

CUMULATIVE QUANTUM MECHANICS (quantum-size effects for nano-, angstrom and femto-technologies)

P.I.Vysikaylo
MRSU, Moscow, RF, filvys@yandex.ru
ID: 0000-0001-9701-5222

Abstract

The world's leading laboratories continue intensive research into the properties of nanocomposites - materials that have unique, both physical and chemical properties that are useful in practice. Along with the discovery of new materials, new technologies are being developed and attempts are being made to create mathematical models capable of describing phenomena in hollow quantum resonators - quantum dots, lines, and other cumulative-dissipative 3D structures of nanometer dimensions. Mathematical models make it possible to develop new materials, classify their properties and discover new patterns. As a result of the study of nanocomposites, the author has discovered and classified more than 31 polarization quantum-size effects. In this paper, it is proved that only by applying the foundations of cumulative quantum mechanics (CQM) can one explain and classify all quantum-size effects discovered by the author. These quantum size effects led to the discovery of the principles of physical doping and the classification of doping into physical and chemical doping. During physical doping, the modification of the properties of the nanocomposite is carried out with the help of nano- and microstructures of foreign material, which have a high affinity for free electrons. In this case, the fractions of foreign (alloying, other) material do not penetrate into the crystal lattice. They only decorate the nano- or microcrystals of the support material in the nanocomposite. A dopant with a high affinity for free electrons is charged with a negative charge, while a doped nanocrystal is charged with a positive charge. Therefore, physical doping of nanocomposites leads to the generation of electric fields that act as catalysts for various reactions, contributes to the strengthening of nanocomposites by Coulomb compression, an increase in the luminescent properties of phosphors, an increase in conductivity up to 10^{10} times, and other properties, due to quantum size effects due to local violation of electrical neutrality. The models we have developed, which are effective for explaining nano-effects, have been used by us to explain angstrom- and femto-phenomena in cumulative-dissipative structures.

Keywords: physical doping, quantum size effects, nanocomposites, cumulative quantum mechanics, de Broglie wave cumulation, unlimited cumulation of de Broglie's wave ψ -functions, cumulation-libration points, endoelectrons, fullerenes, cumulative-dissipative structures.

Introduction

The study of the interaction of polarizable fullerenes with electrons made it possible to discover and analytically describe all the quantum-size effects discussed in this paper. We managed to establish the characteristic dimensions on which these (quantum) effects are formed and to investigate how quantum effects manifest themselves in the mesoworld of nanocomposites and even the macroworld of quantum stars and black holes [1–6]. For this reason, the results obtained by the author on the study of quantum-size effects are successes in the physical and chemical sciences, and even in the astrophysical sciences. The models we have developed, which are effective for explaining nanoeffects, have been used by us to explain angstrom and femto phenomena in cumulative-dissipative structures.

Fullerenes with trapped electrons are a class of negatively charged quantum dots or negative endoions [3–6]. As a comparison of analytical calculations [3–6] with experimental observations [7–9] of the energy spectra of such quantum dots showed, their own energy spectrum is significantly determined not only by the characteristic size of the object, R , capturing an electron, but also by the characteristic polarization size, r_{ind} , where the most effective forces act on the electron (returning it to the trap). These forces are due to the polarization of the quantum trap for electrons. Typically, estimates use the characteristic size of the trap and do not take into account the full characteristic size of the interaction – $R+r_{ind}$. This is an error due to the

neglect of the characteristic polarization size of the nanoobject! For C_{60} fullerene, $R = 0.36$ nm, and $R + r_{ind} = 0.62$ nm [3-6].

The main, fundamental problem of nanochemistry and nanophysics, around which all the interests of researchers revolve, is quantum-size effects. These are intriguing questions: how the properties of individual molecules, when combined, evolve into properties of the phase; how bridges are built between the world of a single, individual molecule and the macroscopic world of matter; how the hierarchy of quantity is transformed into a hierarchy of properties, how the cumulation of various portions of energies leads to a change in the geometry of crystal lattices, a change in their properties, their hardening, enlightenment, polarization when interacting with a charged particle or an electromagnetic field, etc.

The emergence of synergistic electric fields due to the resonant localization of electrons in traps at the boundary of a positively charged nanocrystal leads to the appearance of two types of Vysikaylo's polarization quantum-well resonance effects [3-6]. According to the Vysikaylo's polarization effect of the first type [3-6], the capture of electrons (Fig.1) can occur as a result of the formation of a standing de Broglie wave in a quantum hollow resonator for these waves (Q-particles: quantum dots, wires and wells). The characteristic size of the Vysikaylo's polarization effect of the first type is determined by the characteristic size of the trap (~ 1 nm for $C_{60,70}$ and other fullerenes and nanotubes). According to the Vysikaylo's polarization effect of the second type, depending on the number of traps in the nanocomposite, the positive charge of the nanocrystallite (Fig.2) physically doped with traps will change. And, consequently, the average energy of electrons near the surface of nanocrystals will change on the dimensions of a positively charged crystallite or on the dimensions of a negatively charged nano-modifier. As a result, due to the polarization resonant interaction, by changing the dopant concentration (trap concentration) in the nanocomposite, it is possible to effectively control the macroparameters of bilayer (polarized due to the presence of the modifier) metamaterials with sizes from 10 nm to meso-sizes of the entire material. So, according to [3-6], we come to the discovery of a new method of doping - physical doping or the formation of bilayer (bicharged) metamaterials (Fig.2) and their effective application in practice, for example, when modifying the properties of phosphors [5, 6]. This is how a new nanochemistry arises - nanoelectrochemistry (Fig.1, 2).

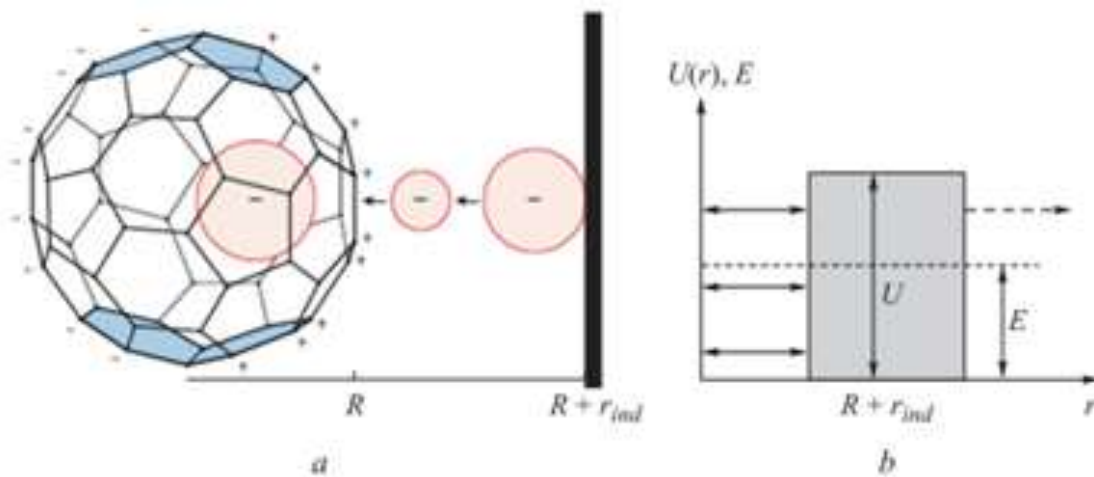


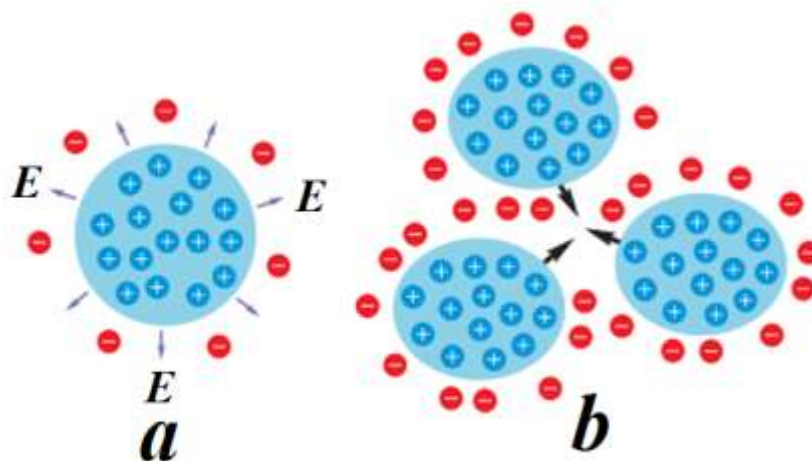
Fig.1. Scheme of cumulative pulsating e-capture with resonant energy electron from 0.2 to 20 eV into the cavity of a $C_{60,70}$ molecule and resonant formation of a negative endo-ion $e_k@C_{N=60,70}$.

This is the first Helmholtz boundary value problem [10, p. 515]. Endoion radius (position of the reflecting "mirror" $R_1 = (R_{C_{60, C70}} + r_{ind})$ [2-6]:

a – 4D (space-time) process of formation of a standing de Broglie's wave in a polarizing quantum resonator with a successive decrease in the de Broglie's wavelength of an electron incident on a fullerene molecule polarized by it [2-6]. The polarization pulsating barrier that returns the electron back to the fullerene is shown as a black rectangle on the right;

b – diagram of the resonance spectrum of a metastable (quasi-open) quantum particle with a polarization partially transparent mirror (of finite dimensions), which captures an electron with energy $E > 0$ in polarization trap of characteristic size $R + r_{ind}$.

The polarizing mirror is darkened.



**Fig.2. Schematic of the physical principle
doping of materials with polarization charge separation [2-6]:**

a - modification of the properties of composite materials by layers of space charge formed by traps for free electrons;

b – diagram of the Coulomb compression of polarized complex structures. The arrows indicate the direction of the Coulomb forces compressing the nanostructured polarizable composite

It is possible that polarization or the formation of electrical potentials can lead to local ice melting. So, due to the heating of water by electrons around positively charged microorganisms in eternal glaciers, a kind of antifreeze can form. It is possible that the accumulation of highly energetic electrons in the structures of microorganisms provides them with the opportunity to survive in such harsh conditions in the almost complete absence of energy coming from outside. In the presence of the Coulomb potential, it is possible to transfer energy from a cold body (for example, a crystal lattice) to a hot one (an electron gas with a large kinetic energy). The cumulative-dissipative structures (CDS) of living organisms are diverse and require detailed studies of electrical neutrality violation, both in living and inanimate plasma polarizable CDS.

In experiments with fullerenes - hollow ideal spherically symmetric polarizable quantum dots capable of capturing free electrons with resonant energy $E_n \sim 0.2 \div 20$ eV [7-9], in this work we will look for experimental confirmation of cumulative quantum mechanics (CQM) formulated in [2-6].

The purpose of this paper is to present 32 quantum-size effects discovered and classified by the author.

Thirty two quantum-size effects during physical doping of nanocomposites

It has long been known that a number of molecules have a high affinity for electrons, such as oxygen molecules. Because of this affinity, negative ions O_2^- and O_4^- are formed. In [2-6], we discussed experimental facts in detail and proved that nanometer-sized carbon structures (like fullerenes, nanotubes, nanodiamonds, and nanographenes (Fig.1)) have the same property of localizing free electrons. Cavities, cracks, etc. have the property to localize free electrons. The use of physical doping of nanocomposites significantly changes their properties in the mesoworld (Fig.2) [2-6].

Violation of electrical neutrality (polarization) at the nanolevel leads to thirty two polarization quantum-size effects discovered in [2-6] and this work. Let's list this effects:

Effect 1 - Coulomb hardening (compression, focusing, reduction, cumulation) of positively charged nanocrystals by nanolayers of negatively charged electron traps (Fig.2b) [2-6]. This quantum size effect was experimentally studied on the example of physical doping with carbon nanostructures of such materials as semiconductors (thermoelectrics), copper, aluminum and transition metal carbides, see references in [4–6]. According to experimental observations, copper and aluminum were physically hardened by doping with fullerenes up to 10 GPa [4–6], and copper films were strengthened by graphite nanostructures up to 4 GPa [4–6]. Hardening of transition metal carbides by physical alloying of films with carbon nanostructures by magnetron sputtering was registered up to 31 GPa. In this case,

strengthening by physical alloying of nanocomposites with a nanocrystal size of ~ 20 nm up to 100 GPa is considered theoretically possible [4–6]. However, maintaining a size of 20 nm for all nanocrystals in a nanocomposite is a complex technological problem.

Effect 2 – according to Newton's third law, see [5] for more details, the cohesive forces and the forces of resistance to compression in crystalline materials are equal to each other. Hardening of the material, including physical alloying (Fig.2b), corresponds to the same increase in the microhardness of the nanocomposite. Nanocomposite materials can be strengthened by creating nanosized graphene shells covering composite nanocrystals.

Effect 3 – generation of electric fields (up to 10^{11} V/m) and potentials that “heat” electrons in the area of the nanocrystal surface (Fig.2a) [4–6]. During physical doping, the properties of the surface of volumetrically charged nanocrystals also change due to the polarization of charges at the nanolevel (sizes of physically alloying nanoadditives), see Fig.2, for example, due to an increase in the average energy of free electrons on the surface of nanocrystals. Experiments have shown that this leads to an increase in the luminescent characteristics of the phosphor by up to 50% [4–6];

Effect 4 – generation of electric fields (up to 10^{11} V/m) and potentials that “heat” electrons in the area of the nanocrystal surface (Fig.2a) [4–6] leads to the appearance of catalytic properties in physically doped nanoparticles. This is due to a local increase in the electrons kinetic energy (decreasing their de Broglie wavelength) in the area of the surface of a positively charged nanocrystal and the possibility of them penetrating through Coulomb's barriers;

Effect 5 - the electric field strength on the surface of a nanocrystal depends on the concentration of nanotraps on the surface of a physically doped material [4–6]. This effect leads to a change in the energy of electrons on the surface of a physically doped nanocomposite with a change in the concentration of traps and, as a consequence, leads to a change in the number of electrons trapped in an electron quantum resonator (nanotrap). If only the resonant spectrum of localized electrons is possible in a quantum resonator, as in fullerene C_{60} [4–6], then the concentration-polarization-quantum-size Vysikaylo's effect of the second type will be observed in the corresponding range of parameters (Fig.3). In this case, the profiles in Fig.3a shift with a change in the characteristic radius of the nanocrystallite, forming a quantum size pair with a relative concentration of traps). With this quantum size effect, with a change in the relative concentration of the physically alloying additive, the parameters of the nanostructured composite in the mesoworld change [4–6]. Using this quantum size effect, the properties of thermoelectrics can be modified by changing the concentration of nanocarbon particles in the nanocomposite [4–6, 11]. In experiments [11], it was possible to increase the efficiency of a thermoelectric by 30% due to physical doping with fullerenes.

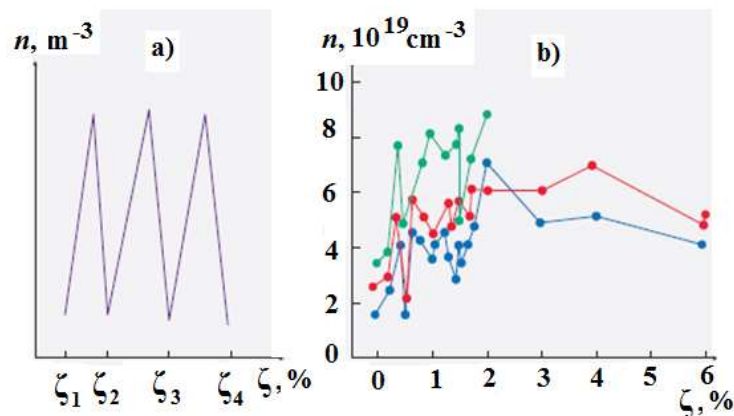


Fig.3. Concentration-quantum-dimensional Vysikaylo's effect:

a - the characteristic dependence of the nanocomposite parameter, for example, the concentration n of ions or electrons, on the volume content of C_{60} in the composite;

b - experimental measurements of resonance profiles of electron density with a change in the relative concentration of traps during physical doping of semiconductor nanocrystals (thermoelectrics) with C_{60} fullerenes at various temperatures (77–295 K); The characteristic size of a nanocrystallite after annealing is $D \approx 17$ nm [11].

Effect 6 – generation of electric fields (up to 10^{11} V/m) and potentials “heating” (decreasing their de Broglie wavelength) electrons locally in the area of the nanocrystal surface (Fig.2a) [4-6] can lead not only to dynamic hardening of physically doped nanocrystals, but weak modification of their crystal lattice for nanocomposites. This effect manifests itself more significantly in quantum stars, as stated in [1, 5], where the strengthening membrane is a layer of electrons or negative mesons rather than fullerenes (Fig.2a);

Effect 7 – in nanocomposites, the pair “eigen-function ψ_n – eigen-energy E_n , describing the quantum state with the principal quantum number n , in the nanoworld, in the mesoworld of nanocomposites physically doped with traps, is replaced by two parameters of the nanoworld: the diameter of the nanocrystal D and the relative concentration of the modifier (traps, for example, C_{60}) – ζ_n (Fig.3) [4-6].

Effect 8 - self-focusing of the charge and electric field in nanostructures due to polarization forces arising from the interaction of nanostructures with electrons in nanostructures with high electron affinity. For example, according to experimental data, see [12], fullerene can accumulate up to six electrons per one C_{60} , which corresponds to electric fields around the fullerene of $\sim 10^{10}$ V/m. This phenomenon is a polarization cumulation (localization) of free electrons in a trap, i.e. quantum resonator.

Effect 9 - cumulation (localization) of the energy of free electrons in a trap - a quantum resonator. For example, according to experiments, see [12], fullerene can accumulate up to six electrons per C_{60} with a resonant kinetic energy from 0.24 to 12 eV. This can lead to the accumulation of the energy of the electron gas up to 30 eV in one fullerene.

Effect 10 - the presence of polarization forces of cumulation (focusing) of charged particles in quantum resonators (traps). The ideal shape of C_{60} allowed the author of [2-6] to use the mathematical model developed by Gamow to describe the penetration of an α -particle through a potential barrier. The Gamow model modified in [2–6] made it possible to analytically calculate the resonant energy levels of trapped electrons for all spherical quantum resonators (C_{60} , C_{70} and higher fullerenes). From a comparison with experiments, in [2-6] we managed to determine the effective polarization sizes for various fullerenes (rind for C_{60} – 0.26 nm and for C_{70} - 0.28 nm), at which a polarization potential barrier with a height about 18 eV, returning the electron back to the fullerene cavity [5] (Fig.1).

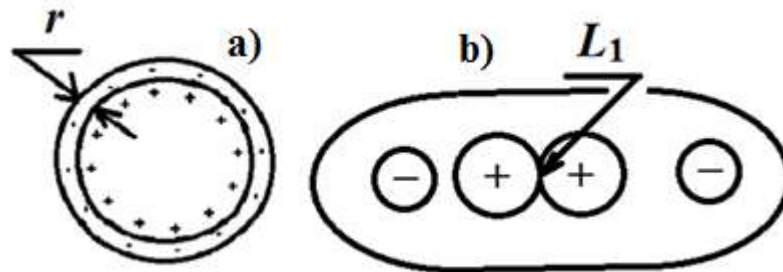


Fig.4. Scheme of quantum self-polarization of molecular structures on nano- and angstrom sizes:

a – polarized drop with e-membrane;

b - an example of a covalent bond of two atoms that share external electrons, for example, in a molecule or a singly ionized hydrogen ion, if one external electron is removed. L_1 is the cumulation point for electrons [5,6] between positively charged elements of the structure

Effect 11 – *resistance to swelling* during physical doping of materials with electron traps. This is a consequence of quantum size effects 8 and 9, i.e. the possibility of capturing free energetic electrons in a high-strength nanoscale trap.

When materials are subjected to: energy loading, for example, when: irradiated with neutrons, electromagnetic radiation or subjected to: bending, friction or other functional energy loads, there is an accumulation of energy coming from outside into the material. In a material, energy is accumulated mainly in a localized electron gas. Let us detail this process of energy accumulation in a functioning material. The electrons in the material are in the Coulomb potential wells of atomic nuclei. And with an increase in the total energy of an electron in the Coulomb well, its kinetic energy decreases. According to the foundations of quantum mechanics, the de Broglie wavelength of such an electron increases;

consequently, an atom with such an electron in the “orbit” swells, and if there are many of them, then the entire functioning material swells significantly. According to Newton's third law, an excited atom is opposed by a medium. Consequently, the excitations will be displaced to the surface and self-focus on the internal inhomogeneities of the functioning material. As the number of excited atoms in the loaded materials increases, a noticeable local swelling is observed, as well as the growth of cracks and a change in other physical and chemical properties, leading to a deterioration in the functional properties of the material and even to its destruction. Expansion and self-focusing processes (cumulation and localization) during swelling promote the formation and growth of cracks and the localization of free electrons in cracks or cavities. Further filling of cracks with free electrons, as the most mobile and bulk gas, is determined by the movement of free electrons from the material into the crack and the reflection of electrons by the crack boundaries back into the crack. This happens in the same way as in the case of polarization capture of an electron by fullerene (Fig.1). In a crack with a large number of electrons, the electron energy distribution function (EDF) is formed. This is due to electron-electron collisions. Since the crack geometry, as a rule, has a conical shape, high-energy electrons are displaced to the crack tip region, where they transfer its kinetic energy into the potential energy of crack growth. An increase in the pressure of the electron gas localized in the crack and the geometry of energy cumulation at the crack tip leads to catastrophic crack growth and destruction of the material, which very quickly dissipates the energy accumulated in the working material into cracks. An increase in the number of excited electrons in the region of a cavity or crack leads first to its slow and then to catastrophic explosive growth, since the material gets tired and excess kinetic energy of electrons accumulates in it, which transforms into potential energy concentrated in cracks. The introduction of the durable nanometer traps for free electrons C_{60} (with a resonant energy spectrum for inner electrons of 0.24–12 eV) into physically doped nanocomposites in small amounts, which do not significantly change the useful properties of nanocomposite materials, makes it possible, under a number of conditions, to reduce the number of free electrons and thereby weaken the catastrophic swelling and crack growth in functional materials subjected to energy loading. These hollow spherical heavy-duty resonators - C_{60} are capable of concentrating up to six electrons [12] with a total energy of up to 30 eV [5,6]. In this case, the strength of the fullerene framework corresponds to the strength of graphene and exceeds the strength of diamond. In this regard, such a spherically symmetric trap, unlike a cavity or a crack, does not increase in volume and does not swell, which significantly increases the catastrophic fracture threshold of a nanocomposite physically doped with high-strength traps for free electrons. Such physical alloying increases the service life of the nanocomposite, its reliability and duration of operation under conditions of intensive operation.

Effect 12 – *levitation* (floating, repulsion) of positively charged nanostructures of a physically alloyed material one above the other (Fig.2b). This effect counteracts the processes of recrystallization in nanocomposites, which significantly increases the characteristic time of their operation [4–6]. Let us note that the violation of electrical neutrality of matter at the level of 10^{-18} (for one proton uncompensated by a negative charge, 10^{18} compensated) stops the gravitational collapse of even black holes [1,5].

Effect 13 – Coulomb’s melting of nanofullerite (fullerene nanocrystals), which occurs when k electrons are captured by fullerene molecules in fullerite. In a fullerite crystal, C_{60} molecules are bound together by weak van der Waals forces. The binding energy of a single fullerene molecule in a fullerite nanocrystal does not exceed 1.6 eV, see [13]. In this case, the potential energy of the Coulomb’s interaction of two negative ions in fullerite with k electrons is of the order $1.4 \cdot k^2$ [eV] (the average distance between fullerenes in fullerite is $D \approx 1$ nm). Thus, the capture of several electrons from a nanocrystal physically doped with fullerenes leads to Coulomb’s sputtering of fullerite nanostructures and a monolayer coating of a positively doped nanocrystal with negatively charged fullerenes (Fig.2b) [4–6]. One C_{60} fullerene molecule is capable of capturing up to six electrons [12]. Note that, in principle, neither nanotubes due to their entanglement or interweaving, nor nanographenes due to their strength, have such a property to decompose into strictly nanosized structures. In experiments, it is this property of fullerites that made it possible to create materials by physically doping thermoelectrics [11], copper [4–6], and aluminum [4–6] with C_{60} fullerenes, which have unique functional and strength properties. Our theory predicts the possibility of increasing the microhardness of a physically doped nanocomposite with fullerenes up to 100 GPa [4–6].

Effect 14 – *cumulation of electron flows* at points (lines and planes) of libration between positively charged nanostructures (Fig.2b, 4b - cumulation-libration point – L_1). These effects have been studied in gas-discharge plasma [14] and are still insufficiently detailed in nanocomposites. This cumulative quantum size effect, caused by the capture of free electrons with resonant energy in a quantum resonator and the formation of positively charged nanocrystals of a doped material, can be used to control local parameters: conductivity, thermal conductivity, and other properties of nanocomposites.

Effect 15 – when a charged particle interacts with a system containing an electron gas or a Fermi liquid, due to the high mobility of electrons in any systems, it leads to the formation of polarization barriers that cumulate these particles to the center of a ***hollow*** polarizing system (Fig.1). This approach now allows us to detail the essence of the polarization barrier introduced by Gammon for the α -decay of atomic nuclei. Such dynamic barriers arise for short times when a charged particle tends to leave a polarizing pulsating system [2-6]. Therefore, such dynamic barriers cannot be modeled using DFT methods [15] (density functional theory). To calculate the resonant energy spectrum, we replaced such a pulsating partially transparent barrier with a stationary barrier surrounding the fullerene (Fig.1a).

Effect 16 is the counteraction of electrons in the trap to external action on the trap and on the entire nanocomposite material (Fig.1a). In this case, electrons having de Broglie wavelengths larger than the dimensions of the polarizing trap counteract the external action, protecting the physically doped material from destruction; R is the radius of the trap; r_{ind} is the characteristic induction size by which the electron leaves the trap (Fig.1b). In experiments, this effect was observed when metal products were coated with graphene sheets. This effect was used to explain the properties of a white dwarf in astrophysics by Frenkel and Fowler. It turns out that any electrons localized in a polarizing Coulomb well in any quantum resonators with sizes from 10^{-15} (in this case, negative mesons play the role of electrons) to 10^{26} m have dual properties - to focus the structure and counteract its compression (Fig.4) [5].

Effect 17 is a significant change in the conductivity of materials physically doped with nanostructures from allotropic forms of carbon. For example, the use of nanotubes can lead to quantum ballistic conduction effects in negative ions from nanotubes that trap electrons. In experiments, the method of physical doping with nanotubes makes it possible to change the conductivity of the initial siloxane by 10 orders of magnitude [4]. As shown in [2-6], if for the polarization length C_{60} of fullerenes, which, according to quantum mechanics, an electron localized by the polarization potential could move, is of the order of $r_{ind} = 0.26$ nm, and for C_{70} - $r_{ind} = 0.28$ nm (Fig.1), then for a carbon nanotube the polarization length r_{ind} is determined by its length L . This leads to a significant decrease the lower eigenlevel (resonant) energy of the quantum resonator for electrons and its tendency to a continuous spectrum with increasing length L .

Effect 18 – increased emission during physical doping with nanotubes. According to effect 8, much more free electrons with resonant energy can be localized around and in the nanotube, which can significantly increase the efficiency of nanotubes in emission phenomena compared to fullerenes. A decrease in the lower energy level of the nanotube leads to an increase in the frequency of ejection of electrons previously captured by the nanotube. Although the analytical calculations given in [2–6] for fullerenes were confirmed in a number of experiments [4, 11], such calculations for nanotubes give ambiguous results. These ambivalent (dual) conclusions for polarized carbon nanotubes should be verified in experiments with nanotubes having a given length $L = \text{const}$.

Effect 19 – quantum size polarization effects should be taken into account in the case of many-particle interaction of atoms and molecules in dense media [5,6]. Such an allowance leads to an increase in the time of capture of several particles into a polarization trap (in the region of the polarization barrier common to several particles) and to an increase in the effective polarization cross section to a value $\sim \pi(R+r_{ind})^2$. Here R is the radius of the cross section without taking into account polarization, and r_{ind} – the polarization length (Fig.1). This leads to an increase in the effective calculated cross sections for three-particle interactions of atoms, molecules, and more complex atomic structures. Accounting for a significant increase in the effective cross sections of reactions at elevated pressures leads to the solution of a number of paradoxes (inconsistencies between experimental results and theoretical concepts) [5].

Effect 20 – comparison of the quantum-size effects of Vysikaylo and Casimir. This effect was discovered by Theo Overbeek. He found that the theory used to explain the van der Waals forces developed by Fritz

London (Fritz London, 1932) could not adequately explain the data of his experiments, which showed the existence of a more significant coupling between long molecules. Casimir suggested [16] that the interaction between two neutral approaching molecules can be described if vacuum fluctuations and the generation of electromagnetic radiation – light (Fig.5a) are taken into account.

Between the mirrors, it is impossible to generate electromagnetic radiation with wavelengths greater than d . But it is possible outside of these mirrors. The pressure of these electromagnetic waves on the mirrors from the outside leads to the Casimir's effect. The attraction of metal plates in the Vysikaylo's effect is explained similarly. "+" - ions of the crystal lattice, "-" - free degenerate electrons with large de Broglie wavelengths, quantum-mechanically squeezed out of the space between the polished plates. The electrons squeezed out of the gap with their pressure, due to their kinetic energy, compress the plates with pressure $P = F/S$ [17, 18] (Fig.5b). For relativistic electrons, the dependence of pressure – P (force – F) in the Vysikaylo's and Casimir's effects is $\sim d^{-4}$, and for the non-relativistic case the Vysikaylo's effect $\sim d^{-5}$ (for Casimir's effect $\sim d^{-4}$) [4,5].

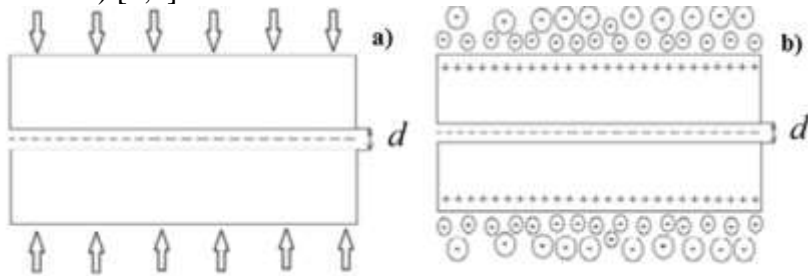


Fig.5. Effect scheme:

a – Casimir's effect. The arrows show the forces compressing the plates;
b – nano- and femto-dimensional quantum Vysikaylo's effect [4,5].

Effect 21 – the valency of an atom in a molecule is determined by the number of possible libration points – L_1 for electrons around an atom in a molecule (Fig.4b) [14].

Effect 22 - Vysikaylo-de Broglie-Fraunhofer's interference of de Broglie waves of electrons in ***hollow*** quantum resonators leads to unlimited cumulation of ψ -functions of de Broglie electron waves in ***hollow*** spherical and cylindrical ***quantum resonators*** (Fig.6) [2-6].

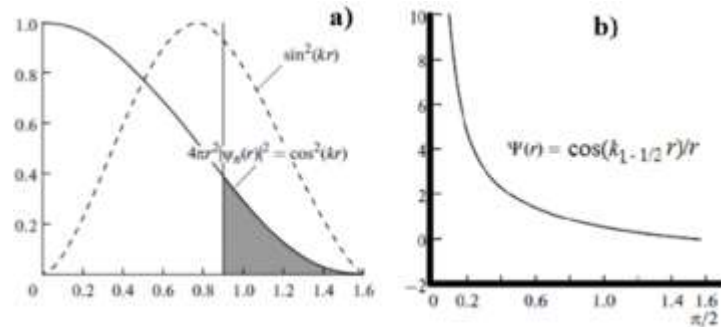


Fig.6. Profiles:

a – relative probability $4\pi r^2 |\psi_n(r)|^2 dr$ to find an electron in the region of a ***hollow*** spherically symmetric molecule, taking into account the geometric normalization coefficient. For an electron with energy, $E_{1/2}$ (Vysikaylo-de Broglie-Fraunhofer's interference of de Broglie waves of electrons in ***hollow*** quantum resonators) is denoted by a solid line, and for an energy resonance, E_1 (Bohr-de Broglie-Fresnel's interference of de Broglie waves of electrons in) is indicated by a dashed line. A straight vertical line separates the internal and external regions of a hollow polarizable spherically symmetric molecule. The relative probability of finding an electron at 1/2 resonance (cos-wave) outside the shell of a hollow molecule is shaded (calculations were made for C_{60}).
b – $\psi_{1/2}$ -functions at 1/2-resonance (cos-wave with $E_{1/2}$).

Effect 23 – eigen-energy spectra of ***hollow*** quantum resonators have a new class of solutions with eigen-energy spectrum $E_{n-1/2} \sim (n-1/2)^{\pm 2}$ for potential wells "-", and for structures with potential barriers "+". This is how the degeneracy discovered by Vysikaylo in [2-6] appeared. This is the degeneracy of the ground quantum level n in sin- and cos-waves in ***hollow*** quantum resonators [2-6].

Effect 24 - in *hollow* quantum resonators, transitions with the corresponding resonance spectrum are possible not only between sin-waves, but also all transitions between cos- and sin-states of de Broglie waves [2-6]. Thus, we discovered several times more allowed spectra in *hollow* quantum resonators than in ordinary atoms, where there are no stationary cos-resonances.

Effect 25 – self-energy spectra of quantum resonators do not depend on the type of their symmetry [2-6]. As shown by us, in accordance with the de Broglie's hypothesis, in QCM the eigen-spectra of quantum *hollow* resonators do not differ from each other for all three types of symmetry: spherical, cylindrical, and planar, and they are determined only by the characteristic size of the resonator [2-6].

Effect 26 – in the theoretical description of cumulation phenomena in spherically and cylindrically symmetric resonators, it is necessary to take into account the *normalization geometric coefficients* that regularize the probability of finding a particle in a layer [2-6]. This is often forgotten in quantum mechanics textbooks. Part of the solutions of the Schrödinger's equations (the first Helmholtz boundary value problem) for this reason are thrown out without any reason for *hollow* quantum resonators.

Effect 27 – in quantum *hollow* resonators with any type of symmetry, de Broglie's standing waves of electrons with half a wavelength are possible [2-6]. This was proved by us in [2,3] by comparing the energy eigenspectra of C_{60} quantum resonators with the resonance cross sections experimentally obtained in [7-9] using the example of C_{60} , C_{70} electron capture (Fig.7).

Effect 28 – we have proved that when applying the de Broglie's hypothesis from the femtodimensional to the mesoworld and the world of galaxies, it is necessary to carry out not a partial, but a complete transfer of mathematical models from areas well studied in the field of natural sciences, studied insufficiently [2-6]. It is necessary to investigate the whole range of similar phenomena, and not be limited only to particular solutions. So in wave physics such phenomena as cavitation and sonoluminescence in acoustic resonators have long been known. These phenomena are due to the cumulation of wave energy towards the center of the acoustic resonator. In this paper, we point out the phenomena of unlimited cumulation of $\psi_{n-1/2}$ -functions of quantum particles in *hollow* quantum resonators with spherical and cylindrical symmetries (Fig.6b).

Effect 29 – during polarization resonant capture of an incident plane de Broglie wave of an electron into a hollow spherically symmetric quantum polarizable resonator with a high electron affinity, the de Broglie plane wave $\psi \sim \exp(ik_{n,n-1/2}x)$ is modified into a spherically symmetric wave with $\psi \sim \sin(\mathbf{k}_n \mathbf{r})/r$ or $\psi \sim \cos(\mathbf{k}_{n-1/2} \mathbf{r})/r$. The reverse transition occurs in the case of penetration of a spherically symmetric de Broglie wave through a potential polarization barrier. This effect is proved by agreement of experimental observations of the resonance spectra of e-capture by fullerenes (Fig.7) with our theoretical studies, see references in [6].

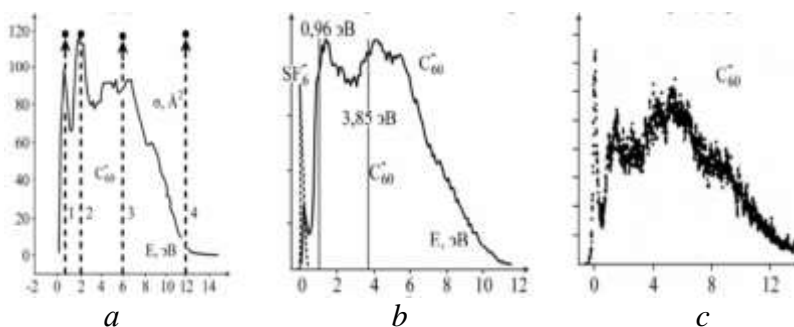


Fig.7. Comparison of theoretical calculations and experimentally measured e-capture cross sections in C_{60} depending on the electron energy:

a – for C_{60} [8]. Straight bold vertical lines mark the eigenenergies for cos-waves in C_{60} (with principal numbers $n = 1, 2, 3, 4$), calculated analytically taking into account the effect of C_{60} polarization forces on the stabilization of the fullerene endion;

b – relative electron attachment cross sections to SF_6 and C_{60} for the C_{60} flow ($T = 673$ K) [9]. Straight vertical lines mark the eigenenergies for sin-waves in C_{60} with principal numbers $n=1, 2$, calculated analytically, taking into account the polarization of C_{60} on stabilization of the fullerene endion;

c – experimental curves of the effective yield of resonant e-capture by C_{60} [7].

Effect 30 – in quantum resonators (complex quantum particles) *are observed to be reflected* from polarization boundaries (Coulomb's barriers) (Fig.1). **Reflection** from atomic nuclei and protons of de Broglie's waves of weakly energetic electrons is proved by the absence of spectra of cos-waves in the hydrogen atom and hydrogen-like atoms. These provisions are proved by comparing analytical calculations of resonant energy spectra with numerous experiments, both with fullerenes (Fig.7) and with hydrogen-like atoms, the spectra of which are described by the Schrödinger's equation [5,29].

Effect 31 – when a free electron (de Broglie plane wave) is captured by *hollow* resonators, for example, fullerenes or nanotubes, a **transformation (partial collapse)** of the electron ψ -function occurs. When a previously free electron is captured, its ψ -function transforms from a plane wave into a spherical or cylindrical ψ -function. As a result, in a hollow quantum resonator with a trapped electron, a complex cumulative-dissipative structure pulsating in time is formed with vibrational states formed due to the presence in the system of a self-consistent electric field, which is constantly present in all quantum systems with charged particles and their energy resonance spectra. All the electrons in these systems pulsate as de Broglie waves inside this electron trap. Quantum traps with trapped wave particles can emit electromagnetic waves or release quantum particles after a certain time into the environment surrounding the trap. The characteristic times of partial collapse ψ - a function of a quantum particle in a polarizable quantum resonator can be estimated from $(R+r_{\text{ind}})/v$. Here $R+r_{\text{ind}}$ is the distance to the polarization barrier, v is the velocity of the particle incident on the resonator. For electrons with an energy of about 1 eV incident on a fullerene with a characteristic size of $R+r_{\text{ind}} \approx 0.62$ nm, the pulsation (partial **collapse**) **time** is about 10^{-15} s. When an electron collapses into a proton (with e-capture of an electron by an atomic nucleus), the **collapse time** is about 10^{-23} s.

Effect 32 – Our analysis of numerous experimental works on the study of collapse or e-capture in [5] allows us to state the following:

1) the phenomena of cumulation and dissipation are not limited by atomism, as stated by E.I. Zababakhin in [30];

2) in [5], we were the first to propose a general mechanism for the collapse (cumulation and transformation) of plane de Broglie waves of electrons into spherical waves in fullerenes, atoms and into protons during the e-capture into atomic nuclei. These three types of spherical e-cumulation are accompanied by corresponding collapses of the electric field of the polarizing fullerene and electron at sizes 10^{-9} m; in the atomic region at sizes $\sim 10^{-10}$ m and similarly in the proton region in the atomic nucleus with the transformation of this proton into a neutron at sizes $\sim 10^{-15}$ m;

3) the probability of finding an electron as a stationary de Broglie wave in the center of hydrogen-like atoms is zero. This suggests that stationary de Broglie waves of an electron in hydrogen-like atoms, like ordinary standing waves, are reflected from a proton or atomic nucleus. And this happens despite the difference between the stationary ψ -function of the de Broglie wave of the electron at the center of the quantum resonator from zero in the ground state [29];

4) in the case of the collapse of plane de Broglie waves of electrons in hollow quantum resonators, where there are no quantum particles in the center of the resonator (as in the case of polarization capture of electrons by fullerenes (C_{60} , C_{70}), Fig.1), unlimited cumulation of $\psi_{n-1/2-}$ is observed functions of de Broglie waves of electrons to the center of a quantum resonator (Fig.6b) and limited cumulation of the probability of finding a particle-wave in the center of a hollow quantum resonator (Fig.6a);

5) as confirmed experimentally, the collapse of de Broglie waves of electrons, according to the virial theorem, is accompanied by the release of half of the potential energy of the electron, in the case of an atom, in the form of electromagnetic radiation (for a hydrogen atom with energy = 13.61 eV, taking into account the entire cascade of transitions to the lower energy level);

6) during e-capture by an atomic nucleus and neutronization of one proton, according to the virial theorem for quantum systems [31], one should expect radiation, for example, in the form of a neutrino (with energy ~ 0.85 MeV, see for more details [5]). When a proton is neutronized and its charge is compensated by an electron, the mass and potential energy of the electron is transformed into the mass of a neutron. According to the fundamentals of quantum mechanics, a light electron cannot be in an atomic nucleus, much less in the region or inside a proton;

7) penetration of an electron into an atomic nucleus and then into a proton is possible only with a dynamic bicyclonic mass exchange [5] between an electron captured by the atomic nucleus and a proton into which this electron is introduced, gaining mass in the process of e-capture. As experiments show, such a bicyclonic mass exchange between an electron and an atomic nucleus is possible in the case of aluminum $Z = 13$, i.e. at sizes $\sim 4 \cdot 10^{-12}$ m, which can be estimated according to $r_1 = a_0/Z = 0.53/Z$ [10^{-10} m] – the first Bohr orbit of the de Broglie wave of an electron in a hydrogen-like atom with the charge of the atomic nucleus – Z [29];

8) the model of bicumulation of de Broglie waves of electrons into atomic nuclei proves the ineffectiveness of the debate about cold or hot nuclear fusion [5]. In nature, this synthesis is a structural synthesis, where temperature plays an insignificant role. Here, a significant role is played by pressure (density of the number of particles) and the processes of structural bicumulation of ψ -functions of de Broglie waves of electrons into protons and the probability of penetration of an electron, like a de Broglie wave, into a proton in the atomic nucleus. This is possible when the proton is polarized to the captured electron in the form of a mass jet, increasing the effective mass of the electron collapsing into the proton;

9) neutron instability in the meso world (~ 15 min, which is very stable for the femto world, the next stable state is a million times less) suggests the existence of cumulative-dissipative structures based on neutrinos and antineutrinos, taking part in the resonant collisional destruction of neutrons.

10) gravity leads to the accumulation of enormous mass into galaxies, quantum stars and black holes. This occurs as a result of the processes of neutronization of matter and is accompanied by the collapse of de Broglie waves of electrons from sizes 10^{-10} m to 10^{-15} m. In this case, the characteristic volume of a quantum star changes by a factor of 10^{15} . The gravitational energy released in this case is hundreds of times higher than the electrical energy [5]. In complex cumulative-dissipative interference of Coulomb, gravitational and centrifugal forces occurs in cumulative-dissipative structures with sizes from 10^{-15} to 10^{27} m.

Conclusion

We have obtained original analytical results on the generalization of the general model of structural stationary and dynamic limited and unlimited cumulation of de Broglie waves of electrons and electric field for various types of e-capture and neutronization of matter on the characteristic sizes of cumulative-dissipative structures from 10^{-15} m to 10^{26} m [5]. When electrons are captured by fullerenes, the partial collapse of the ψ -function of electrons obeys the general laws of collapse. The discoveries and classification of quantum-size effects of nanometer sizes presented by us will undoubtedly find their application in nanotechnologies for their application in strengthening the world's human potential.

Quantum polarization resonators for de Broglie waves of electrons and self-forming negative endoions (with endo-electrons inside them) based on fullerenes and carbon nanotubes have enormous strength (exceeding the strength of diamond). These strength properties make it possible (without destroying endoions) to move them in electric fields, to study such structures and determine in experiments and theories: the lifetime of negative endoions of fullerenes with electrons inside, the resonance energy of trapped electrons [7-9], the profile of the probability of finding endoelectrons in the cavity fullerene [5], etc. This can be done without significantly modifying their parameters, given that the polarization forces that hold the electron in the endoion are significantly greater than the forces acting on the entire endoion. The knowledge gained from studying the polarization forces between charged quantum particles and spherically symmetric structures with an electron gas on their surface is easily transferred to their interactions with cylindrically symmetric quantum resonators - nanotubes [5] and other systems of nucleons with electrons. Such studies carried out in experiments [7-9] allowed us to:

- 1) describe in time the process of transformation (limited and unlimited collapse, or reduction according to von Neumann) of the ψ -function of the de Broglie wave of a free electron when it is captured by a spherically symmetric hollow polarization barrier, see Fig.1, for $C_{60,70}$;
- 2) prove that in quantum mechanics there are analogues of cumulative classical wave processes;
- 3) prove analytically and on the basis of these experimental observations [7-9] that Dirac's requirement that the ψ -function is limited for de Broglie waves everywhere is excessive and can be replaced by the requirement that the probability of finding a particle in any volume is limited and the presence of quantum

reflection (rebound) de Broglie waves of electrons with low kinetic energy from atomic nuclei and protons.

Based on experiments [7-9], we have proven, using the energy spectra of captured de Broglie waves of electrons, that in nature it is possible to realize half a de Broglie wave in hollow quantum spherically and cylindrically symmetric resonators (a wave with one antinode and two nodes).

The unlimited cumulation of de Broglie waves cannot be described within the framework of the old (Bohr) theory, which involves the orbital model of the hydrogen atom (cos waves are impossible in it). In the Bohr model, electrons move in orbits rather than converging as waves towards the center of a quantum hollow resonator. The author in the textbook [5] provides all the irrefutable evidence of his statement that "Cumulative quantum mechanics is a logical consequence of Schrödinger's new (wave) quantum mechanics, capable of describing all known cumulative and dissipative phenomena and structures in hollow quantum resonators," both with potential wells and with potential polarization barriers. We have considered the process in time of unlimited and limited collapse and modification of the ψ -function of quantum particles during their capture in hollow spherically or cylindrically symmetric quantum resonators (Fig. 6). It has been shown that phenomena in quantum (wave) mechanics have their analogues in conventional (wave) hydrodynamics of liquids and gases [5].

As a result of the research, the author discovered and classified more than 32 polarization quantum-size effects. This review proves that only by applying the fundamentals of cumulative quantum mechanics can one explain and classify all quantum-size effects discovered by the author of the textbook [5]. These quantum size effects led to the discovery of the principles of physical doping and the classification of doping into physical and chemical doping. During physical alloying, modification of the properties of a composite is carried out using nano- and microstructures of a foreign material. In this case, fractions of foreign material are not introduced into the crystal lattice. They only decorate the nano- or microcrystals of the supporting material in the composite. Physical doping promotes strengthening of composites by Coulomb compression of physically doped materials, increasing the luminescent properties of phosphors, increasing conductivity up to 10^{10} times and other properties due to quantum-size effects.

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