

# Persistence, extinction and blowup in a generalized logistic model with impulses and regional evolution\*

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**Abstract.** To explore the impacts of regional evolution and impulses on the persistence or extinction of species, a generalized logistic model with impulses in an evolving domain is proposed and researched. Firstly, the ecological reproduction index, which is regarded as a threshold value, is introduced and characterized. Secondly, in the case of monotone or non-monotone impulsive function, the asymptotic behavior of population is fully investigated and the sufficient conditions for the solution to persist, be extinct or blow up are given. Finally, numerical simulations results indicate that whatever impulse is, larger periodic evolution rates are more favorable for species. However, impulsive harvesting has a negative impact on persistence of species, while birth pulse admits a positive impact and even results in blowup.

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## 1 Introduction

Mathematical models have been extensively employed as credible models to explore the development of theories of spaces ecology for quite some time [2, 8, 21, 34], in which the core problem is how the distribution and existence of species are influenced by growth, death [12], diffusion [24] and so on. When the population dynamics is composed of growth, diffusion, continuous components as well as discrete components, the spread and sustainable existence of the population are not apparently characterized by the classical logistic models, and for such a case, the impulsive reaction diffusion models offer a more reasonable description for these hybrid systems [20]. Besides, pulse perturbation, such as Tang and Chen in [28] proposed pesticide application, Kot and Lewis et al. in [17] introduced species invasion and the release of natural enemies at some fixed points, can also affect persistence or extinction of species, so the understanding of the role of impulses is of extreme importance. An ordinary differential equation with birth pulse and impulsive harvesting was investigated in [15]; a nonlocal dispersal model with a birth pulse was researched in [30]; the impulsive reaction-diffusion predator-prey systems was set up in [22] to study the population dynamics; and time delayed equations with impulses were proposed in [4, 11]. Taking these into account, partial differential reaction-diffusion equations

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with impulses play a vital role in population dynamics and in such various forms of impulses, population growth in most cases is accomplished by the birth pulse of new individuals, and impulsive harvesting also affects the density of population, so we introduce here a generalized logistic equation with birth pulse or impulsive harvesting to research the persistence, extinction and blowup of species.

When researchers set up reaction-diffusion models with or without impulses to study space ecology, the domain considered in most cases is fixed, while it turns out that regional variations can have an effect on species too. For example, in 1952, Turing [29] first found pattern generation in a fixed domain, and Turing patterns in a growing domain were observed and studied. Taking this fact into consideration, in 1995, the changes in the stripe pattern of the angelfish *Pomacanthus* observed by Kondo and Asia [18] were more consistent with the actual observed pattern, which means regional growth plays an important role in model generation. What's more, as in [10], the size of habits will change with abiotic factors such as temperature, humidity and soil environment, which also exert an effect on population dynamics; please refer to, for example, [13, 25] and references therein. To describe the effects of regional expansion on population dynamics, some shifting domains are gradually introduced in mathematical models. For example, Crampin et al. in [6, 7] discussed a reaction-diffusion pattern on a growing domain; Jiang et.al [14] established a diffusive logistic equation on a periodically evolving domain and found that domain evolution rate  $\rho(t)$  has impacts on persistence of species, that is: if  $\rho^{-2}(t) < 1$ , the periodical domain evolution has a positive effect on population, otherwise, it has a negative effect on survival of species. The region variations mentioned above are already known, however, more complex free boundary problems, which mean the shifting boundaries of habit are unknown, are proposed. For example, Kawai and Yamada in [19] proposed a free boundary problem and characterized spreading phenomena; Du and Ni in [9] analysed free boundary problems with nonlocal diffusion.

Based on the above, we introduce an impulsive reaction-diffusion equation in a periodically evolving domain to consider the impacts of evolving domain and impulses on persistence, extinction and blowup of species. With the condition of Dirichlet boundary, a one-dimensional generalized logistic model with impulses is as follows:

$$\begin{cases} u_t = du_{xx} + \frac{r(t)u(K-u)}{K+Vu}, & t \in ((nT)^+, (n+1)T], \ x \in (0, l), \\ u(t, 0) = u(t, l) = 0, & t > 0, \\ u(0, x) = u_0(x), & x \in [0, l], \\ u((nT)^+, x) = g(u(nT, x)), & x \in (0, l). \end{cases} \quad (1.1)$$

All meanings of variables and parameters in model (1.1) are given below:

- $u(t, x)$  expresses the density of species at time  $t$  and space  $x$ ;
- $d(> 0)$  is the diffusion rate;
- $r(t)(> 0)$  means intrinsic growth rate of population;
- $K(> 0)$  denotes the environmental capacity;
- $V(-1 < V < 0)$  is a negative constant;
- $g(u)$  is the impulsive function. If  $0 < g(u)/u < 1$ , then  $g(u)$  represents impulsive harvesting function, if  $g(u)/u > 1$ , then  $g(u)$  denotes birth pulse function.

Given that the evolution of domain is isotropic, similar to the method [26] used, we now convert problem (1.1) into problem (1.2) in a fixed domain  $(0, l_0)$

$$\begin{cases} v_t = \frac{d}{\rho^2(t)} v_{yy} - \frac{\dot{\rho}(t)}{\rho(t)} v + \frac{r(t)v(K-v)}{K+Vv}, & t \in ((nT)^+, (n+1)T], \ y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v(0, y) = v_0(y), & y \in [0, l_0], \\ v((nT)^+, y) = g(v(nT, y)), & y \in (0, l_0), \end{cases} \quad (1.2)$$

where  $\rho(t)$  is the evolution rate, and  $T$  represents one period, satisfying  $\rho(t) = \rho(t + T)$ .

Assuming that an impulse occurs at every time  $t = nT$  within the successive stage of growth and disperse. Let's make the following assumption for  $g(u)$  ( $u \geq 0$ ).

**Hypothesis 1.1**  $g(u)$  is a continuous and once differentiable function for  $u \geq 0$ ,  $g(u) > 0$  and  $g(u)/u$  is decreasing monotonically in respect to  $u$  for  $u > 0$ ; while for  $u = 0$ ,  $g(0) = 0$ ,  $g'(0) > 0$ .

Here we introduce three commonly used impulsive functions, and all three functions satisfy the above-mentioned Hypothesis 1.1.

Firstly, the Beverton-Holt function [5]

$$g(u) = \frac{mu}{a+u} \quad (m, a > 0)$$

applies when there is competition between individuals who need to reproduce. Secondly, the Ricker function [27]

$$g(u) = ue^{r-bu} \quad (r, b > 0)$$

applies to more scramble competition between adults who attack the young within every unit of resource in the process of reproduction, as was shown in [20]. Finally we introduce linear impulse function

$$g(u) = cu \quad (c > 0).$$

If  $c > 1$ , then  $g(u)$  represents birth pulse function, if  $0 < c < 1$ , then  $g(u)$  represents impulsive harvesting function.

On condition that  $g(v) = v$  ( $c = 1$ ), i.e.  $g'(0) = 1$ , which means there is no impulse. If  $V = 0$ , (1.2) is transformed into the classical logistic model in an evolving domain and has been studied by Jiang and Wang in [14]. When  $-1 < V < 0$ , (1.2) is the generalized logistic model and the threshold value utilized to determine the persistence, extinction and blowup of species is calculated as [32]

$$D = \frac{\int_0^T r(t)dt}{\lambda_1 \int_0^T \rho^{-2}(t)dt}. \quad (1.3)$$

If  $d < D$ , then  $v$  decays to 0 and species vanishes; If  $d > D$  and initial value is small, then  $v$  stabilizes to a positive steady state, and species persists; if  $d > D$  and initial value is big, then blowup takes place.

In comparison with the aforementioned problem without impulse, we are interested in two essential aspects: On account of the presence of impulse, whether a new threshold value of problem (1.2) to affect the persistence, extinction and blowup of species will be generated? and what positive or negative impacts do evolution rate and impulses have on the longtime behaviors of population dynamics? The ecological reproduction index, which is related to the impulsive function  $g(u)$  and evolution rate  $\rho(t)$ , is introduced in the following section for the sake of dealing with the first issue. Sections 3 and 4 deal with the sufficient conditions for persistence, extinction and blowup of species. Numerical simulations and ecological explanations are presented in Section 5.

## 2 The ecological reproduction index

Widely,  $\mathcal{R}_0$ , regarded as the basic reproduction number, is a key indicator for disease transmission, seen in [1, 16, 33], and references therein. Additionally,  $\mathcal{R}_0$ , seen as the ecological reproduction index, is a threshold value and can be employed to describe the asymptotic behavior of population in ecology and demography. Compared with (1.3), in this section, a threshold value related to impulse and evolution rate is introduced and investigated.

Linearizing problem (1.2) at  $v^* = 0$ , which is an equilibrium solution of periodic problem (1.2), we obtain

$$\begin{cases} v_t = \frac{d}{\rho^2(t)} v_{yy} + [r(t) - \frac{\dot{\rho}(t)}{\rho(t)}]v, & t \in ((nT)^+, (n+1)T], \ y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v((nT)^+, y) = g'(0)v(nT, y), & y \in (0, l_0), \end{cases} \quad (2.1)$$

and take the following periodic eigenvalue problem into account

$$\begin{cases} \phi_t = \frac{d}{\rho^2(t)} \phi_{yy} + [\frac{r(t) + |\ln g'(0)|/T}{\mathcal{R}_0} - \frac{\dot{\rho}(t)}{\rho(t)}] \phi - \frac{|\ln g'(0)|}{T} \phi, & t \in ((nT)^+, (n+1)T], \ y \in (0, l_0), \\ \phi(t, 0) = \phi(t, l_0) = 0, & t > 0, \\ \phi(0, y) = \phi(T, y), & y \in [0, l_0], \\ \phi((nT)^+, y) = g'(0)\phi(nT, y), & y \in (0, l_0). \end{cases} \quad (2.2)$$

The eigenvalue problem is written in this form to ensure the positivity of  $\mathcal{R}_0$ , which can be seen in (2.3). Similar as [23] in a growing domain, [31] in a fixed domain and [25] in a periodic evolving domain, the principal eigenvalue  $\mathcal{R}_0$  of problem (2.2) is regarded as the ecological reproduction index, and a corresponding eigenfunction  $\phi(t, y)$  is positive. The explicit expression of  $\mathcal{R}_0$  will be given in the following.

**Theorem 2.1** *Assume that the environment changes periodically with respect to  $t$ . The ecological reproduction index of problem (1.2) can be precisely expressed as*

$$\mathcal{R}_0 = \frac{\int_0^T r(t)dt + |\ln g'(0)|}{d\lambda_1 \int_0^T \rho^{-2}(t)dt - \ln g'(0) + |\ln g'(0)|}, \quad (2.3)$$

where  $\lambda_1 (> 0)$  is the principal eigenvalue of  $-\Delta$  in  $(0, l_0)$  that meets the requirement of homogeneous Dirichlet boundary condition at  $x = 0$  and  $x = l_0$ .

**Proof:** Let

$$\phi(t, y) = \alpha(t)\psi(y),$$

where  $\alpha(t)$  is to be determined,  $\psi(y)$  is the corresponding eigenfunction of  $\lambda_1$ , which satisfies the eigenvalue problem

$$\begin{cases} -\psi''(y) = \lambda_1 \psi(y), & y \in (0, l_0), \\ \psi(0) = \psi(l_0) = 0. \end{cases} \quad (2.4)$$

Substituting  $\phi(t, y) = \alpha(t)\psi(y)$  into the reaction diffusion equation in (2.2), and then integrating both sides from  $0^+$  to  $t$ , yield

$$\alpha(t) = ce^{\int_0^t -\frac{d\lambda_1}{\rho^2(\tau)} + (\frac{r(\tau) + |\ln g'(0)|/T}{\mathcal{R}_0} - \frac{\dot{\rho}(\tau)}{\rho(\tau)})d\tau - \frac{|\ln g'(0)|}{T}t}, \quad (2.5)$$

where  $c = \alpha(0^+)$ . Since

$$\alpha(0^+) = g'(0)\alpha(0),$$

taking  $t = T$  at (2.4), we have

$$\begin{aligned}\alpha(T) &= \alpha(0^+)e^{\int_0^T -\frac{d\lambda_1}{\rho^2(t)} + \frac{r(t) + |\ln g'(0)|/T}{\mathcal{R}_0} dt - |\ln g'(0)|} \\ &= g'(0)\alpha(0)e^{\int_0^T -\frac{d\lambda_1}{\rho^2(t)} + \frac{r(t) + |\ln g'(0)|/T}{\mathcal{R}_0} dt - |\ln g'(0)|}.\end{aligned}$$

On account of  $\alpha(T) = \alpha(0)$ , we can eventually figure out that

$$\mathcal{R}_0 = \frac{\int_0^T r(t)dt + |\ln g'(0)|}{d\lambda_1 \int_0^T \rho^{-2}(t)dt - \ln g'(0) + |\ln g'(0)|} (> 0).$$

□

### 3 The case of monotone impulsive function

In this section, we consider the case where impulsive function  $g(u)$  is nondecreasing, and then take more general case of  $g(u)$  into consideration in the next section.

**Hypothesis 3.1**  $g(u)$  is nondecreasing for  $u \geq 0$ .

**Example 3.1** *Beverton-Holt function:*  $g(u) = mu/(a + u)$  ( $m, a > 0$ ).

In order to research the impacts of the basic reproduction index  $\mathcal{R}_0$  on population dynamics, we first make the following Hypothesis 3.2 about monotonic impulsive function  $g(u)$ .

**Hypothesis 3.2** *There exist positive constants  $D, \delta < \sigma$ , and  $\nu > 1$ , such that  $g(u) \geq g'(0)u - Du^\nu$  for  $0 \leq u \leq \delta$ .*

**Theorem 3.1** *Assume that Hypothesis 1.1 and  $\mathcal{R}_0 < 1$  hold. For small initial value  $v_0(y)$  and  $-1 < V < 0$ , the solution of problem (1.2) satisfies  $\lim_{t \rightarrow \infty} v(t, y) = 0$  uniformly for  $y \in [0, l_0]$ .*

**Proof:** Constructing

$$\tilde{v}(t, y) = Me^{(1-1/\mathcal{R}_0) \int_0^t r(\tau) + |\ln g'(0)|/T d\tau} \phi(t, y),$$

where  $\phi(t, y) (\leq 1)$  is a positive eigenfunction of problem (2.2) corresponding to the reproduction index  $\mathcal{R}_0$ , and  $M$  is to be chosen later. To all appearances,  $\tilde{v}(t, y)$  is a solution of the following linear problem

$$\begin{cases} v_t = \frac{d}{\rho^2(t)} v_{yy} + [r(t) - \frac{\dot{\rho}(t)}{\rho(t)}]v, & t \in ((nT)^+, (n+1)T], \ y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v(0, y) = M\phi(0, y), & y \in (0, l_0), \\ v((nT)^+, y) = g'(0)v(nT, y), & y \in (0, l_0). \end{cases} \quad (3.1)$$

It is easy to see that

$$\tilde{v}_t - \frac{d}{\rho^2(t)} \tilde{v}_{yy} + \frac{\dot{\rho}(t)}{\rho(t)} \tilde{v} - \frac{r(t)\tilde{v}(K - \tilde{v})}{K + V\tilde{v}} = r(t)\tilde{v}(V + 1)\left(\frac{1}{K/\tilde{v} + V}\right).$$

Due to  $0 < \tilde{v}(t, y) \leq M$ , we obtain

$$\frac{K}{\tilde{v}} + V \geq \frac{K}{M} + V.$$

If  $0 < M \leq K$ , combined with  $-1 < V < 0$ , then

$$\frac{K}{\tilde{v}} + V \geq \frac{K}{M} + V \geq 1 + V > 0,$$

so

$$\tilde{v}_t - \frac{d}{\rho^2(t)} \tilde{v}_{yy} + \frac{\dot{\rho}(t)}{\rho(t)} \tilde{v} - \frac{r(t) \tilde{v}(K - \tilde{v})}{K + V \tilde{v}} = r(t) \tilde{v}(V + 1) \left( \frac{1}{K/\tilde{v} + V} \right) > 0.$$

Recalling that  $g(v) \leq g'(0)v$  by the Hypothesis 1.1, we have  $\tilde{v}((nT)^+, y) = g'(0)\tilde{v}(nT, y) \geq g(\tilde{v}(nT, y))$ .

We now choose  $M = K$ , if the initial value  $v_0(y) \leq K\phi(0, y)$ , we acknowledge that  $\tilde{v}(t, y)$  is an upper solution of problem (1.2), that means

$$v(t, y) \leq \tilde{v}(t, y), \quad t > 0, \quad y \in [0, l_0].$$

Consequently, by the fact that  $\lim_{t \rightarrow \infty} \tilde{v}(t, y) = 0$  for  $\mathcal{R}_0 < 1$ , it is clear to see that  $\lim_{t \rightarrow \infty} v(t, y) = 0$  uniformly for  $y \in [0, l_0]$ .  $\square$

In order to explore the asymptotic behavior of solution to problem (1.2) in the case of  $\mathcal{R}_0 > 1$ , the following periodically auxiliary problem (3.2) is first given, then the relationship between solutions of periodic problem (3.2) and initial boundary problem (1.2) are investigated.

$$\begin{cases} v_t = \frac{d}{\rho^2(t)} v_{yy} - \frac{\dot{\rho}(t)}{\rho(t)} v + \frac{r(t)v(K-v)}{K+Vv}, & t \in ((nT)^+, (n+1)T], \quad y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v(0, y) = v(T, y), & y \in [0, l_0], \\ v((nT)^+, y) = g(v(nT, y)), & y \in (0, l_0). \end{cases} \quad (3.2)$$

**Theorem 3.2** Assume that impulsive function  $g(u)$  satisfied Hypotheses 1.1, 3.1, 3.2 and  $\mathcal{R}_0 > 1$ . For  $-1/(1+M) < V < 0$ , the auxiliary periodic problem (3.2) admits a unique positive periodic solution, where

$$M = (g'(0)e^{\int_0^T r(t)dt} - 1)e^{\int_0^T r(t)dt} \rho^{-m} / \int_0^T \frac{r(t)e^{\int_0^t r(s)ds}}{\rho(t)} dt.$$

**Proof:** We first construct an upper solution  $\tilde{v}(t, y)$  of perodic problem (3.2). Let  $\tilde{v}(t, y) = W(t)$ , we have  $K + VW(t) < K$ , and  $W(t)$  meets the equations set

$$\begin{cases} W_t = (r(t) - \frac{\dot{\rho}(t)}{\rho(t)})W(t) - \frac{r(t)}{K}(V+1)W^2(t), & t \in ((nT)^+, (n+1)T], \\ W(t) = W(t+T), & t > 0, \\ W((nT)^+) = g'(0)W(nT) \geq g(W(nT)), & n = 1, 2, \dots \end{cases} \quad (3.3)$$

thus

$$\begin{aligned} & \tilde{v}_t - \frac{d}{\rho^2(t)} \tilde{v}_{yy} + \frac{\dot{\rho}(t)}{\rho(t)} \tilde{v} - \frac{r(t) \tilde{v}(K - \tilde{v})}{K + V \tilde{v}} \\ &= W_t + \frac{\dot{\rho}(t)}{\rho(t)} W - r(t)W \left( 1 - \frac{(V+1)W}{K+VW} \right) \\ &= W_t + \frac{\dot{\rho}(t)}{\rho(t)} W - r(t)W + r(t) \frac{V+1}{(K+VW)} W^2. \end{aligned}$$

If  $K + VW > 0$ , then

$$\tilde{v}_t - \frac{d}{\rho^2(t)} \tilde{v}_{yy} + \frac{\dot{\rho}(t)}{\rho(t)} \tilde{v} - \frac{r(t) \tilde{v}(K - \tilde{v})}{K + V \tilde{v}} > W_t + \left( \frac{\dot{\rho}(t)}{\rho(t)} - r(t) \right) W + \frac{r(t)}{K} (V + 1) W^2 = 0.$$

Adding to the fact that  $\tilde{v}((nT)^+, y) - g(\tilde{v}(nT, y)) = W((nT)^+) - g(W(nT)) \geq 0$ , obviously,  $\tilde{v}(t, y) = W(t)$  is an upper solution of problem (3.2).

Next, let's take full advantage of equations in (3.3) to figure out  $W(t)$ .

Integrating both sides of the first equation of problem (3.3) from  $(nT)^+$  to  $t$ , yields

$$W(t) = \frac{e^{\int_0^t r(\tau) d\tau} W((nT)^+) \rho^{-1}(t)}{e^{n \int_0^T r(t) dt} + W((nT)^+) \frac{V+1}{K} \int_{nT}^t \frac{r(\tau) e^{\int_0^\tau r(s) ds}}{\rho(\tau)} d\tau}, \quad t \in ((nT)^+, (n+1)T], \quad (3.4)$$

therefore

$$\begin{aligned} W((n+1)T) &= \frac{e^{\int_0^{(n+1)T} r(t) dt} g'(0) W(nT)}{e^{n \int_0^T r(t) dt} + g'(0) W(nT) \frac{V+1}{K} \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt} \\ &= \frac{e^{\int_0^T r(t) dt} g'(0) W(nT)}{1 + g'(0) W(nT) \frac{V+1}{K} \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt}. \end{aligned}$$

On account of  $W((n+1)T) = W(nT)$ , we derive that

$$W(nT) = \frac{g'(0) e^{\int_0^T r(t) dt} - 1}{\frac{V+1}{K} g'(0) \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt}. \quad (3.5)$$

Since  $\mathcal{R}_0 > 1$ , then  $g'(0) e^{\int_0^T r(t) dt} > 1$ . It is observed that  $W(nT) > 0$  and

$$W(t) = \frac{K e^{\int_0^t r(\tau) d\tau} (g'(0) e^{\int_0^T r(t) dt} - 1) \rho^{-1}(t)}{(V+1) [(g'(0) e^{\int_0^T r(t) dt} - 1) \int_{nT}^t \frac{r(\tau) e^{\int_0^\tau r(s) ds}}{\rho(\tau)} d\tau + e^{n \int_0^T r(t) dt} \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt]}.$$

Finally, we should verify that  $K + VW > 0$ .

In fact, for the simplicity and transparency of the result, we denote

$$\rho^m = \min_{t \in ((nT)^+, (n+1)T]} \rho(t),$$

and

$$M = (g'(0) e^{\int_0^T r(t) dt} - 1) e^{\int_0^T r(t) dt} \rho^{-m} / \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt,$$

direct calculation gives that

$$\begin{aligned} W(t) &< \frac{K e^{\int_0^t r(\tau) d\tau} (g'(0) e^{\int_0^T r(t) dt} - 1) \rho^{-1}(t)}{(V+1) e^{n \int_0^T r(t) dt} \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt} \\ &< \frac{K (g'(0) e^{\int_0^T r(t) dt} - 1) e^{\int_0^T r(t) dt} \rho^{-1}(t)}{(V+1) \int_0^T \frac{r(t) e^{\int_0^t r(s) ds}}{\rho(t)} dt} \\ &< \frac{KM}{V+1}. \end{aligned}$$

Paying attention to the condition that

$$-1/(M+1) < V < 0,$$

it is easy to verify that  $W(t) < -K/V$  i.e.  $W < -K/V$  holds.

Next, we construct its lower solution. Let

$$\hat{v}(t, y) = \begin{cases} \varepsilon\phi(nT, y), & t = nT, \\ \varepsilon\frac{\rho_1}{g'(0)}\phi((nT)^+, y), & t = (nT)^+, \\ \varepsilon\frac{\rho_1}{g'(0)}e^{[G(t)-\delta](t-nT)}\phi(t, y), & t \in ((nT)^+, (n+1)T], \end{cases} \quad (3.6)$$

where  $G(t) = e^{-\frac{t}{T}} \int_0^t \frac{1}{T}(1 - 1/\mathcal{R}_0)r(t)e^{\frac{t}{T}}dt > 0$ ,  $\phi(t, y) (> 0)$  is an eigenfunction to problem (2.2) corresponding to  $\mathcal{R}_0$  and  $\varepsilon (> 0)$  is sufficiently small. Furthermore, we choose  $\delta = G((n+1)T)/2 (> 0)$  and  $\rho_1 = g'(0)e^{-T\frac{G((n+1)T)}{2}} (< g'(0))$ , such that  $\hat{v}(nT, y) = \hat{v}((n+1)T, y)$ . Since  $Te^{\frac{t}{T}}G(t) = \int_0^t (1 - 1/\mathcal{R}_0)r(t)e^{\frac{t}{T}}dt$ , then  $TG'(t) + G(t) = (1 - 1/\mathcal{R}_0)r(t)$ . For any  $t \in ((nT)^+, (n+1)T]$  and  $y \in (0, l_0)$ , we derive that

$$\begin{aligned} & \hat{v}_t - \frac{d}{\rho^2(t)}\hat{v}_{yy} + \frac{\rho'(t)}{\rho(t)}\hat{v} - \frac{r(t)\hat{v}(K-\hat{v})}{K+V\hat{v}} \\ &= \varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}[(G'(t)(t-nT) + G(t) - \delta)\phi \\ &+ \phi_t - \frac{d}{\rho^2(t)}\phi_{yy} + \frac{\rho'(t)}{\rho(t)}\phi - \frac{r(t)\phi(K-\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi)}{K+V\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi}] \\ &\leq \varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}[(TG'(t) + G(t))\phi - \delta\phi \\ &+ \phi_t - \frac{d}{\rho^2(t)}\phi_{yy} + \frac{\rho'(t)}{\rho(t)}\phi - \frac{r(t)\phi(K-\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi)}{K+V\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi}]. \end{aligned}$$

On account of

$$\begin{aligned} & (TG'(t) + G(t))\phi + \phi_t - \frac{d}{\rho^2(t)}\phi_{yy} + \frac{\rho'(t)}{\rho(t)}\phi \\ &= (1 - \frac{1}{\mathcal{R}_0})r(t)\phi + \frac{r(t)+|\ln g'(0)|/T}{\mathcal{R}_0} - \frac{|\ln g'(0)|}{T}\phi \\ &= r(t)\phi + \frac{|\ln g'(0)|/T}{\mathcal{R}_0}\phi - \frac{|\ln g'(0)|}{T}\phi \\ &\leq r(t)\phi, \end{aligned}$$

we have

$$\begin{aligned} & \hat{v}_t - \frac{d}{\rho^2(t)}\hat{v}_{yy} + \frac{\rho'(t)}{\rho(t)}\hat{v} - \frac{r(t)\hat{v}(K-\hat{v})}{K+V\hat{v}} \\ &\leq \varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi[(r(t)(1 - \frac{K-\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi}{K+V\varepsilon\frac{\rho_1}{g'(0)}e^{(G(t)-\delta)(t-nT)}\phi}) - \delta] \\ &< 0. \end{aligned}$$

Based on Hypothesis 3.2, for given positive constants  $D, \delta (< \sigma)$ ,  $\nu (> 1)$ , there exists sufficiently small  $\varepsilon (> 0)$ , such that

$$\begin{aligned} & \hat{v}((nT)^+, y) - g(\hat{v}(nT, y)) \\ &= \varepsilon\frac{\rho_1}{g'(0)}\phi((nT)^+, y) - g(\hat{v}(nT, y)) \\ &= \varepsilon\rho_1\phi(nT, y) - g(\hat{v}(nT, y)) \\ &\leq \varepsilon\rho_1\phi(nT, y) - g'(0)\varepsilon\phi(nT, y) + D(\varepsilon\phi(nT, y))^\nu \\ &= [(\rho_1 - g'(0)) + D(\varepsilon\phi(nT, y))^{\nu-1}]\varepsilon\phi(nT, y) \\ &< 0. \end{aligned}$$

thus,  $\hat{v}(t, y)$  is a lower solution of problem (3.2).

In the following, the upper solution  $\tilde{v}(t, y)$  and lower solution  $\hat{v}(t, y)$  mentioned above are employed to prove the existence of periodic solution to problem (3.2).

Denote  $f(v, t) = -\frac{\rho(t)}{\rho(t)}v + \frac{r(t)v(K-v)}{K+Vv}$ , we choose  $M^*$ , which is sufficiently large, such that  $F(v, t) = f(v, t) + M^*v$  is increasing with respect to  $v$ . In the following, initial value iteration  $\bar{v}^{(0)} = \tilde{v}$  and  $\underline{v}^{(0)} = \hat{v}$  are utilized to construct monotone iteration sequence  $\{\bar{v}^{(k)}\}$  and  $\{\underline{v}^{(k)}\}$  ( $k = 1, 2, \dots$ ).

$$\left\{ \begin{array}{l} \bar{v}_t^{(k)} - \frac{d}{\rho^2(t)}\bar{v}_{yy}^{(k)} + M^*\bar{v}^{(k)} = M^*\bar{v}^{(k-1)} - \frac{\dot{\rho}(t)}{\rho(t)}\bar{v}^{(k-1)} \\ \quad + \frac{r(t)\bar{v}^{(k-1)}(K-\bar{v}^{(k-1)})}{K+V\bar{v}^{(k-1)}}, \quad t \in ((nT)^+, (n+1)T], y \in (0, l_0), \\ \underline{v}_t^{(k)} - \frac{d}{\rho^2(t)}\underline{v}_{yy}^{(k)} + M^*\underline{v}^{(k)} = M^*\underline{v}^{(k-1)} - \frac{\dot{\rho}(t)}{\rho(t)}\underline{v}^{(k-1)} \\ \quad + \frac{r(t)\underline{v}^{(k-1)}(K-\underline{v}^{(k-1)})}{K+V\underline{v}^{(k-1)}}, \quad t \in ((nT)^+, (n+1)T], y \in (0, l_0), \\ \bar{v}^{(k)}(t, 0) = \bar{v}^{(k)}(t, l_0) = \underline{v}^{(k)}(t, 0) = \underline{v}^{(k)}(t, l_0) = 0, \quad t \in ((nT)^+, (n+1)T], \\ \bar{v}^{(k)}(0, y) = \bar{v}^{(k-1)}(T, y), \underline{v}^{(k)}(0, y) = \underline{v}^{(k-1)}(T, y), \quad y \in (0, l_0), \\ \bar{v}^{(k)}((nT)^+, y) = g(\bar{v}^{(k-1)}((n+1)T, y)), \quad y \in (0, l_0), \\ \underline{v}^{(k)}((nT)^+, y) = g(\underline{v}^{(k-1)}((n+1)T, y)), \quad y \in (0, l_0). \end{array} \right. \quad (3.7)$$

which yields

$$\hat{v} \leq \underline{v}^{(k)} \leq \underline{v}^{(k+1)} \leq \bar{v}^{(k+1)} \leq \bar{v}^{(k)} \leq \tilde{v}.$$

Next, it follows from the monotone bounded convergence theorem that the limits of sequences  $(\bar{v}^{(k)}, \underline{v}^{(k)})$  exist. Let

$$\lim_{m \rightarrow \infty} \bar{v}^{(k)} = \bar{V}^* \text{ and } \lim_{m \rightarrow \infty} \underline{v}^{(k)} = \underline{V}^*,$$

substituting into (3.7), we can see that  $\bar{V}^*$  and  $\underline{V}^*$  are two periodic solutions to periodic problem (3.2), satisfying

$$\hat{v} \leq \underline{v}^{(k)} \leq \underline{v}^{(k+1)} \leq \underline{V}^* \leq \bar{V}^* \leq \bar{v}^{(k+1)} \leq \bar{v}^{(k)} \leq \tilde{v}.$$

Ultimately, we prove the uniqueness of  $T$ -periodic solution. To proceed, we suppose that  $v_1$  and  $v_2$  are two solutions of problem (3.2) and define

$$S = \{s \in [0, 1], sv_1 \leq v_2, t \in [0, T], y \in [0, l_0]\}.$$

Explicitly,  $F(v, t)$  is nondecreasing and  $f(v, t)/v$  is non-increasing with respect to  $v$  in  $[0, \max_{[0, T] \times [0, l_0]} v_2]$ .

In the following, we prove the fact that  $1 \in S$ , otherwise, suppose that  $s_0 = \sup S < 1$ , then

$$\begin{aligned} & (v_2 - s_0v_1)_t - \frac{d}{\rho^2(t)}(v_2 - s_0v_1)_{yy} + M^*(v_2 - s_0v_1) \\ &= f(v_2, t) + M^*v_2 - s_0(f(v_1, t) + M^*v_1) \\ &\geq f(s_0v_1, t) + M^*s_0v_1 - s_0(f(v_1, t) + M^*v_1) \\ &\geq 0 \end{aligned}$$

for  $t \in (0^+, T]$ ,  $y \in (0, l_0)$ .

For  $y \in (0, l_0)$ ,  $v_2(0^+, y) - s_0v_1(0^+, y) = g(v_2(0, y)) - s_0g(v_1(0, y)) \geq 0$  is inferred by using Hypotheses 1.1 and 3.1, as well as for  $t > 0$ ,  $v_2(t, 0) - s_0v_1(t, 0) = v_2(t, l_0) - s_0v_1(t, l_0) = 0$ . Hence the strong maximum principle asserts that

$$v_2 - s_0v_1 > 0 \text{ or } v_2 - s_0v_1 \equiv 0, \text{ for } t \in (0^+, T] \text{ and } y \in (0, l_0).$$

(i) If  $v_2 - s_0v_1 > 0$  is true for  $t \in (0^+, T]$  and  $y \in (0, l_0)$ . Since  $v_1$  and  $v_2$  are two periodic solutions, we have  $v_1(0, y) = v_1(T, y)$  and  $v_2(0, y) = v_2(T, y)$ , which signifies  $v_2 - s_0v_1 > 0$

for  $t \in [0, T]$ ,  $y \in (0, l_0)$ . Owing to Hopf boundary lemma, with  $\frac{\partial}{\partial \eta} |_{y=0} (v_2 - s_0 v_1) > 0$ ,  $\frac{\partial}{\partial \eta} |_{y=l_0} (v_2 - s_0 v_1) < 0$  for  $t \in [0, T]$ , where  $\eta$  delegates outward unit normal vector, there holds a certain positive constant  $\varepsilon$  such that  $v_2 - s_0 v_1 > \varepsilon v_1$ . It follows that  $s_0 + \varepsilon \in S$ , which is not in accordance with the fact that  $s_0 = \sup S$ .

(ii) If  $v_2 - s_0 v_1 \equiv 0$  is correct for  $t \in (0^+, T]$  and  $y \in (0, l_0)$ . By means of  $v_t - \frac{d}{\rho^2(t)} v_{yy} = f(v, t)$ , there holds  $f(v_2, t) = s_0 f(v_1, t)$ . While  $f(v, t)/v$  is non-increasing with respect to  $v$ , which yields  $f(v_2, t) = f(s_0 v_1, t) > s_0 f(v_1, t)$ . Consequently this case does not make sense either.  $\square$

The existence and uniqueness of positive periodic solution  $V^*(t, y) (= \bar{V}^*(t, y) = \underline{V}^*(t, y))$  to problem (3.2) have been specifically demonstrated above. In the next, the relationships between solutions to problems (3.2) and (1.2) are characterized in the case of  $\mathcal{R}_0 > 1$ .

**Theorem 3.3** *Suppose that Hypotheses 1.1, 3.1, 3.2 and  $\mathcal{R}_0 > 1$  hold. For nonnegative and nontrivial small initial value  $v_0(y)$ , the solution of problem (1.2) yields*

$$\lim_{m \rightarrow \infty} v(t + mT, y) = V^*(t, y), \quad t \geq 0, y \in [0, l_0],$$

where  $V^*(t, y)$  is the unique solution to periodic problem (3.2).

**Proof: Step1**  $v_0(y) > 0$  holds for  $y \in (0, l_0)$ , otherwise, substituting  $t_0 > 0$  for  $t = 0$ , such that  $v_0(t_0, y) > 0$ . Taking notice of  $\phi_y(0, 0) > 0$ ,  $\phi_y(0, l_0) < 0$ , by using Hopf boundary lemma, a positive and sufficiently small  $\varepsilon$  can be selected to enable  $\varepsilon \phi(0, y) \leq v(0, y)$ . For such a given  $\varepsilon$ , we choose the upper solution  $\tilde{v}(t, y) = W(t)$  defined in (3.3) and the lower solution  $\hat{v}(t, y)$  defined in (3.6) satisfying

$$\hat{v}(0, y) \leq v(0, y) \leq \tilde{v}(0, y), \quad y \in [0, l_0]. \quad (3.8)$$

Hence,  $g(\hat{v}(0, y)) \leq g(v(0, y)) \leq g(\tilde{v}(0, y))$ , which is equivalent to  $\hat{v}(0^+, y) \leq v(0^+, y) \leq \tilde{v}(0^+, y)$ .

It follows from the comparison principle that  $\hat{v}(t, y) \leq v(t, y) \leq \tilde{v}(t, y)$  for  $t \in (0^+, T]$  and  $y \in [0, l_0]$ , and induction shows that

$$\hat{v}(t, y) \leq v(t, y) \leq \tilde{v}(t, y), \quad t = nT, \quad t \in [(nT)^+, (n+1)T], \quad y \in [0, l_0].$$

i.e.  $\underline{v}^{(0)}(t, y) \leq v(t, y) \leq \bar{v}^{(0)}(t, y)$ ,  $t \geq 0$ ,  $y \in [0, l_0]$ .

**Step2** Owing to the initial value iteration process in Theorem 3.2, we obtain

$$\begin{cases} \bar{v}^{(1)}((nT)^+, y) = g(\bar{v}^{(0)}((n+1)T, y)), & y \in (0, l_0), \\ \underline{v}^{(1)}((nT)^+, y) = g(\underline{v}^{(0)}((n+1)T, y)), & y \in (0, l_0), \\ \bar{v}^{(1)}(0, y) = \bar{v}^{(0)}(T, y), & y \in (0, l_0), \\ \underline{v}^{(1)}(0, y) = \underline{v}^{(0)}(T, y), & y \in (0, l_0). \end{cases} \quad (3.9)$$

There holds

$$\underline{v}^{(1)}(0, y) = \underline{v}^{(0)}(T, y) \leq v(T, y) \leq \bar{v}^{(0)}(T, y) = \bar{v}^{(1)}(0, y),$$

for  $t = 0, y \in [0, l_0]$ , and

$$\underline{v}^{(1)}(0^+, y) = g(\underline{v}^{(0)}(T, y)) \leq g(v(T, y)) = v(T^+, y) \leq g(\bar{v}^{(0)}(T, y)) = \bar{v}^{(1)}(0^+, y),$$

for  $t = 0^+, y \in [0, l_0]$ . Thus,  $\underline{v}^{(1)}(t, y) \leq v(t + T, y) \leq \bar{v}^{(1)}(t, y)$  holds for  $t \in (0^+, T], y \in [0, l_0]$ , by comparison argument, and induction asserts that  $\underline{v}^{(1)}(t, y) \leq v(t + T, y) \leq \bar{v}^{(1)}(t, y)$  for  $t \geq 0$  and  $y \in [0, l_0]$ .

**Step3** We conclude that for any  $m$ ,

$$\underline{v}^{(m)}(t, y) \leq v(t + mT, y) \leq \bar{v}^{(m)}(t, y), \quad t \geq 0, y \in [0, l_0].$$

In fact,  $m = 0$  is valid, as is seen in *Step1*, and  $m = 1$  is correct, which is presented by *Step2*. The above result can be obtained by iteration. This, together with the fact that  $\lim_{m \rightarrow \infty} \underline{v}^{(m)}(t, y) = \lim_{m \rightarrow \infty} \bar{v}^{(m)}(t, y) = V^*(t, y)$  in Theorem 3.2, where  $V^*(t, y)$  is the unique T-periodic solution of problem (3.2), yields the desired result.  $\square$

## 4 The case of non-monotone impulsive function

Compared with monotone impulsive function in Section 3, we will use the conclusion obtained above to discuss more general case where  $g(u)$  is non-monotonic, and the core point is to find two monotone functions to approach  $g(u)$  from above and below, respectively.

**Example 4.1** *Ricker function:*  $g(u) = ue^{r-bu}$  ( $r, b > 0$ ).

Motivated by work of Lewis and Li in [20], to study the dynamics of the population when the impulse function is non-monotonic, we first make the following assumption.

**Hypothesis 4.1** *There is a positive constant  $\sigma$  such that  $g(u)$  is nondecreasing with respect to  $u$  for  $0 \leq u \leq \sigma$ .*

Choosing  $d = 0$  in problem (3.2), now we take the following problem into consideration

$$\begin{cases} v_t = -\frac{\rho(t)}{\rho(t)}v + \frac{r(t)v(K-v)}{K+Vv}, & t \in ((nT)^+, (n+1)T], \\ v(0) = v(T), \\ v((nT)^+) = g^+(v(nT)), & y \in (0, l_0), \end{cases} \quad (4.1)$$

where  $g^+(u)$  is defined as

$$g^+(u) = \max_{v \in [0, u]} g(v).$$

We can not say that the solution of problem (4.1) is unique because the monotonicity of  $\frac{g^+(u)}{u}$  is unknown, but it can be obtained according to Theorem 3.2 that the above problem has a minimum positive solution  $v_{min}^+(t)$  for  $\mathcal{R}_0 > 1$ , then we define

$$\beta^+ = v_{min}^+(0), \quad g^-(u) = \min_{v \in [u, \beta^+]} g(v).$$

It is effortless to see from the definition that  $g^\pm(u)$  are nondecreasing for  $u \geq 0$ ,  $g^-(u) \leq g(u) \leq g^+(u)$  and  $g^\pm(0) = g'(0)$ , simultaneously there exists  $\sigma_0(> 0)$  such that  $g^\pm(u) = g(u)$  for  $\sigma_0 < \sigma$ .

In the following, we consider the auxiliary systems

$$\begin{cases} v_t = \frac{d}{\rho^2(t)}v_{yy} - \frac{\rho(t)}{\rho(t)}v + \frac{r(t)v(K-v)}{K+Vv}, & t \in ((nT)^+, (n+1)T], \quad y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v(0, y) = v_0(y), & y \in [0, l_0], \\ v((nT)^+, y) = g^+(v(nT, y)), & y \in (0, l_0) \end{cases} \quad (4.2)$$

and

$$\begin{cases} v_t = \frac{d}{\rho^2(t)} v_{yy} - \frac{\rho(t)}{\rho(t)} v + \frac{r(t)v(K-v)}{K+Vv}, & t \in ((nT)^+, (n+1)T], \quad y \in (0, l_0), \\ v(t, 0) = v(t, l_0) = 0, & t > 0, \\ v(0, y) = v_0(y), & y \in [0, l_0], \\ v((nT)^+, y) = g^-(v(nT, y)), & y \in (0, l_0). \end{cases} \quad (4.3)$$

Since  $g^-(u) \leq g(u) \leq g^+(u)$ , if  $0 \leq v_0(y) \leq \beta^+$ , then for any arbitrary solution  $v(t, y)$  of problem (1.2), the comparison principle gives that

$$0 \leq v^-(t, y) \leq v(t, y) \leq v^+(t, y) \leq v_{min}^+(t), \quad (4.4)$$

where  $v^-(t, y)$  and  $v^+(t, y)$  are solutions to problems (4.3) and (4.2), respectively.

The above two problems have the same ecological reproduction index  $\mathcal{R}_0$ , which is determined by Theorem 2.1, and the solutions of problems (4.2) and (4.3) have the same asymptotic behavior in accordance with Theorems 3.1, 3.2 and 3.3. Combining this with (4.4), we derive that if  $\mathcal{R}_0 < 1$ , the solution to problem (1.2) meets  $\lim_{t \rightarrow \infty} v(t, y) = 0$  for  $y \in [0, l_0]$ , and if  $\mathcal{R}_0 > 1$ , the solution yields

$$\underline{V}^+(t, y) \leq \liminf_{m \rightarrow \infty} v^+(t + mT, y) \leq \limsup_{m \rightarrow \infty} v^+(t + mT, y) \leq \bar{V}^+(t, y), \quad t \geq 0, \quad y \in [0, l_0],$$

where  $\underline{V}^+(t, y)$  and  $\bar{V}^+(t, y)$  are maximum and minimum positive  $T$ -periodic solutions of problem (4.2), respectively, with the initial value condition replaced by the periodic condition. Similarly,  $\bar{V}^-(t, y)$  and  $\underline{V}^-(t, y)$  are defined as the maximum and minimum positive  $T$ -periodic solutions to the corresponding  $T$ -periodic problem, which meets

$$\underline{V}^-(t, y) \leq \liminf_{m \rightarrow \infty} v^-(t + mT, y) \leq \limsup_{m \rightarrow \infty} v^-(t + mT, y) \leq \bar{V}^-(t, y), \quad t \geq 0, \quad y \in [0, l_0].$$

These two inequations, together with (4.4), yield

$$\underline{V}^-(t, y) \leq \liminf_{m \rightarrow \infty} v(t + mT, y) \leq \limsup_{m \rightarrow \infty} v(t + mT, y) \leq \bar{V}^+(t, y).$$

Based on the aforementioned analysis, the asymptotic behavior of solution to problem (1.2) in the non-monotonic case of the impulsive function is given below.

**Theorem 4.1** *Assume that Hypotheses 1.1, 3.1, 3.2, 4.1 hold, the following assertions are true:*

- (i) *If  $\mathcal{R}_0 < 1$ , the solution  $v(t, y)$  to problem (1.2) satisfies  $\lim_{t \rightarrow \infty} v(t, y) = 0$  uniformly for  $y \in [0, l_0]$ .*
- (ii) *If  $\mathcal{R}_0 > 1$ , for nonnegative and nontrivial initial value  $0 \leq v_0(y) \leq \beta^+$ , the solution  $v(t, y)$  to problem (1.2) admits*

$$\underline{V}^-(t, y) \leq \liminf_{m \rightarrow \infty} v(t + mT, y) \leq \limsup_{m \rightarrow \infty} v(t + mT, y) \leq \bar{V}^+(t, y)$$

*for  $t \geq 0, y \in [0, l_0]$ , where  $\bar{V}^+(t, y)$  and  $\underline{V}^-(t, y)$  are the maximum and minimum positive periodic solutions of the corresponding  $T$ -periodic solutions to problems (4.2) and (4.3), respectively.*

## 5 Numerical simulation and biological explanation

In this section, for small initial value  $v_0(y)$ , Beverton-Holt function and Ricker function are firstly chosen to research the impacts of evolving domain and impulses on dynamics of population. For these two functions mentioned above, their solutions have upper bounds ( $g(u) \leq m$ ,  $g(u) \leq \frac{e^{r-1}}{b}$ ), so that blowup is impossible in the usually case, while blowup occurs in generalized logistic model without impulse when the initial value is big and  $\mathcal{R}_0 > 1$ . So we further introduce impulsive function  $g(u) = cu (c > 0)$  and consider whether blowup occurs with impulses.

We consider the interval  $[0, l(t)] = [0, \rho(t)l_0]$ ,  $l_0 = \pi$ , which subsequently implies that  $\lambda_1 = (\pi/l_0)^2 = 1$ . Some parameters such as  $d = 1, K = 2.5, r = 1.2$  and  $V = -0.2$  in model (1.2) are chosen. The initial value is fixed as  $v_0(y) = 0.5 \sin x + 0.2 \sin 3x$ . Assuming that impulse occurs at each time  $T = 2$ , numerical simulations to Beverton-Holt function and Ricker function are presented below.

### 5.1 The impact of evolving domain

Small evolution rate  $\rho_1(t) = e^{-0.1(1-\cos \pi t)}$  and big evolution rate  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$  are chosen here to research the impacts of regional evolution on species survival.

We take impulsive function  $g(u)$  which is non-monotonic into consideration, such as Ricker function with  $r = 0.05$  and  $b = 1.2$ .

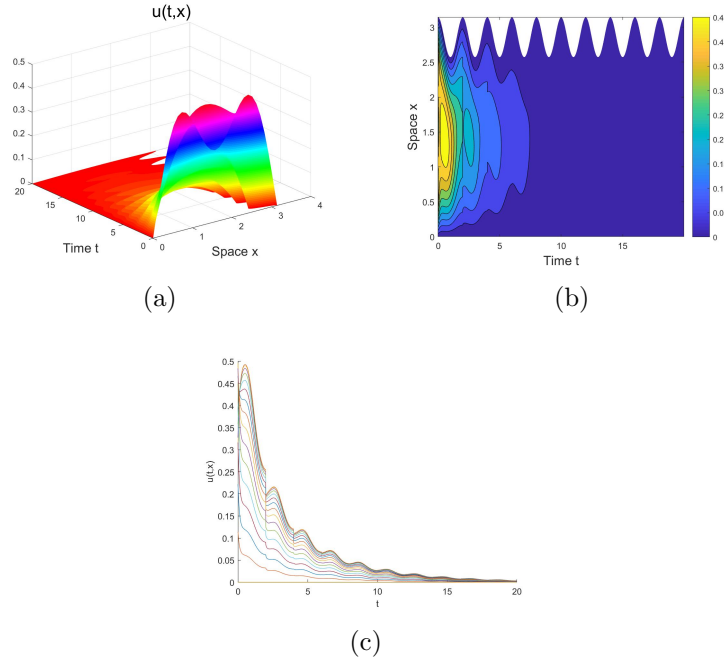


Figure 1: For a smaller evolution rate  $\rho_1(t) = e^{-0.1(1-\cos \pi t)}$ , we obtain  $\mathcal{R}_0 \approx 0.993 < 1$ . Graphs (a)-(c) show that  $u(t, x)$  gradually tends to zero with time  $t$ . It can be seen from Graph (c) that the impulse occurs at each time  $T = 2$ .

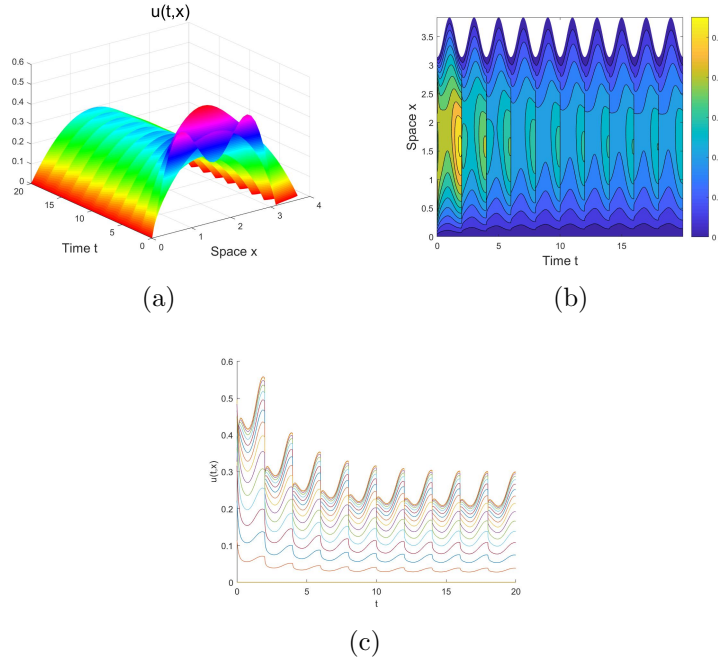


Figure 2: For bigger evolution rate  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$ ,  $\mathcal{R}_0 \approx 1.496 > 1$  is calculated. Graphs (a)-(c) characterize the population density stabilizes to a equilibrium state in a periodic evolving domain.

When we choose monotonic impulsive function  $g(u)$ , such as Beverton-Holt function, we can also find the similar asymptotic behavior, that is, the species will extinct with small evolution rate, and will persist with big evolution rate.

We finally draw a conclusion that no matter what impulsive harvesting function  $g(u)$  is, the population in a periodically evolving domain has similar dynamic behavior, that is, the population finally decays to zero under a smaller evolution rate while persists at a larger evolution rate. In other words, when impulsive harvesting occurs, the larger the evolution rate is, the more favorable for the persistence of species.

## 5.2 The impact of impulse

For the purpose of researching the biological effects produced by impulses in an evolving domain, the occurrence and non-occurrence of impulses, as well as the monotone or non-monotone impulsive functions when impulses take place, are considered. Here a table 1 is made to classify the impacts of impulse.

Table 1. Classification of different situations caused by impulsive harvesting

Small evolution rate	No impulse	Figure 3
	Monotone impulse	Figure 4
	Non-monotone impulse	Figure 1
Big evolution rate	No impulse	Figure 5
	Monotone impulse	Figure 6
	Non-monotone impulse	Figure 7

First of all, under a small evolution rate, we consider the case where there is no impulse.

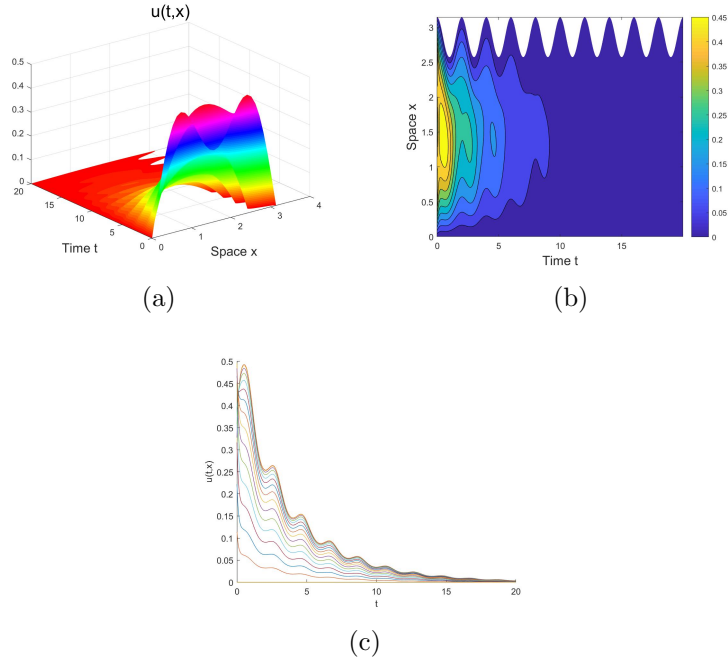


Figure 3: A numerical simulation to the graph of  $u(t, x)$  without impulses by choosing  $\rho_1(t) = e^{-0.1(1-\cos \pi t)}$ .  $\mathcal{R}_0 \approx 0.973 < 1$ , Graphs (a)-(c) depict that the species finally tends to vanish.

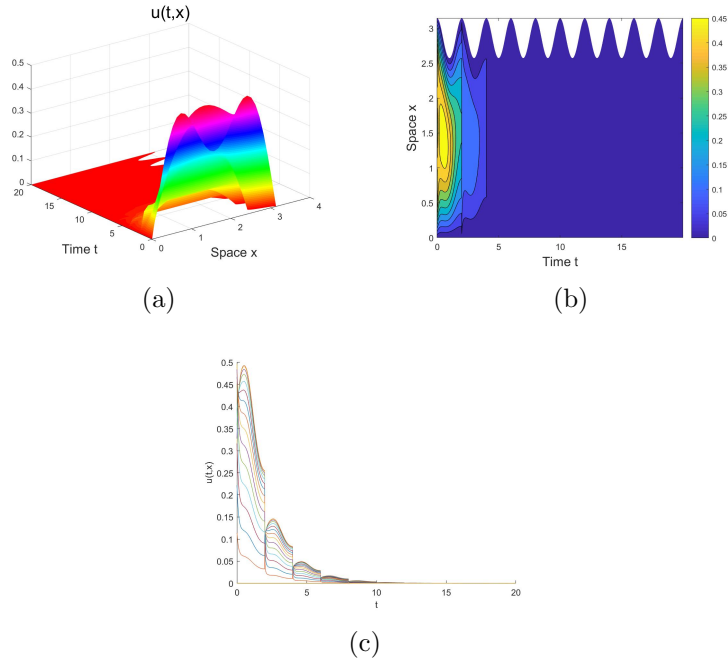


Figure 4: For the smaller evolution rate, Beverton-Holt impulse with  $m = 8, a = 10$ , we acquire  $\mathcal{R}_0 \approx 0.892 < 1$ . Graphs (a)-(c) describe that the population density decays to zero. It can be seen from Graph (c) that the impulse arises at each time  $T = 2$ .

Comparing Fig 3 with Figs 4 and 1, it is clear that regardless of monotone or non-monotone functions  $g(u)$  are selected, Beverton-Holt and Ricker impulses all accelerate species extinction at a small evolution rate.

Secondly, on the basis of big evolution rate, we consider the situation where there is no impulse.

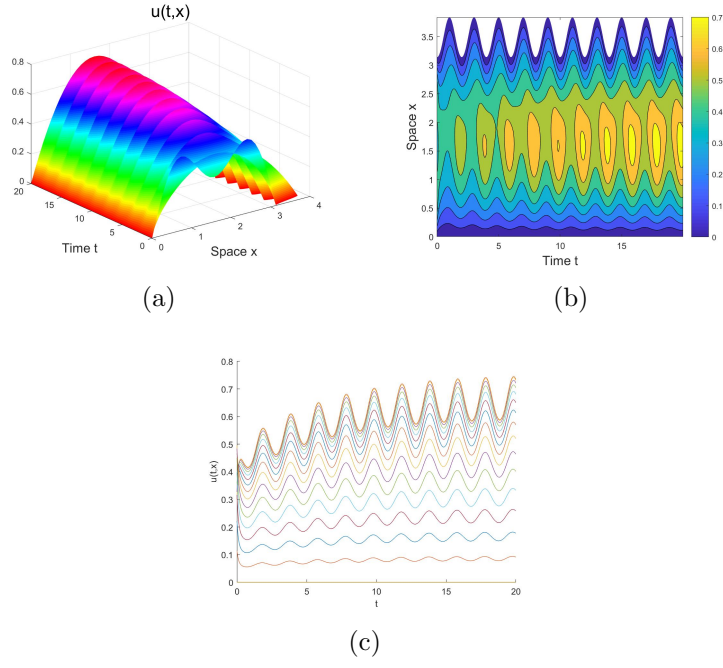


Figure 5: Without impulses, a numerical approximation to the graph of  $u(t, x)$ . By choosing  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$ ,  $\mathcal{R}_0 \approx 1.451 > 1$ . Graphs (a)-(c) show that the population gradually tends to a steady state.

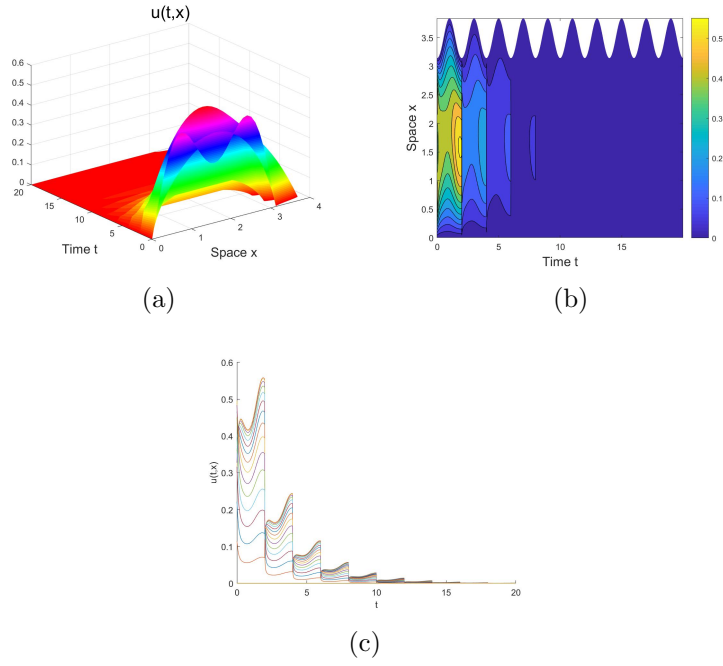


Figure 6:  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$  and Beverton-Holt function with  $m = 4, a = 10$  are given, such that  $\mathcal{R}_0 \approx 0.934 < 1$ . Graphs (a)-(c) portray that population density  $u(t, x)$  decays ultimately to zero.

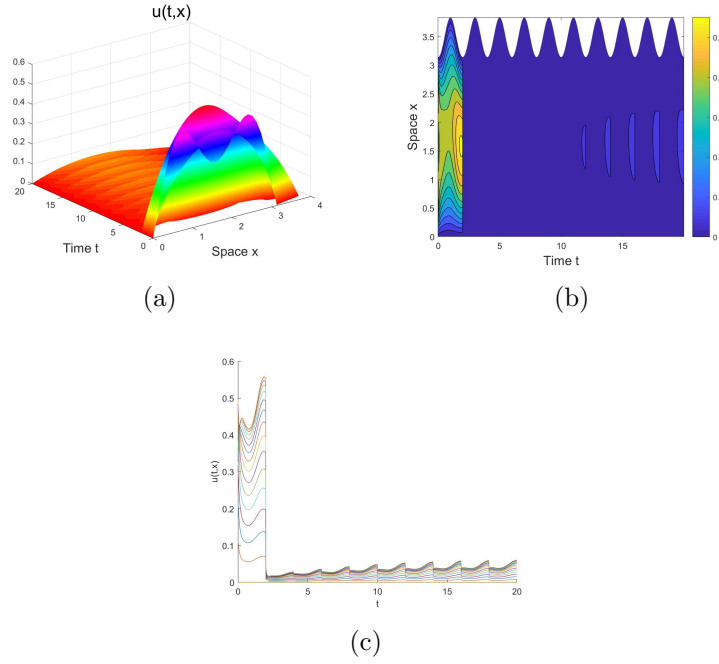


Figure 7:  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$ , Ricker function with parameter values  $r = 0.05$  and  $b = 8$  are given such that  $\mathcal{R}_0 \approx 1.496 > 1$ . Graphs (a)-(c) characterize that the population stabilizes to a smaller positive steady state.

In comparison with Figs 5 for no impulse and 6 for monotonic impulse, we conclude that population is still alive under the large evolution rate, and finally turns into extinction when impulse occurs in the form of Beverton-Holt function.

In contrast to Figs 5 for no impulse and 7 for non-monotonic impulse, we observe that population continues to persistently exist under the large evolution rate. When impulse takes place in the form of Ricker function, the population gradually goes down, and finally struggles to survive.

To sum up, the influence of Beverton-Holt and Ricker impulses on survival of species are showed below: under a small evolution rate, the rate of population extinction will be accelerate with impulse, and at large evolution rate, the addition of impulses make population struggle to survive, or even totally extinct. In other words, Beverton-Holt and Ricker impulses both have adverse effects on population dynamics.

### 5.3 The case of blowup

Compared with classical logistic model, the solution of generalized logistic model may blow up. Therefore, we choose linear impulse function  $g(u) = cu (c > 0)$  to research the impacts of impulses on population dynamics. When  $0 < c < 1$ , impulsive harvesting will take place, while  $c > 1$ , birth pulse may occurs.

We make a table 2 to summarize the sorts the following numerical simulation may cause and focus on the situation of blowup.

Table 2. Situations caused by evolution rate, initial value, and impulse

		With impulse( $c > 0, \neq 1$ )
Big evolution rate	Big initial value	Figure 8
	Small initial value	Figure 9
Small evolution rate	Samll initial value	Figure 10

Now we use numerical approximation to show specific graphs and choose a period  $T = 0.02$ . Under a big evolution rate  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$ , numerical simulations about population dynamics for big initial value  $v_0(y) = 14 \sin x + 10 \sin 3x$  are firstly given in Figure 8.

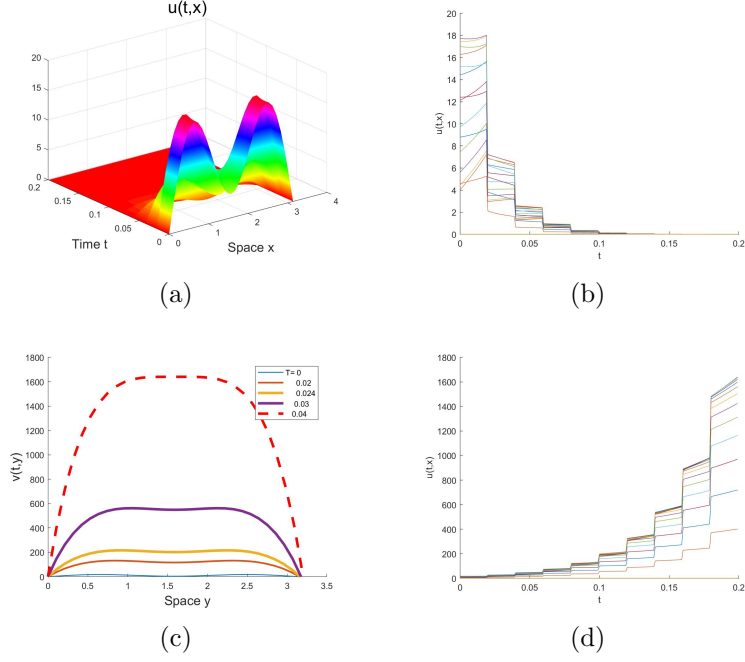


Figure 8:  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$  and big initial value  $v_0(y) = 14 \sin x + 10 \sin 3x$ . Graphs (a) and (b) with  $c = 0.4(< 1)$  characterizes the solution tends to zero, while  $c = 1.5(> 1)$  in Graphs (c) and (d) means  $u(t, x)$  still blow up.

Secondly, under a big evolution rate  $\rho_2(t) = e^{0.1(1-\cos \pi t)}$ , a numerical simulation about population dynamics for small initial value  $v_0(y) = 0.5 \sin x + 0.2 \sin 3x$  is presented in Figure 9.

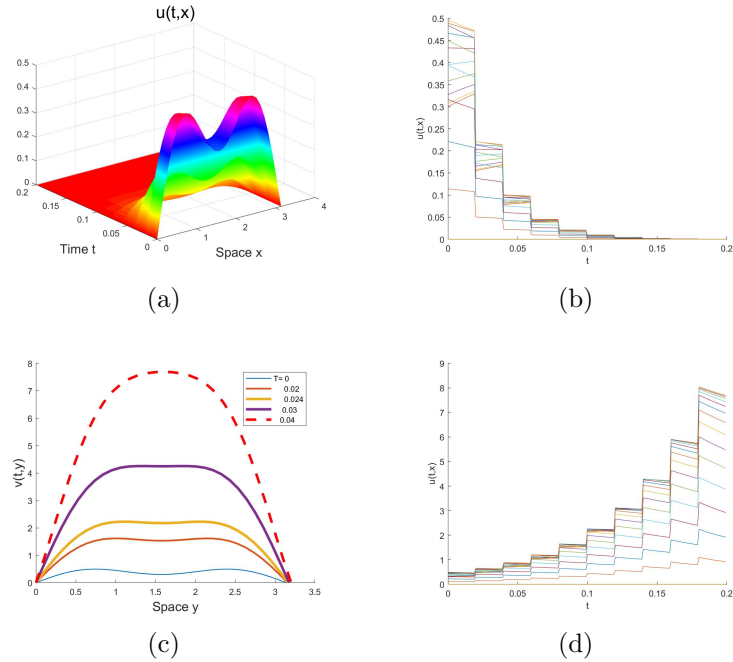


Figure 9: Graphs (a) and (b) means the population will be extinct with  $c = 0.47(< 1)$ , while Graphs (c)-(d) indicate the solution will blow up with  $c = 1.4(> 1)$ .

Finally, under a small evolution rate  $\rho_1(t) = e^{-0.1(1-\cos \pi t)}$ , we choose small initial value  $v_0(y) = 12 \sin x + 3 \sin 3x$  to research the impacts of impulses on population dynamics.

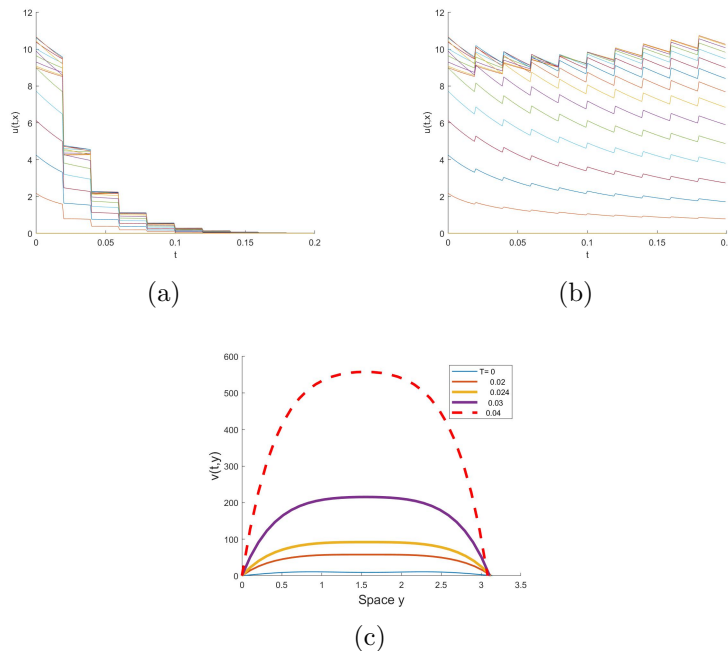


Figure 10: Graph (a) means  $u(t, x)$  decays to zero with  $c = 0.5 (c < 1)$ , Graph (b) characterizes the population stabilizes to a steady state with  $c = 1.07 (> 1)$ , and Graph (c) indicates that the solution will blow up when  $c = 1.4 (> 1)$ .

In combination with Figures 8-10, we eventually show the impacts of impulse on blowup and population dynamics in Remark 5.1 below.

**Remark 5.1** For linear impulse function  $g(u) = cu (c > 0, \neq 1)$ , impulsive harvesting with  $0 < c < 1$  plays a negative effect on population dynamics, while birth pulse with  $c > 1$  takes a positive effect on species. Moreover,  $u(t, x)$  either blows up, or it skips over the point of blowup and goes to zero.

Most previous works on population dynamics have been set up on a fixed domain, here we draw a conclusion that both regional evolution and impulses have influence on species. Especially, persistence, extinction and blowup all are possible when impulse and regional evolution are introduced in generalized logistic model.

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