1$, and then

$$
r_\iota(t) = \mathfrak{k}_\iota \|\kappa\|_\varrho h(t)h^{-1}(t_0)
$$

It is shown below that: in $t \in (t_0, +\infty)$, $\iota \in [1, m]$, (5) is true.

Proof by contradiction: assume that (5) is not true, and there are $t^* > t_0$ and $s' \in [1, m]$, $\imath \in [1, m]$, $t \in [t_0 - \varrho, t^*]$ at this time

$$r_\imath(t) \leq \mathfrak{k}_\imath \|\kappa\|_\varrho h(t)h^{-1}(t_0), \tag{7}$$

$$r_{s'}(t^*) \leq \mathfrak{k}_{s'} \|\kappa\|_\varrho h(t^*)h^{-1}(t_0), \tag{8}$$

based on (3), we can get:

$$\begin{aligned} r_{s'}(t^*) =& e^{\int_{t_0}^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} r_{s'}(t_0)h(t^*)h^{-1}(t_0) \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}[-c_{s'}(r_{s'}(\varepsilon))d_{s'}(r_{s'}(\varepsilon)) \\ &+ w_{s's'}c_{s'}(r_{s'}(\varepsilon))z_{s's'}(r_{s'}(\varepsilon)) - \partial_{s's'}(\varepsilon)r_{s'}(\varepsilon)]h^{-1}(t^*)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s'\neq s}^{m} w_{s's}z_{s's}(r_s(\varepsilon))h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{s's}Z_{s's}(r_s(\varepsilon - \varrho_{s's}(\varepsilon))) \\ &h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} r_{s'}(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \end{aligned}$$

Here, based on the assumption $Q_1 - Q_5$

$$\begin{aligned} &- c_{s'}(r_{s'}(\varepsilon))d_{s'}(r_{s'}(\varepsilon)) + w_{s's'}c_{s'}(r_{s'}(\varepsilon))z_{s's'}(r_{s'}(\varepsilon)) \\ &- \partial_{s's'}(\varepsilon)r_{s'}(\varepsilon) \\ \leq& - c_{s'}(r_{s'}(\varepsilon))\varphi_\imath r_{s'}(\varepsilon) + w_{s's'}c_{s'}(r_{s'}(\varepsilon))z_{s's'}(r_{s'}(\varepsilon)) \\ &- z_{s's'}(r_{s'}(\varepsilon)) \\ \leq& - \underline{\epsilon}_\imath\varphi_\imath r_{s'}(\varepsilon) + w_{s's'}\overline{\epsilon}_\imath z_{s's'}(r_{s'}(\varepsilon)) - z_{s's'}(r_{s'}(\varepsilon)) \\ \leq& - \underline{\epsilon}_\imath\varphi_\imath r_{s'}(\varepsilon) + (w_{s's'}\overline{\epsilon}_\imath - 1)z_{s's'}(r_{s'}(\varepsilon)) \\ \leq& 0 \end{aligned}$$

where $w_{s's'}\overline{\epsilon}_\imath - 1 \geq 0$.
therefore

$$\begin{aligned} &r_{s'}(t^*) \\ \leq& \|\kappa\|_\varrho h(t^*)h^{-1}(t_0) \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s'\neq s}^{m} w_{s's}\partial_{s's}(\varepsilon)r_s(\varepsilon)h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{s's}\acute{\hbar}_{s's}(\varepsilon)r_s(\varepsilon - \varrho_{s's}(\varepsilon))h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} r_{s'}(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \end{aligned}$$

$$\begin{aligned} \leq& \|\kappa\|_\varrho h(t^*)h^{-1}(t_0) \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s'\neq s}^{m} w_{s's}\partial_{s's}(\varepsilon)\mathfrak{k}_s \|\kappa\|_\varrho h(\varepsilon) \\ &h^{-1}(t_0)h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{s's}\acute{\hbar}_{s's}(\varepsilon)\mathfrak{k}_s \|\kappa\|_\varrho \\ &h(\varepsilon - \varrho_{s's}(\varepsilon))h^{-1}(t_0)h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ h(t^*)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\mathfrak{k}_\imath \|\kappa\|_\varrho h(\varepsilon)h^{-1}(t_0)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \\ \leq& \|\kappa\|_\varrho h(t^*)h^{-1}(t_0) \\ &+ \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s'\neq s}^{m} w_{s's}\mathfrak{k}_s\partial_{s's}(\varepsilon)\mathrm{d}\varepsilon \\ &+ \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{s's}\mathfrak{k}_s\acute{\hbar}_{s's}(\varepsilon) \\ &h(\varepsilon - \varrho_{s's}(\varepsilon))h^{-1}(t_0)h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ \mathfrak{k}_{s'} \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} h(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \\ \leq& \|\kappa\|_\varrho h(t^*)h^{-1}(t_0) + \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} \\ &\sum_{s'\neq s}^{m} w_{s's}\mathfrak{k}_s\partial_{s's}(\varepsilon)\mathrm{d}\varepsilon \\ &+ \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\sum_{s=1}^{m} l_{s's}\mathfrak{k}_s\acute{\hbar}_{s's}(\varepsilon) \\ &h(\varepsilon - \varrho_{s's}(\varepsilon))h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ &+ \mathfrak{k}_{s'} \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} h(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \end{aligned}$$

furthermore,

$$\begin{aligned} r_{s'}(t^*) \leq& \mathfrak{k}_{s'} \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\Big(\frac{1}{\mathfrak{k}_{s'}} + \sum_{s'\neq s}^{m} \frac{\mathfrak{k}_s}{\mathfrak{k}_{s'}} w_{s's}\hat{\partial}_{s's}(\varepsilon)\Theta_{1s'} \\ &+ \sum_{s=1}^{m} \frac{\mathfrak{k}_s}{\mathfrak{k}_{s'}} l_{s's}\hat{\acute{\hbar}}_{s's}(\varepsilon)\Theta_{2s's} + \Theta_{3s'}\Big) \end{aligned}$$

where

$$\begin{aligned} \Theta_{1s'} &= \int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta}\mathrm{d}\varepsilon \\ \Theta_{2s's} &= \int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} h(\varepsilon - \varrho_{s's}(\varepsilon))h^{-1}(\varepsilon)\mathrm{d}\varepsilon \\ \Theta_{3s'} &= \int_{t_0}^{t^*} e^{\int_\varepsilon^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} h(\varepsilon)\mathfrak{Q}(\varepsilon)\mathrm{d}\varepsilon \end{aligned}$$

because of $e^{\int_{t_0}^{t^*} \partial_{s's'}(\theta)\mathrm{d}\theta} \leq e^{\hat{\partial}_{s's'}(t^*-t_0)}$, by assuming A_1 and A_2, we have:

$$\begin{aligned}
\Theta_{1s'} &\leq \int_{t_0}^{t^*} e^{\hat{\partial}_{s's'}(t^*-\varepsilon)}\mathrm{d}\varepsilon = -\frac{1}{\hat{\partial}_{s's'}}(1 - e^{\hat{\partial}_{s's'}(t^*-t_0)}) \\
&\leq -\frac{1}{\hat{\partial}_{s's'}} \\
\Theta_{2s's} &\leq \int_{t_0}^{t^*} e^{\hat{\partial}_{s's'}(t^*-\varepsilon)}(\eta + \rho\varrho_{s's(\varepsilon)})e^{\iota\varrho_{s's(\varepsilon)}}\mathrm{d}\varepsilon \\
&\leq (\eta + \rho\varrho_{s's(\varepsilon)})e^{\iota\hat{\partial}_{s's'}} \int_{t_0}^{t^*} e^{\hat{\partial}_{s's'}(t^*-\varepsilon)}\mathrm{d}\varepsilon \\
&\leq -\frac{(\eta + \rho\varrho_{s's(\varepsilon)})e^{\iota\hat{\varrho}_{s's}}}{\hat{\partial}_{s's'}} \\
\Theta_{3s'} &\leq -\frac{\chi}{\hat{\partial}_{s's'}} e^{\int_{t_0}^{t^*} \partial_{s's'}(\theta)}\mathrm{d}\theta \leq -\frac{\chi}{\hat{\partial}_{s's'}}
\end{aligned}$$

to sum up

$$r_{s'}(t^*) \leq \mathfrak{k}_\iota \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)\Xi$$

from (6), $\Xi < 1$, then $r_{s'}(t^*) < \mathfrak{k}_\iota \|\kappa\|_\varrho h(t^*)h^{-1}(t_0)$, which contradicts (8), then (i) is established.

(ii) because

$$\begin{aligned}
\|r(t)\| &\leq \sum_{\iota=1}^{m} |r_\iota(t)| \leq \sum_{\iota=1}^{m} \mathfrak{k}_\iota \|\kappa\|_\varrho h(t)h^{-1}(t_0) \\
&\leq \mathfrak{k} \|\kappa\|_\varrho h(t)h^{-1}(t_0), \quad t \geq t_0.
\end{aligned}$$

and $\mathfrak{k} = \sum_{\iota=1}^{m} \mathfrak{k}_\iota$, so the system (1) is global h-stability.

Remark 1: In Theorem 1, we make $h(t) = e^{-\varepsilon t}$, $t \geq t_0$, where $\varepsilon > 0$ is a scalar. If on the basis of assuming A_1 and A_2, let $\eta = 1$, $\rho = 0$ and $\iota = \chi = \varepsilon$, then the system (1) is globally exponentially stable through Theorem 1. (Equivalent to the conclusion of Definition 1 and Theorem 1 in the reference [25])

Remark 2: Assuming that A_1 and A_2 are true, if $\eta = 1$, $\rho = 0$ and $\iota = \chi = \frac{\sigma}{\mathfrak{x}}$, make $h(t) = e^{-\sigma t} + \frac{\mathfrak{x}e^{-\sigma t_0}}{\|\kappa\|_\varrho}$, $\mathfrak{x} > 0$, $\sigma > 0$, at this time, the Theorems 2 and 3 in reference [31] are equivalent to the Lagrangian exponential stability criterion obtained from Theorem 1 in the paper.

Remark 3: In addition, when Theorem 1 is $h = \frac{e^{-\varepsilon t}}{1+\varrho+t}$, $t \geq 0$, we find that its convergence speed is faster than $h = e^{-\varepsilon t}$, and it converges to zero first, as can be seen in the following Example 1 and Figure 1.

4. Examples

Example 1: Take $\iota = 3$, $m = 3$, then

$$\begin{aligned}
\dot{r}_\iota(t) =& c_\iota(r_\iota(t))\Big[-d\iota(r_\iota(t)) + \sum_{s=1}^{m} w_{\iota s}z_{\iota s}(r_\iota(t)) \\
&+ \sum_{s=1}^{m} l_{\iota s}Z_{\iota s}(r_\iota(t - \varrho_{\iota s}(t))) + \mu_\iota\Big] \\
r_\iota(t) =& \kappa_\iota(t), \iota = 1, 2, 3. \quad t \in [t_0 - \varrho, t_0],
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
r_\iota(t) &= \kappa_\iota(t) = [0.71, 0.42, 0.42]^T, t \in [-\varrho, 0] \\
c_\iota(r_\iota(t)) &= diag[1.2 + 0.1cos(r_1), 1.2 + 0.1sin(r_2), \\
&\quad 1.2 + 0.1tanh(r_3)] \\
d\iota(r_\iota(t)) &= [6r_1(t), 4.5r_2(t), 3.5r_3(t)]^T \\
z_{\iota s}(r_\iota(t)) &= Z_{\iota s}(r_\iota(t - \varrho_{\iota s}(t))) = \frac{1}{2}(|r_\iota(t) + 1| - |r_\iota(t) - 1|)
\end{aligned}$$

$$W = \begin{bmatrix} 4.6 & -1.2 & -1.1 \\ -1.6 & 3.5 & -1.2 \\ -1.2 & -1.1 & 1.9 \end{bmatrix}, \quad L = \begin{bmatrix} 0.04 & 0.02 & 0.03 \\ 0.07 & 0.04 & 0.02 \\ 0.02 & 0.03 & 0.05 \end{bmatrix},$$

to satisfy Assumptions Q_3 and Q_4, let $\grave{\hbar}_{\iota s} = l_{\iota s}$, $\acute{\hbar}_{\iota s} = 2l_{\iota s}$, and

$$\delta_{\iota s} = \begin{bmatrix} -4.6 & 1.2 & 1.1 \\ 1.6 & -7 & 1.2 \\ 1.2 & 1.1 & -2 \end{bmatrix}, \quad \partial_{\iota s} = \begin{bmatrix} -4.6 & 2.4 & 2.2 \\ 3.2 & -3.5 & 2.4 \\ 2.4 & 2.2 & -1.9 \end{bmatrix}.$$

To satisfy the Assumptions A_1 and A_2, let $h = \frac{e^{-\varepsilon t}}{1+\varrho+t}$, $t \geq -\varrho$, $\eta \geq 1$, $\rho \geq \frac{1}{1+\varrho}$, $\chi \geq \varepsilon + 1$, $\iota \geq \varepsilon$, $\varrho_{\iota s} = \frac{1}{2}|sint|$. Through MATLAB calculation (4), we can get:

$$\begin{aligned}
\chi =& 1.1221, \iota = \varepsilon = 0.1221, \eta = 1.8402, \rho \geq 1.667, \\
\bar{\bar{\partial}} =& [0.9245, 1.1921, 1.0342]^T.
\end{aligned}$$

According to the Theorem 1, it is concluded that the PCGNN system under consideration satisfies $\|x\| \leq 1.84814e^{-0.1221t}$, $t \geq -0.5$, then the system has globally h-stability.

In addition, we order $h(t) = e^{-\varepsilon t}, t \geq -\varrho, \eta \geq 1, \rho \geq 0, \chi = \iota \geq \varepsilon$. It satisfies the assumptions A_1 and A_2, and combines Theorem 1, calculated by MATLAB (4) to get the maximum value of $\varepsilon = 0.1643$, and at the same time,

$$\begin{aligned}
&\chi = 0.4764, \eta = 1.7554, \rho = 0.7824, \iota = \varepsilon = 0.1643, \\
&\bar{\bar{\partial}} = [0.9367, 1.0124, 0.9876]^T.
\end{aligned}$$

An inequality has been derived with respect to the time variable t, $\|x\| \leq 1.23209e^{-0.1643t}$, where x represents the state vector of the positive system. Notably, this inequality holds for all $t \geq -0.5$, establishing the system's global exponential stability. The implication is that the system exhibits a decay rate faster than the exponential function, ensuring robust and reliable performance across a wide range of time intervals.

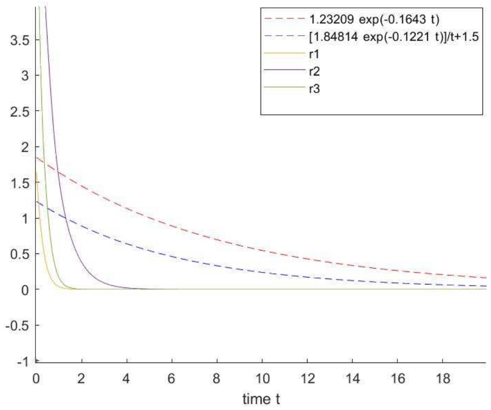

Figure 1: The state responses of the PCGNN system in Example 1.

Based on the observations from Figure 1, it becomes apparent that as t approaches infinity, the function h(t) converges to zero at a significantly faster rate compared to the exponential function. In other words, the convergence speed of h(t) surpasses that of the exponential function, highlighting the practical advantages of the h(t) function in real-world applications.

Example 2: In system (1), we choose $\iota = 2, s = 2, t_0 = 0, \kappa_\iota(t) = [0.7, 0.88]^T$,

$$\begin{aligned}
\varrho_{\iota s}(t) &= \frac{e^t}{1+e^t}, \quad t \geq 0,\\
z_{1s}(t, r_s(t)) &= w_{1s} r_s(t),\\
Z_{1s}(t, r_s(t-\varrho_{1s}(t))) &= l_{1s}[-r_s(t-\varrho_{1s}(t)) + sint],\\
z_{2s}(t, r_s(t)) &= w_{2s} r_s(t),\\
Z_{2s}(t, r_s(t-\varrho_{2s}(t))) &= l_{2s}[-r_s(t-\varrho_{2s}(t)) + cost],\\
d_\iota(r_\iota(t)) &= r_\iota(t), \quad \iota = 1, 2.
\end{aligned}$$

$$c_\iota(r_\iota(t)) = diag[0.8 + 0.1cos(r_1(t)), 0.8 + 0.1sin(r_2(t))]$$

$$w_{\iota s} = \begin{bmatrix} 0.5 & -0.1 \\ 0.45 & 0.3 \end{bmatrix}, \quad l_{\iota s} = \begin{bmatrix} 0.65 & 0.37 \\ 0.75 & 0.52 \end{bmatrix}$$

$$\delta_{\iota s} = \begin{bmatrix} 1.0 & -0.1 \\ 0.45 & 0.6 \end{bmatrix}, \quad \partial_{\iota s} = \begin{bmatrix} -2.5 & 4.1 \\ -3.1 & -2.2 \end{bmatrix}.$$

To satisfy the assumptions A_1 and A_2, take $\grave{\hbar}_{\iota s} = l_{\iota s}$, $\acute{\hbar}_{\iota s} = 2l_{\iota s}$, $h(t) = e^{-\sigma t} + \frac{\mathfrak{r} e^{-\sigma t_0}}{\|\kappa\|_\varrho}$, $t \geq 0$, $\eta \geq 1$, $\rho \geq 0$, $\chi = \iota = \frac{\sigma}{\mathfrak{r}}$. When $\varrho = 50$, $\mathfrak{r} = 5.6773$, $\chi = \iota = \frac{\sigma}{\mathfrak{r}} = 0.00022017$, $\eta = 1.0789$, and $\rho = 0.0012$, we bring it into (4) to calculate $\bar{\eth} = [0.0004370, 0.0003156]^T$. Based on the derivation of Theorem 1, we can conclude that the positive system under consideration is global h-exponential stability in the sense of Lagrange. From Figure 2, it can be observed that under the conditions of Example 2, the state response of system (1) gradually converges to a periodic orbit after an initial perturbation.

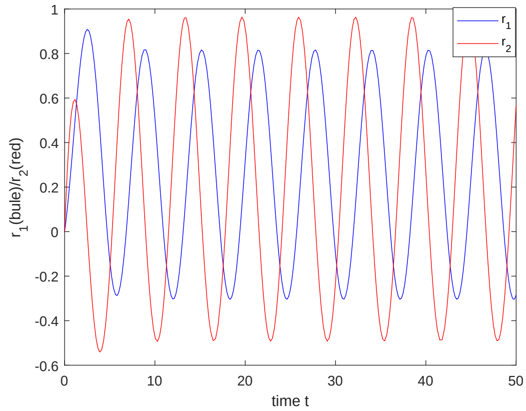

Figure 2: The state responses of the PCGNN system in Example 2.

5. Conclusions

This paper investigates the global h-stability of PCGNN systems with time-varying delays, which is more suitable for practical applications in our daily lives. Due to the presence of time delays, the current state variables are influenced by both current information and past input information in a nonlinear manner, making it impossible to directly analyze the relationships between system states using vector matrices. Therefore, by taking advantage of the stability of h-system, the appropriate h function is determined, and the differential equation in system (1) is Laplace transformed by direct analysis method, and the analytical solution of the equation is obtained. Then, the obtained analytical solution is reverted to the time domain and sufficient conditions for global h-stability of the system are derived. By changing the differential expression of the h function, we can obtain the special h-stability, namely the Lagrange exponential stability of the system (1) and the global exponential stability. The experimental results show that the fitting effect of the system state trajectory can be optimized by changing the differential expression of h function, which further shows that h function can ensure the validity and accuracy of the stability results.

At present, sliding mode control methods have received extensive attention in the field of studying the stability of neural networks [32]. The sliding surface is introduced to offset various system errors caused by uncertainty and ensure the stability of the system. This is a subject of great concern and worthy of further study, and further research work can be done in this direction.

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